Math 1220-006 (Calculus II), Fall 2016

Homework 9 Grading Key

Problems

9.6 # 17, 19, 21, 23, 25, 27
9.7 # 1, 3, 5, 7, 9, 13, 15, 17, 19, 21, 25, 27
9.8 # 1, 3, 5, 7, 9, 11
Total: 24

- Total score = completeness points + correctness points

<u>Completeness</u> (5 points)

- Points are awarded based on the following table:

Points	0	1	2	3	4	5
Percentage of problems attempted	<60%	60-69%	70-79%	80-89%	90-99%	100%

- In order for a problem to count as attempted, there must be some kind of work present. Simply writing down the problem doesn't count.

<u>Correctness</u> (5 points)

- Every week certain problems are selected for individual grading. Correct answers are only worth a small portion of the points. The majority of the points come from demonstrating conceptual knowledge and showing the calculations or reasoning that led to an answer.

- This week, the following problems were selected for grading:

<u>9.6 # 21</u> (1.25 points)

Find the convergence set for $1 + 2x + \frac{2^2x^2}{2!} + \frac{2^3x^3}{3!} + \dots$

- Find a formula for the n^{th} term (0.25 pt.):

$$1 + 2x + \frac{2^2x^2}{2!} + \frac{2^3x^3}{3!} + \dots = \boxed{\sum_{n=0}^{\infty} \frac{2^n x^n}{n!}}$$

- Apply Ratio Test (0.25 pt.):

$$\lim_{n\to\infty}\left|\frac{2^{n+1}x^{n+1}}{(n+1)!} \div \frac{2^n x^n}{n!}\right| = \lim_{n\to\infty}\left|\frac{2^{n+1}}{2^n}\cdot\frac{x^{n+1}}{x^n}\cdot\frac{n!}{(n+1)!}\right| = \lim_{n\to\infty}\left|2x\cdot\frac{1}{n+1}\right| = 0$$

$\therefore$ The series converges $\boxed{\text{for all } x.}$

<u>9.7 # 5</u> (1.25 points)

Find the power series representation for $f(x) = \frac{1}{2-3x} = \frac{\frac{1}{2}}{1-\frac{3}{2}x}$ and specify the radius of convergence.

- Recognize that $\frac{\frac{1}{2}}{1-\frac{3}{2}x}$ is of the form $\frac{1}{1-x}$ and use the power series for the latter (0.75 pt.):

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots \longrightarrow \frac{1}{2}\left(\frac{1}{1-\frac{3}{2}x}\right) = \frac{1}{2}\left(1 + \frac{3}{2}x + \frac{9}{4}x^2 + \frac{27}{8}x^3 + \dots\right) = \boxed{\frac{1}{2} + \frac{3}{4}x + \frac{9}{8}x^2 + \frac{27}{16}x^3 + \dots}$$

- Find the radius of convergence (0.5 pt.):

$$-1 < \frac{3}{2}x < 1 \longrightarrow -\frac{2}{3} < x < \frac{2}{3}$$ Radius of convergence is $\boxed{\frac{2}{3}}$

9.7 #21 (1.25 points)

Find a power series for $f(x) = (\tan^{-1}x)(1+x^2+x^4)$

- Find the power series for $\tan^{-1}x$ (0.75 pt.):

$$\tan^{-1}x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$$

- Multiply that series by $1+x^2+x^4$ (0.5 pt.):

$$\left(x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots\right)(1+x^2+x^4) = \boxed{x + \frac{2x^3}{3} + \frac{13x^5}{15} - \frac{29x^7}{105} + \dots}$$

9.8 # 7 (1.25 points)

Find the fifth-order Maclaurin series for $f(x) = e^x + x + \sin x$

- Find the Maclaurin series for e^x and $\sin x$ individually (0.75 pt.):

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$$

- Add those series together with x (0.5 pt.):

$$e^x + x + \sin x = x + \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots\right) + \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots\right)$$

$$= \boxed{1 + 3x + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{2x^5}{5!} + \dots}$$