Midterm 2 **Name:** _____

MATH1220, Section 06 November 17, 2016

This test is:

- closed-book
- closed-notes
- no-calculator
- 120 minutes

Indicate your answers clearly, and show your work. Partial credit will be awarded based on work shown. Full credit will not be awarded without some work shown.

Fun fact of life: if your work is not legible, I will not be able to read it. The ramifications of such a situation should be clear.

There are 4 questions with multiple parts; each question is worth a total of 25 points.

All pages are one-sided. If on any problem you require more space, use the back of the page.

DO NOT TURN THIS PAGE UNTIL DIRECTED TO BEGIN

1. (25 pts total) Compute the following definite integals, or show that they diverge by any method.

a.) (8 pts)

$$\int_0^\infty e^{-3x}\,\mathrm{d}x$$

b.) (8 pts)

$$\int_2^4 \frac{x}{\sqrt{16-x^2}}\,\mathrm{d}x$$

c.) (9 pts)

$$\int_0^1 \frac{1}{x^2 - x^3}\,\mathrm{d}x$$

2. (25 pts total) Evalute the following limits, or show that they diverge.

a.) (6 pts)

$$\lim_{x\to\infty} \frac{x - x^3}{3x^3 + 3}$$

b.) (6 pts)

$$\lim_{x\to 0} \frac{\tan x}{x}$$

c.) (6 pts)

$$\lim_{x\to\infty} x\left(2\tan^{-1} x - \pi\right)$$

d.) (7 pts)

$$\lim_{x\to 0^+} x^{(\sin x)}$$

3. (25 pts total) The following question concerns sequences and series. *In each case you must justify your answer.*

a.) (6 pts) Determine whether or not the sequence $\{a_n\}_{n=1}^{\infty}$ converges, and if so, find $\lim_{n\to\infty} a_n$, with

$$a_n = \frac{\sin n}{n}$$

b.) (6 pts) Determine whether or not the following series converges:

$$\sum_{n=0}^{\infty} \left(\frac{\pi}{4}\right)^n$$

c.) (6 pts) Determine whether or not the following series converges:

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$$

d.) (7 pts) Determine whether or not the following series converges:

$$\sum_{n=1}^{\infty} \frac{n!}{(2n)^n}$$

4. (25 pts total) This question concerns power series.

a.) (12 pts) Compute the Maclaurin series for the following function, *and* determine its set of convergence.

$$f(x) = x^2 \ln(1 + x)$$

b.) (13 pts) Compute the order-4 Taylor polynomial for the following function around $x = 1$:

$$f(x) = \frac{1}{2-x} \exp(-x)$$