Name:		
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29 Aug 2018

Instructions and due date:

- **Due:** 5 Sep 2019 at the start of class.
- If extra paper is necessary, please staple it at the end of the packet.
- For full credit, show all of your work.
- Work together! However, your work should be your own (not copied from a group member).
- My email: @math.utah.edu. If you have any questions, please visit me during office hours or the tutors in the math tutoring center.

1. Match each differential equation with the corresponding slope field. Then for each slope field sketch the solution to the differential equation that passes through the initial condition. Justify each answer by verifying specific points.

Differential Equations with initial condition:

1.
$$\frac{dy}{dx} = 1 - yx; \ y(1) = 1$$

2. $\frac{dy}{dx} = \frac{x}{1 - y}; \ y(0) = -3$
3. $\frac{dy}{dx} = x^2 + y^2 - 1; \ y(1) = 0$
4. $\frac{dy}{dx} = y - \sin(x); \ y(0) = 1$

Slope fields:



- 2. An initial value problem can have zero, finitely many, or infinitely many solutions, as explored in this problem.
 - (a) Consider the initial value problem

$$y' = \frac{1}{x}$$
 $y(0) = 0.$

How many solutions are there? Justify your answer.

(b) Consider the initial value problem

$$y' = 2\sqrt{y} \qquad y(0) = 0.$$

How many solutions are there? Justify your answer by sketching solution curves.

(c) Consider again the differential equation from part (b):

$$y' = 2\sqrt{y}.$$

Verify that if c is a constant, then the following piecewise function satisfies the differential equation for all x:

$$y(x) = \begin{cases} 0 & x \le c \\ (x-c)^2 & x > c. \end{cases}$$

(d) For what values of b does the initial value problem $y' = 2\sqrt{y}, y(0) = b$ have no solution? For what values does it have a unique solution that is defined for all x? For what values does it have infinitely many solutions? Justify your answers.

3. Assume that when a rain drop begins to fall, both its radius and velocity are zero. Then as the rain drop falls, its radius increases linearly with respect to time. That is, r = kt where k is constant. The mass of the rain drop is

$$m = \frac{4}{3}\pi r^3$$

where the radius r is the function of t as assumed above.

- (a) Write the formula for m(t), the mass of the raindrop as a function of time.
- (b) Newton's second law of motion states

$$\frac{dp}{dt} = \sum f$$

where p = mv is the momentum and the sum of the forces equals mg. Therefore, we have the following initial value problem:

$$\frac{dp}{dt} = mg \qquad v(0) = 0$$

Using the product rule, show that this initial value problem is equivalent to:

$$tv' + 3v = tg$$
 $v(0) = 0.$ (1)

(c) Using the integrating factor method, solve the initial value problem (1) for v(t).

(d) Verify that the acceleration of the raindrop is $\frac{1}{4}$ the acceleration of due to gravity.

4. Mixture Problem: Consider the *cascade* of two tanks shown in the following figure. Pure water flows into the first tank. The contents of the first tank flow into the second, and the contents of the third tank flow out. All three of these flow rates are 10 gal/min. $V_1 = 150$ gal is the fixed volume of the first tank, and $V_2 = 200$ gal is the fixed volume of the second tank. Each tank initially contains 25 lb of salt, and the rest is filled with water. Therefore, each tank contains a brine solution, with a greater initial concentration of salt in the first tank than in the second. Denote x(t) as the amount of salt in the first tank at time t, and y(t) as the amount of salt in the second tank at time t.



(a) Find x(t). Hint: Start by finding $\frac{dx}{dt}$.

(b) Find y(t). Hint: Show first that

$$\frac{dy}{dt} = \frac{x}{15} - \frac{y}{20}$$

and then solve for y(t), using the function x(t) found in part (a).

(c) The water in the second tank is used for brine-curing leather. In order to be prepared properly, the leather must be placed in the tank when the concentration of salt is at its maximum. Find the time at which this maximal concentration occurs. (Use a calculator to round your answer to the nearest hundredth of a minute.)