Math 1220-006 (Calculus II), Fall 2016

Homework 7 Grading Key

- Total score = completeness points + correctness points

Completeness (5 points)

- Points are awarded based on the following table:

Points	0	1	2	3	4	5
Percentage of problems attempted	<60%	60-69%	70-79%	80-89%	90-99%	100%

- In order for a problem to count as attempted, there must be some kind of work present. Simply writing down the problem doesn't count.

Correctness (5 points)

- Every week certain problems are selected for individual grading. Correct answers are only worth a small portion of the points. The majority of the points come from demonstrating conceptual knowledge and showing the calculations or reasoning that led to an answer.

- This week, the following problems were selected for grading:

9.1 # 9 (1.25 points)

Write first five terms of $a_n = \frac{\cos(n\pi)}{n}$, determine whether the series converges or diverges, and if it converges, find $\lim_{n\to\infty} a_n$.

- Write down first five terms (0.25 pt.): $\boxed{a_1 = -1,\ a_2 = \frac{1}{2},\ a_3 = -\frac{1}{3},\ a_4 = \frac{1}{4},\ a_5 = -\frac{1}{5}}$
- Recognize that a_n is the alternating harmonic series or use another test to conclude that a_n $\boxed{\text{converges}}$ (0.5 pt.).
- Find $\lim_{n\to\infty} a_n$ using Squeeze Theorem or another limit test (0.5 pt.):

$\cos(n\pi) = (-1)^n$, so $\frac{-1}{n} \le \frac{\cos(n\pi)}{n} \le \frac{1}{n}$. $\lim_{n\to\infty} \frac{-1}{n} = \lim_{n\to\infty} \frac{1}{n} = 0$, so $\lim_{n\to\infty} \frac{\cos(n\pi)}{n} = \boxed{0}$

9.2 # 5 (1.25 points)

Does $\sum_{k=1}^{\infty} \frac{k-5}{k+2}$ converge or diverge? If it converges, find the sum.

- Use the n^{th} term test or another relevant test (1.25 pts.):

$\lim_{k\to\infty} \frac{k-5}{k+2} = 1$, the ratio of the leading coefficients. Since $\lim_{k\to\infty} \neq 0$, the sum $\boxed{\text{diverges}}$.

9.3 # 19 (1.25 points)

Does $\sum_{k=1}^{\infty} k^2 e^{-k^3}$ converge or diverge?

- Use the integral test or another relevant test (1.25 pts.):

$$\int_1^{\infty} x^2 e^{-x^3} dx = \lim_{b \to \infty} \int_1^{b} x^2 e^{-x^3} dx = \lim_{b \to \infty} \left[-\frac{1}{3} e^{-x^3} \right]_1^b$$

$$= \lim_{b \to \infty} \left[-\frac{1}{3} e^{-b^3} + \frac{1}{3} e^{-(1)^3} \right] = 0 + \frac{1}{3e}$$

Since $\int_1^{\infty} x^2 e^{-x^3} dx$ converges, $\sum_{k=1}^{\infty} k^2 e^{-k^3}$ $\boxed{\text{converges}}$

* Note that in order to use the integral test you need to check the function (i.e. the series) is continuous, positive, and nonincreasing on the interval $[1, \infty)$. *

9.4 # 15 (1.25 points)

Does $\sum_{n=1}^{\infty} \frac{n^2}{n!}$ converge or diverge?

- Use the ratio test or another relevant test (1.25 pts.):

$$\frac{a_{n+1}}{a_n} = \frac{\left(\frac{(n+1)^2}{(n+1)!}\right)}{\left(\frac{n^2}{n!}\right)} = \frac{(n+1)^2}{(n+1)!} \cdot \frac{n!}{n^2} = \frac{(n+1)^2}{(n+1)\,n^2} = \frac{n^2+2n+1}{n^3+n^2}$$

$\lim_{n \to \infty} \frac{n^2+2n+1}{n^3+n^2} = 0$ since the ratio is bottom heavy, so by the Ratio test the series $\boxed{\text{converges}}$ since $0 < 1$.