Math 2250 Lab 5 $\,$

26 Sep 2019

Instructions and Due Date:

- Due: 3 Oct 2019 at the start of the lab
- Please staple all plots to the end of this packet.
- Work together!
- My current office hours: Mon 9:30am-10:30am WEB 1705 Tues 9am-10am LCB basement Math Center, 10am-11am WEB 1705,
- Additional help for the lab: Lab TA's will be in WEB 1705 M, T, W, Th from 8:30am-1pm. If you go there outside of my office hours, you can ask another section's TA for help. Otherwise, you can ask for help from tutors in the Math Center in the LCB basement.

1. Traffic flow

The following image represents an area of Sweet River City. The streets are all one-way, with the arrows indicating the direction of traffic flow. Each number represent the flow of traffic, measured in vehicles per hour.

Law of traffic: At each intersection the incoming traffic has to match the outgoing traffic.



(a) Write the equations describing the amount of traffic at each intersection.

(b) Write the corresponding augmented matrix and find its row reduced echelon form.

(c) The Sweet River Outdoor Association is organizing the traditional Water Balloon Fight at Big Park and they require that one of the surrounding street segments of the park be closed to cars. You are in charge of the Sweet River City Traffic Control Center and you want that the traffic flow remains unaltered during the event. The Outdoor Association suggest to close the segment of 8th East, i.e. to close down x_2 . Is it possible to close it and mantain a normal traffic flow? (Hint: Assume that if all of the new traffic flows remain positive, it is possible to maintain normal traffic flow. And assume that if any of the new traffic flows are negative, it is not possible to maintain normal traffic flow.)

(d) What street segment would you close? To decide this see what happens with the traffic flow when you close the different street segments, list all possible solutions, and pick one solution that you think is better. Explain your reasons, and use the same assumption as in the previous part.

2. Mixture Model

Consider the system of tanks shown below. Into tank 1 is a steady flow(means the flow rate does not change) of water with 20g of compound X per liter. Into tank 3 is a steady flow of distilled water.



(a) Use the Mixture model to write a system of differential equations modeling the amount of compound X in each tank at a given time.

(b) Then assume that the system reaches an equilibrium. (Hint: At equilibrium, the amount of compound X in each tank is not changing, so set set $x'_1 = x'_2 = x'_3 = x'_4 = 0$.) How many grams of compound X is in each tank at equilibrium? (That is, what are x_1, x_2, x_3 , and x_4 at equilibrium?)

3. Roller Coaster

Morgan just got a new job in a roller coaster design company, and her first task is to design a small part of the new roller coaster in the Sweet River City theme park. She wants to model the track with a polynomial equation of degree three, with the following requirements:

- The roller coaster starts at the level x = 0.
- The height is 400 ft (.4 thousands of feet) above ground level 1000 ft from the starting position.
- The height is 200ft (.2 thousands of feet) above ground level 2000ft from the starting position.
- The roller coaster ends 3000 ft from the starting position at a platform 60 ft (.06 thousands of feet) above the ground.
- (a) Write the general form of the function that describes the height of the roller coaster in thousands of feet.

(b) Write down the system of linear equations that represents all the requirements above. Show your work.

(c) Write the corresponding augmented matrix and using Gaussian elimination, reduce the matrix into its echelon form. Label each elementary row operation.

(d) Finally, solve the system of linear equations from (a) by means of back substitution, and write the equation that describes the roller coaster height in thousands of feet.