Midterm 2
MATH3150, Section 04

Name: ____________________

March 29, 2017

This test is:

- closed-book
- closed-notes
- no-calculator
- 80 minutes

Indicate your answers clearly, and show your work. Partial credit will be awarded based on work shown. Full credit will not be awarded without some work shown.

Fun fact of life: if your work is not legible, I will not be able to read it. The ramifications of this outcome should be clear.

There are 3 questions with multiple parts; questions 1 and 3 are worth 20 points, and question 2 is worth 10 points. Thus the entire exam is scored out of 50 points.

All pages are one-sided. If on any problem you require more space, use the back of the page.

Do not turn this page until directed to begin

1. (20 pts total) This question involves sketching Fourier Series and computing coefficients on the interval $x \in [-L, L]$.

Your sketches <u>**must**</u> include (i) illustration of behavior *outside* the interval $[-L, L]$ (along with behavior inside this interval), (ii) explicit markings showing jump discontinuities and the value of the series at those discontinuous locations.

a.) (5 pts) Sketch the Fourier Series for the function

$$f(x) = -x$$

b.) (5 pts) Sketch the Fourier Series for the function

$$f(x) = \begin{cases} -1, & x \geq 0 \\ 1, & x < 0 \end{cases}$$

c.) (10 pts) Compute the Fourier Series coefficients for the function

$$f(x) = \begin{cases} -1, & x \geq 0 \\ 1, & x < 0 \end{cases}$$

2. (10 pts) Consider the function

$$f(x) = \begin{cases} 1 - x, & x \geq 0 \\ -1, & x < 0 \end{cases}$$

Sketch (a) the Fourier Series on $[-L, L]$, (b) the Fourier Cosine series on $[0, L]$, and (c) the Fourier Sine series on $[0, L]$. Your sketches must adhere to the requirements outlined in the previous question.

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3. (20 pts) Compute the solution $u(x,t)$ to the following one-dimensional wave equation:

$$u_{tt} = u_{xx}, \qquad 0 < x < 1, \quad t > 0$$

subject to the initial conditions

$$\begin{aligned} u(x,0) &= x^2, \\ \frac{\partial u}{\partial t}(x,0) &= \sin(\pi x) \end{aligned}$$

along with the boundary conditions

$$\begin{aligned} \frac{\partial u}{\partial x}(0,t) &= 0, \\ u(1,t) &= 1 \end{aligned}$$

Show all work. Your solution must be written down in terms of explicit, computable expressions or integrals, but you need not compute the values of these integrals.

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