Math 1220-006 (Calculus II), Fall 2016

Homework 8 Grading Key

- Total score = completeness points + correctness points

Completeness (5 points)

- Points are awarded based on the following table:

Points	0	1	2	3	4	5
Percentage of problems attempted	<60%	60-69%	70-79%	80-89%	90-99%	100%

- In order for a problem to count as attempted, there must be some kind of work present. Simply writing down the problem doesn't count.

Correctness (5 points)

- Every week certain problems are selected for individual grading. Correct answers are only worth a small portion of the points. The majority of the points come from demonstrating conceptual knowledge and showing the calculations or reasoning that led to an answer.

- This week, the following problems were selected for grading:

9.4 # 29 (1.25 points)

Determine whether $\sum_{n=1}^{\infty} \frac{n^n}{(2n)!}$ converges or diverges.

(0.25 pt.) $-1 \le \cos n \le 1$ for all n so $3 \le 4+\cos n \le 5 \rightarrow \frac{3}{n^3} \le \frac{4+\cos n}{n^3} \le \frac{5}{n^3}$ for all n.

Using the limit comparison test (1 pt.)

$\sum_{n=1}^{\infty} \frac{5}{n^3}$ converges and since $\sum_{n=1}^{\infty} \frac{4+\cos n}{n^3} < \sum_{n=1}^{\infty} \frac{5}{n^3}$, $\sum_{n=1}^{\infty} \frac{4+\cos n}{n^3}$ $\boxed{\text{converges}}$

9.5 # 9 (1.25 points)

Show that $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{2^n}$ converges absolutely.

Using the Ratio Test: (1.25 pt.)

$$\frac{|u_{n+1}|}{|u_n|} = \frac{\frac{n+1}{2^{n+1}}}{\frac{n}{2^n}} = \frac{n+1}{2n}$$

$\lim_{n\to\infty} \frac{n+1}{2n} = \frac{1}{2} < 1$, so the series converges absolutely.

9.5 #21 (1.25 points)

Find whether $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^2+1}$ is absolutely convergent, conditionally convergent, or divergent.

Using the Alternating Series test:

1) (0.5pt.) Let $a_n = \frac{n}{n^2+1}$. Then $a_{n+1} = \frac{n+1}{(n+1)^2+1}$.

Comparing a_n to a_{n+1}: $\frac{n}{n^2+1} > \frac{n+1}{(n+1)^2+1}$ $\forall n > 1$

Since $n^2+n-1 > 0$ $\forall n > 1$.

(0.5pt.)

2) $\lim_{n\to\infty} \frac{n}{n^2+1} = 0$ Hence the series converges.

Let $b_n = \frac{1}{n}$ which diverges (b_n is the Harmonic series).

(0.25 pt.) $\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n^2}{n^2+1} = 1$, $0<1<\infty$, so a_n also diverges. Thus a_n is only $\boxed{\text{conditionally Convergent}}$.

9.6 #7 (1.25 points)

Find the convergence set for $\sum_{n=1}^{\infty} (-1)^n \frac{(x-2)^n}{n}$.

Let $u = x-2$. Then we have $\sum_{n=1}^{\infty} (-1)^n \frac{u^n}{n}$.

Applying the Ratio Test: (0.5 pt.)

$\lim_{n\to\infty} \left| \frac{n x^{n+1}}{(n+1)x^n} \right| = \lim_{n\to\infty} \left| x\left(\frac{n}{n+1}\right) \right| = |x|$

$-1 < x < 1$ $\Rightarrow$ interval of convergence is $(-1,1)$

Testing the end points: (0.75pt.)

At $x=1$, $a_n = \frac{(-1)^n}{n}$ which converges (it is the Alternating Harmonic Series)

At $x=-1$, $a_n = \frac{1}{n}$ which diverges (it is the Harmonic Series)

So the series converges when $u \in (-1,1]$.

Substituting back into $u = x-2$ the final answer is that the series converges when $\boxed{x \in (1,3]}$