This test is:

- closed-book

- closed-notes

- no-calculator

- 120 minutes

Indicate your answers clearly, and show your work. Partial credit will be awarded based on work shown. Full credit will not be awarded without some work shown.

Fun fact of life: if your work is not legible, I will not be able to read it. The ramifications of such a situation should be clear.

There are 4 questions with multiple parts; each question is worth a total of 25 points.

All pages are one-sided. If on any problem you require more space, use the back of the page.

Do not turn this page until directed to begin

1. (25 pts total) Compute the following definite or indefinite integrals by any method.

a.) (8 pts) Compute

$$\int \frac{2x}{x^2 - 4} \, \mathrm{d}x$$

b.) (8 pts) Compute

$$\int \frac{\ln(x^2)}{x} \, \mathrm{d}x$$

c.) (9 pts) Compute

$$\int_0^3 \frac{2x + 3}{x^2 + 1} \, \mathrm{d}x$$

2. (25 pts total) This problem concerns properties of exponentials and logarithms.

a.) (6 pts) Compute

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(2^{\left(x^2\right)} + 3^{\sqrt{x}}\right)$$

b.) (6 pts) Compute

$$\int_1^4 \frac{2^{\sqrt{x}}}{\sqrt{x}}\,\mathrm{d}x$$

c.) (6 pts) Compute

$$\frac{\mathrm{d}}{\mathrm{d}x} \log_2 \left(5^x\right)$$

d.) (7 pts) Compute

$$\frac{\mathrm{d}}{\mathrm{d}x} x^{\left(2^x\right)}$$

3. (25 pts total) This question concerns exponential growth models and solutions to ordinary differential equations.

a.) (10 pts) The popluation of a country grows at a rate proportional to its size, and it is observed that the population grows by 10% every 2 years. I.e., if the population is P today, then it is $1.1P$ after two years. Assume the population is 1000 now. Write down (1) an explicit expression for the population as a function of time, and (2) an expression for the population 7 years from now.

b.) (15 pts) Solve the following differential equation for $y(x)$. You must find an explicit expression for y as a function of x.

$$\frac{dy}{dx} + \frac{2}{x}y = \frac{\sin x}{x}$$

4. (25 pts total) Compute the following integrals by any method.

a.) (8 pts) Compute

$$\int \sin 3x \cos 5x \, \mathrm{d}x$$

b.) (8 pts) Compute

$$\int \frac{x-3}{x^4 - 4x^3 + 4x^2}\, dx$$

c.) (9 pts) Compute

$$\int \frac{\ln(x^2 + 1)}{x^3} \, \mathrm{d}x$$