Math 2250 Lab 10

November 21, 2019

Name: ____________________

uNID: ____________________

Due Date and Office Hours:

- Due December 5, 2019 at the start of the lab
- I have REDUCED office hours for the next two weeks. They are:

  Tuesday November 26: 9am-11am LCB basement Math Center,

  Wednesday December 4: 9:30am - 10:30am LCB basement Math Center,

  or by appointment.
- Additional help for the lab:

  Lab TA's will be in WEB 1705 M, T, W, Th from 8:30am-1pm. If you go there outside of my office hours, you can ask another section's TA for help. Otherwise, you can ask for help from tutors in the Math Center in the LCB basement.
- Happy Thanksgiving!

1. **Eigenvalues/Eigenvectors**

   (a) Consider the matrix

$$A = \begin{bmatrix} -2 & -3 & 0 \\ 0 & 1 & 0 \\ -3 & -3 & 1 \end{bmatrix}.$$

   i. Find the Eigenvalues of A, and the Eigenvectors associated to them.

   ii. If possible, diagonalize A. That is, find a matrix P such that $P^{-1}AP = D$, where D is a diagonal matrix.

(b) Consider the matrix

$$B = \begin{bmatrix} 4 & -2 & -1 \\ 2 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}.$$

i. Find the Eigenvalues of B, and the Eigenvectors associated to them.

ii. If possible, diagonalize B.

2. **Lake Pollution**

A river runs through three lakes, Lake Dustin, Lake Hayley and Lake Tristan. The flow at all points of the river is a constant 4 km^3 per unit of time, and the water entering Lake Dustin is pure. The volumes of the lakes are 24, 60 and 48 km^3 respectively.

(a) At time $t = 0$, some pollutant is discovered in the lakes, x kilograms in Lake Dustin, y kilograms in Lake Hayley and z kilograms in Lake Tristan. Write down the system of differential equations satisfied by $x(t)$, $y(t)$ and $z(t)$, being careful to state what each term of your equations represents and any assumptions you have made.

(b) Show that your equation in matrix form looks like

$$\mathbf{x}' = \begin{bmatrix} -a & 0 & 0 \\ a & -b & 0 \\ 0 & b & -c \end{bmatrix} \mathbf{x} \quad \text{where } \mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

(c) Find the general solution of the above equation.

(d) If the initial values are $x(0) = 100$, $y(0) = z(0) = 0$ (i.e. initially there is no pollutant in Lake Hayley and Lake Tristan), find the arbitrary constants in the general solution obtained in part (c).

(e) At what time is the pollutant level in Lake Hayley at its maximum value?