

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF UTAH  
**PDEs for Engineering Students**  
**MTH3150 – Section 004 – Spring 2017**

**Final exam formula sheet**

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**Note:** The final exam is cumulative!

The following are some standard thermal properties of materials. Units follow in [brackets].

- $u(x, t)$  — Temperature as a function of space and time [temperature]
- $e(x, t)$  — Thermal energy density [energy/volume]
- $\phi(x, t)$  — Thermal heat flux [energy/(time  $\times$  area)]
- $\rho(x)$  — Mass density [mass/volume]
- $c(x)$  — Specific heat [energy/(mass  $\times$  temperature)]
- $K_0$  — Thermal conductivity [energy/(time  $\times$  temperature  $\times$  length)]

You may find the following integrals helpful. In all the following,  $n$  and  $m$  are non-negative integers, and  $L$  is any positive number.

$$\int_0^L \sin\left(\frac{n\pi x}{L}\right) \sin\left(\frac{m\pi x}{L}\right) dx = \begin{cases} 0, & n \neq m \\ L/2, & n = m \end{cases}$$

$$\int_0^L \cos\left(\frac{n\pi x}{L}\right) \cos\left(\frac{m\pi x}{L}\right) dx = \begin{cases} 0, & n \neq m \\ L/2, & n = m \neq 0 \\ L, & n = m = 0 \end{cases}$$

$$\int_0^L \cos\left(\frac{n\pi x}{L}\right) \sin\left(\frac{m\pi x}{L}\right) dx = 0 \quad (n > 0)$$

$$\int_0^L \sin\left(\frac{(2n+1)\pi x}{2L}\right) \sin\left(\frac{(2m+1)\pi x}{2L}\right) dx = \begin{cases} 0, & n \neq m \\ L/2, & n = m \end{cases}$$

$$\int_0^L \cos\left(\frac{(2n+1)\pi x}{2L}\right) \cos\left(\frac{(2m+1)\pi x}{2L}\right) dx = \begin{cases} 0, & n \neq m \\ L/2, & n = m \end{cases}$$

A Fourier Series on the interval  $[-L, L]$  takes the form

$$\sum_{n=0}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right).$$

The coefficients of the Fourier Series associated to a function  $f(x)$  are

$$\begin{aligned} a_0 &= \frac{1}{2L} \int_{-L}^L f(x) \, dx \\ a_n &= \frac{1}{L} \int_{-L}^L f(x) \cos\left(\frac{n\pi x}{L}\right) \, dx, & n \geq 1 \\ b_n &= \frac{1}{L} \int_{-L}^L f(x) \sin\left(\frac{n\pi x}{L}\right) \, dx, & n \geq 1 \end{aligned}$$

Let  $f(x)$  and  $F(\omega)$  be functions defined over the real number line. If  $f$  and  $F$  are Fourier transform pairs, then

$$\begin{aligned} F(\omega) &= \mathcal{F}[f] = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(x) e^{i\omega x} \, dx \\ f(x) &= \mathcal{F}^{-1}[F] = \int_{-\infty}^{\infty} F(\omega) e^{-i\omega x} \, d\omega, \end{aligned}$$

where  $x$  is the physical variable and  $\omega$  is the frequency variable. If  $f$  and  $F$  are Fourier Transform pairs, and  $h$  and  $H$  are Fourier Transform pairs, then the following table describes the relationship between  $g$  and its Fourier Transform  $G$ :

$g(x)$	$G(\omega)$
$e^{-ax^2}$	$\frac{1}{\sqrt{4\pi a}} e^{-\omega^2/4a}$
$\sqrt{\frac{\pi}{a}} e^{-x^2/4a}$	$e^{-a\omega^2}$
$\delta(x)$	$\frac{1}{2\pi}$
$f(x-a)$	$e^{ia\omega} F(\omega)$
$e^{-iax} f(x)$	$F(\omega-a)$
$\frac{df}{dx}$	$-i\omega F(\omega)$
$ixf(x)$	$\frac{dF}{d\omega}$
$(f * h)(x)$	$F(\omega)H(\omega)$
$\frac{1}{2\pi} f(x)h(x)$	$(F * H)(\omega)$

Above, the convolution  $*$  between two functions  $f$  and  $h$  is defined as

$$(f * h)(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(s)h(x-s) \, ds.$$