Math 2250Lab 1

Name: ______________________

22 Aug 2019

uNID: ______________________

Instructions and due date:

- **Due:** 29 Aug 2019 at the start of class.
- If extra paper is necessary, please staple it at the end of the packet.
- For full credit: Show all of your work, and simplify your final answers.
- Work together! However, your work should be your own (not copied from a group member).
- My email: jiao@math.utah.edu.
  If you have any questions, please email me or visit me during office hours.

1. Solve the following directly integrable DEs of the form

$$\frac{dx}{dt} = f(t).$$

If there is no initial condition, find the general solution. If there is an an initial condition, solve the IVP and find a particular solution.

(a) $f(t) = t^n,\ n \neq -1,\ x(1) = 1.$

(b) $f(t) = \frac{1}{t}$, where $t > 0$.

(c) $f(t) = e^{-t}$, $x(0) = 1$.

(d) $f(t) = a\sin(bt) + b\cos(at)$, where a, b are two real numbers.

(e) $f(t) = \dfrac{1}{(t+1)(t+2)}$, $x(0) = 0$.

(f) $f(t) = \dfrac{1}{t(t+a)+b}$, $a^2 > 4b$, $t > 0$. (Hint: express $t(t+a)+b$ as $(t+x_1)(t+x_2)$ fisrt)

2. Verify that for any **fixed numbers** C_1, C_2,

$$y(x) = C_1 \int_x^\infty \frac{e^{-u}}{u} du + C_2$$

is a solution to the differential equation

$$x\frac{d^2y}{dx^2} + (x+1)\frac{dy}{dx} = 0.$$

Hint: the integral identities $\int_a^b f(u)du = -\int_b^a f(u)du$ *and* $\frac{d}{dx}\int_a^x f(u)du = f(x)$ *may be useful.*

The exponential integral and its ODE are important in modeling water flow to a pumping well through a porous ground medium.

3. Daniel is working in a restaurant. He has just boiled a large pot of soup. However, in order to serve customers, he need to cool the soup down.

   The soup had just boiled at 100 $^\circ C$, and the recommended temperature to serve the soup is $25^\circ C$. Daniel wants to cool the soup down by using a refrigerator in the restaurant, which holds the temperature constant at $0^\circ C$.

   Daniel puts the pot of soup into the refrigerator at time $t = 0$.

   (a) Let $T(t)$ be the temperature of the pot at any time t, measured in minutes. Use Newton's Law of cooling to write down a differential equation for $T(t)$.

   Hint: Newton's Law of cooling states that the rate of change of the temperature of an object is proportional to the difference between the object's temperature and the surrounding temperature. You do ***not*** *need to write down the constant of proportionality in this step.*

   (b) After 10 minutes, Daniel opened the door of the refrigerator and find that the soup is now of 50 $^\circ C$.

   Use this information to find the constant of proportionality in your differential equation in part (a).

   (c) How long Daniel still need to wait before he can serve the soup to the customers?

4. Consider this second order linear differential equation:

$$x^2 \frac{d^2y}{dx^2} - 5x\frac{dy}{dx} + 8y = 0$$

(a) Find numbers m such that $y_1(x) = x^m$ solve the DE. Show that there are exactly two such m, and we call them m_1, m_2

(b) Show that $y(x) = c_1 x^{m_1} + c_2 x^{m_2}$ also solves the DE, where c_1, c_2 are arbitrary constants.

(c) Can you find c_1, c_2 so that y(x) solves the following initial value problem?

$$y(1) = 0 \quad y'(1) = 2$$

(d) Can you find c_1, c_2 so that y(x) solves the following initial value problem?

$$y(0) = 1 \quad y(1) = -1$$