Math 2250 Lab 6

17 Oct 2019

Name: ____________________

uNID: ____________________

Instructions and Due Date:

- **Due:** 24 Oct 2019 at the start of the lab
- Work together!
- My current office hours: Mon 9:30am-10:30am WEB 1705 Tues 9am-10am LCB basement Math Center, 10am-11am WEB 1705,
- Additional help for the lab: Lab TA's will be in WEB 1705 M, T, W, Th from 8:30am-1pm. If you go there outside of my office hours, you can ask another section's TA for help. Otherwise, you can ask for help from tutors in the Math Center in the LCB basement.

1. (a) Find a basis for the vector space V spanned by the vectors,

$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} \ \mathbf{v}_2 = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} \ \mathbf{v}_3 = \begin{bmatrix} 4 \\ 6 \\ 2 \end{bmatrix} \ \mathbf{v}_4 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

What is the dimension of this space?

(b) Find a basis for the vector space spanned by S where

$$S = \left\{ \begin{bmatrix} 2 \\ 6 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ -6 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ 18 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix} \right\}.$$

What is the dimension of this vector space?

2. If f, g are two functions, then $af, f+g$ are also functions. That means functions can be vectors in a linear space. We call this linear space the function space.

   (a) Are the functions $f_1(x) = 1,\ f_2(x) = \cos(2x)$ and $f_3(x) = \sin^2(x)$ linearly independent? Why or why not?

   (b) How about the functions $f_1(x) = 1,\ f_2(x) = \sin(x)$ and $f_3(x) = \sin^2(x)$? Why or why not? (Hint: Plug in 3 different values of x to get a linear system for the coefficients in the definition of linear independence.)

(c) Verify that for every $n \geq 1$, the functions $f_1(x) = 1, f_2(x) = x, f_3(x) = x^2 .... f_n(x) = x^{n-1}$ are linearly independent.

3. Let $\{u, v\}$ be a basis for a two-dimensional vector space.
   (a) Establish that $p = u + v$ and $q = u - v$ is also a basis; hint: "establishing" means you show that the pair of vectors have all the properties that define them as a basis.

   (b) Establish that ap and bq also form a basis provided the scalars $a, b \neq 0$.

(c) Let $x'' + x = 0$. Verify that $u(t) = e^{it}$ and $v(t) = e^{-it}$ are linearly independent solutions.

(d) Use Euler's formula ($e^{i\theta} = \cos(\theta) + i\sin(\theta)$) and parts (a) and (b), with $a = 1/2$ and $b = 1/(2i)$ to find a new basis $p(t)$ and $q(t)$ for the DE in (c) that is real-valued and simpler to use.