Midterm 1 **Name:** ______________________________
MATH2250, Section 04 September 6, 2019

This test is:

- closed-book
- closed-notes
- no-calculator
- 50 minutes

Indicate your answers clearly, and show your work. Partial credit will be awarded based on work shown. Full credit will not be awarded without some work shown.

For questions with multiple parts, **clearly indicate** your solution to each portion of the question.

Fun fact of life: if your work is not legible, I will not be able to read it. The ramifications of this outcome should be clear.

There are 4 questions with multiple parts; each question is worth a total of 10 points.

All pages are one-sided. If on any problem you require more space, use the back of the page.

DO NOT TURN THIS PAGE UNTIL DIRECTED TO BEGIN

1. (10 pts total) Consider the differential equation

$$y' = y + \sqrt{\exp(2x) - y^2}$$

a) Show that $y(x) = e^x \sin(x + C)$, for an arbitrary constant C, is a solution to this differential equation.
b) Compute the particular solution that satisfies both the differential equation and the initial condition $y(0) = 1$.

2. (10 pts total) Solve the following initial value problems for $y(x)$:

a)

$$y' = y^2 \sin x, \qquad\qquad y(0) = 2$$

b)

$$y' + 4xy = x, \qquad\qquad y(0) = 1$$

3. (10 pts total) Consider the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = x^2 + (k-1)x - k$$

where k is a real-valued parameter.

a) Find all critical points x^* as function(s) of k. Specify for which values of k each critical point exists.
b) Determine the stability of each critical point from the previous part as a function of k.
c) Determine all bifurcation points k^* where existence and/or stability of any critical points changes.
d) Sketch a bifurcation diagram corresponding to your findings from the previous parts.

4. (10 pts total) A species has a population $x(t)$. The time evolution of this population obeys a logistic equation of the form,

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \pm\left(x^2 + bx\right)$$

where b is an unknown constant and $\pm$ indicates that the multiplicative sign (either $+$ or $-$) of the right-hand side is unknown.

a) It is known that the species population has an unstable equilibrium at $x = 1$. Use this to compute the value of the constant b and whether $\pm$ should be $+$ or $-$.

b) Using the DE determined from part a), identify the other equilibrium point and classify its stability.

c) Compute the general solution to the differential equation from part a).

d) Compute the particular solution to the initial value problem with initial data $x(0) = \frac{1}{2}$ using the general solution from c).