Math 2250 Lab 4

19 Sep 2019

Name: ____________________

uNID: ____________________

Instructions and Due Date:

- **Due:** 26 Sep 2019 at the start of the lab
- Please staple all plots to the end of this packet.
- Work together!
- My *LATEST* office hours: Mon 9:30am-10:30am WEB 1705 Tues 9am-10am LCB basement Math Center, 10am-11am WEB 1705,
- Additional help for the lab: Lab TA's will be in WEB 1705 M, T, W, Th from 8:30am-1pm. If you go there outside of my office hours, you can ask another section's TA for help. Otherwise, you can ask for help from tutors in the Math Center in the LCB basement.

1. **Numerical Methods and Linear Air Resistance**

   Consider the following IVP:

$$v'(t) = 9.8 - 0.1v \qquad v(0) = 0.$$

   Use the Euler, Heun (Improved Euler) and Runge–Kutta methods to numerically approximate the solution function on the time interval $0 \le t \le 1$.

   Use the NumericalApprox.ipynb to complete these computations.

   The exact particular solution is:

$$v(t) = 98(1 - e^{-t/10}) \text{ and thus}$$
$$v(1) = 98(1 - e^{-1/10}) \approx 9.32593.$$

   (a) Euler: $h = 0.5$, $n = 4$

   (b) Heun (Improved Euler): $h = 0.5$, $n = 4$

   (c) Runge–Kutta: $h = .5$, $n = 4$

   (d) Compute the relative errors in each case by using the following formula

$$\left| \frac{v_{\text{approx}} - v_{\text{exact}}}{v_{\text{exact}}} \right|$$

   (e) Plot the results of these methods on one graph so that they can be compared. Add the error into the table below:

Method	Relative Error
Euler	
Heun	
Runge–Kutta	

Method	(t_0, v_0)	(t_1, v_1)	(t_2, v_2)	Relative Error
Euler	(0,0)			
Improved Euler	(0,0)			
Runge–Kutta	(0,0)		N/A	

(a) Euler ($k = 10 - 0.2 \cdot v_n$)

n	t_n	v_n	k	$v_n + k \cdot h$
0	0	0		
1	0.5			
2	1.0			

(b) Heun (Improved Euler)

n	t_n	v_n	k_1	k_2	$v_n + \frac{1}{2}(k_1 + k_2) \cdot h$
0	0	0			
1	0.5				
2	1.0				

(c) Relative Errors

Method	Relative Error
Euler	
Heun	

2. **Numerical Approximation Sometimes Fails**

When we use numerical approximation techniques we are mainly concerned with how accurate a given numerical algorithm is, i.e. how small the error is for a given step size h. In some cases though, a numerical method might result in a solution that is completely wrong. To see this, consider the IVP:

$$y' = -8y, \qquad y(0) = 1,$$

where y is a function of time, t, with domain $0 \le t \le 2$.

Use the Euler and Runge-Kutta methods to create one plot for each part below. Every plot should have the exact solution curve: `y(t) = exp(-8*t)` plotted in blue. In Python it will be `y(t) = np.exp(-8*t)`.

(a) Explain why Euler's method fails so badly when $h = 0.5$.

Method	t_0	y_0	h	n	color
`euler`	0.0	1.0	0.5	4	red

(b) Explain why Euler's method fails when $h = 0.25$, but suddenly starts working fairly well when $h = 0.1$.

Method	t_0	y_0	h	n	color
`euler`	0.0	1.0	0.25	8	orange
`euler`	0.0	1.0	0.1	20	green

(c) Explain why Runge–Kutta fails so badly when $h = 0.5$.

Method	t_0	y_0	h	n	color
`runge_kutta`	0.0	1.0	0.5	4	red

(d) If you decrease h a little, Runge–Kutta starts working correctly again. No explanation needed here, just the plot.

integrator	t_0	y_0	h	n	color
`runge_kutta`	0.0	1.0	0.25	8	orange
`runge_kutta`	0.0	1.0	0.1	20	green

Print your plots and staple them to this lab.

3. If we drop a package from a helicopter with a parachute attached, the wind resistance provided by the parachute is not really linearly dependent on the velocity. Assume that for a certain type of parachute the velocity initial value problem is modeled by:

$$v'(t) = 9 - 0.2v - 0.27v^{1.5}$$
$$v(0) = 1$$

(a) What is the terminal velocity in this model?

(b) This is a differential equation which does NOT have an elementary solution. Use Euler, improved Euler, and Runge-Kutta, and with time steps 0.5 and 0.05. Plot your results and comment about apparent accuracy of the three methods with these different choices of time step size.

(c) Based on your work above, approximately what percent of the terminal velocity is attained at $t = 1$ and $t = 4$ seconds?