# Math 1220-006 (Calculus II), Fall 2016

## Homework 6 Grading Key

- Total score = completeness points + correctness points

Completeness (5 points)

- Points are awarded based on the following table:

| Points | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Percentage of problems attempted | <60% | 60-69% | 70-79% | 80-89% | 90-99% | 100% |

- In order for a problem to count as attempted, there must be some kind of work present. Simply writing down the problem doesn't count.

Correctness (5 points)

- Every week certain problems are selected for individual grading. Correct answers are only worth a small portion of the points. The majority of the points come from demonstrating conceptual knowledge and showing the calculations or reasoning that led to an answer.

- This week, the following problems were selected for grading:

8.1 # 15 (1.25 points)

Use l'Hôpital's Rule to find $\lim_{x\to 0} \frac{\tan x - x}{\sin 2x - 2x}$

- Check that the limit is of the form $\frac{0}{0}$ (0.25 pt.):

$$\frac{\tan(0) - (0)}{\sin 2(0) - 2(0)} = \frac{0}{0} \checkmark$$

- Use l'Hôpital's Rule 3 times (1 pt.):

$\tan^2\theta = \sec^2\theta - 1$

$$\lim_{x\to 0} \frac{\tan x - x}{\sin 2x - 2x} = \lim_{x\to 0} \frac{\sec^2 x - 1}{2\cos 2x - 2} = \lim_{x\to 0} \frac{\tan^2 x}{2\cos 2x - 2} = \lim_{x\to 0} \frac{2\tan x \sec^2 x}{-4\sin 2x}$$

$$= \lim_{x\to 0} \frac{2\sec^4 x + 4\sec^2 x \tan^2 x}{-8\cos 2x} = \frac{2+0}{-8} = \boxed{-\frac{1}{4}}$$

8.2 # 23 (1.25 points)

Find $\lim_{x\to\infty} x^{\frac{1}{x}}$

- Check that the limit is an indeterminate form (0.25 pt.):

$$\lim_{x\to\infty} x^{\frac{1}{x}} = \infty^0 \checkmark$$

- Evaluate the limit using natural logarithms and l'Hôpital's Rule (1 pt.):

$L = \lim_{x\to\infty} x^{\frac{1}{x}}$ $\quad \ln L = \lim_{x\to\infty} \ln(x^{\frac{1}{x}}) = \lim_{x\to\infty} \frac{\ln x}{x}$ which is of the form $\frac{\infty}{\infty}$,

so applying l'Hôpital's Rule: $\ln L = \lim_{x\to\infty} \frac{\frac{1}{x}}{1} = 0$ $\quad L = e^0 = \boxed{1}$

8.3 # 11 (1.25 points)

Evaluate the improper integral $\int_e^\infty \frac{1}{x\ln x}\,dx$

- Rewrite the integral as a limit (0.25 pt.):

$$\int_e^\infty \frac{1}{x\ln x}dx = \lim_{b\to\infty}\int_e^b \frac{1}{x\ln x}dx$$

- Find the antiderivative (0.5 pt.):

$$u = \ln x \qquad du = \frac{dx}{x} \qquad \lim_{b\to\infty}\int_e^b \frac{1}{x\ln x}dx = \lim_{b\to\infty}\int_0^b \frac{du}{u}$$

$$= \lim_{b\to\infty}\left[\ln u\right]_0^b = \lim_{b\to\infty} \ln(\ln b) - \overbrace{\cancel{\ln(\underbrace{\ln e}_{1})}}^{0}$$

- Evaluate the limit (0.5 pt.)

$$\lim_{b\to\infty} \ln(\ln b) = \infty \qquad \boxed{\text{The integral diverges}}$$

8.4 #11 (1.25 points)

Evaluate the improper integral $\int_0^4 \frac{dx}{(2-3x)^{1/3}}$

- Identify point of discontinuity in the set $[0,4]$ (0.25 pt.):

$$(2-3x)^{1/3} = 0 \qquad 2-3x=0 \qquad x = \frac{2}{3}$$

- Rewrite integral as a limit (0.25 pt.):

$$\int_0^4 \frac{dx}{(2-3x)^{1/3}} = \lim_{b\to\frac{2}{3}^-}\int_0^b \frac{dx}{(2-3x)^{1/3}} + \lim_{b\to\frac{2}{3}^+}\int_b^4 \frac{dx}{(2-3x)^{1/3}}$$

- Find the antiderivative (0.5 pt.):

$$u = 2-3x \qquad du = -3\,dx \qquad dx = \frac{-1}{3}du$$

$$\lim_{b\to\frac{2}{3}^-}\int_0^b \frac{dx}{(2-3x)^{1/3}} + \lim_{b\to\frac{2}{3}^+}\int_b^4 \frac{dx}{(2-3x)^{1/3}} = \lim_{b\to\frac{2}{3}^-}\int_0^b \frac{-du}{3u^{1/3}} + \lim_{b\to\frac{2}{3}^+}\int_b^4 \frac{-du}{3u^{1/3}}$$

$$= \lim_{b\to\frac{2}{3}^-}\left[-\frac{1}{2}u^{2/3}\right]_0^b + \lim_{b\to\frac{2}{3}^+}\left[-\frac{1}{2}u^{2/3}\right]_b^4$$

$$= \lim_{b\to\frac{2}{3}^-}\left(-\frac{1}{2}(2-3b)^{2/3} + \frac{1}{2}(2)^{2/3}\right) + \lim_{b\to\frac{2}{3}^+}\left(-\frac{1}{2}(-10)^{2/3} + \frac{1}{2}(2-3b)^{2/3}\right)$$

- Evaluate the limit (0.5 pt.):

$$0 + \frac{1}{2}(2)^{2/3} + 0 - \frac{1}{2}(-10)^{2/3} = \boxed{\frac{1}{2}\left(2^{2/3} - 10^{2/3}\right)}$$