Math 1220-006 (Calculus II), Fall 2016

Homework 5 Grading Key

- Total score = completeness points + correctness points

Completeness (5 points)

- Points are awarded based on the following table:

Points	0	1	2	3	4	5
Percentage of problems attempted	<60%	60-69%	70-79%	80-89%	90-99%	100%

- In order for a problem to count as attempted, there must be some kind of work present. Simply writing down the problem doesn't count.

Correctness (5 points)

- Every week certain problems are selected for individual grading. Correct answers are only worth a small portion of the points. The majority of the points come from demonstrating conceptual knowledge and showing the calculations or reasoning that led to an answer.

- This week, the following problems were selected for grading:

  7.4 # 3, 19
  7.5 # 3, 21

7.4 #3 (1.25 points)

Evaluate $\int \frac{t\,dt}{\sqrt{3t+4}}$

- Use the u-substitution $u = \sqrt{3t+4}$ (0.75 pt.):

$u = (3t+4)^{1/2}$ $\quad 2u\,du = 3\,dt$

$u^2 = 3t+4$ $\quad dt = \frac{2}{3}u\,du$

$t = \frac{1}{3}(u^2-4)$

$$\int \frac{t\,dt}{\sqrt{3t+4}} = \int \frac{\frac{1}{3}(u^2-4)\frac{2}{3}u\,du}{u} = \frac{2}{9}\int (u^2-4)\,du$$

- Find antiderivative (0.5 pt.):

$$\frac{2}{9}\int (u^2-4)\,du = \frac{2}{9}\left(\frac{1}{3}u^3\right) - \frac{2}{9}(4u) + C = \frac{2}{27}(3t+4)^{3/2} - \frac{8}{9}(3t+4)^{1/2} + C$$

7.4 #19 (1.25 points)

Evaluate $\int \frac{3x}{\sqrt{x^2+2x+5}}\,dx$

- Complete the square of x^2+2x+5 (0.25 pt.):

$x^2+2x+5 = x^2+2x+1+4 = (x+1)^2+4$

- Use the u-substitution $u = x+1$ (0.5 pt.):

$u = x+1$, $x = u-1$, $du = dx$

$$\int \frac{3x}{\sqrt{x^2+2x+5}}\,dx = \int \frac{3x}{\sqrt{(x+1)^2+4}}\,dx = \int \frac{3u-3}{\sqrt{u^2+4}}\,du = 3\int \frac{u}{\sqrt{u^2+4}}\,du - 3\int \frac{du}{\sqrt{u^2+4}}$$

- Take antiderivative (0.5 pt.):

$$3\int \frac{u}{\sqrt{u^2+4}}\,du - 3\int \frac{du}{\sqrt{u^2+4}} = 3\sqrt{u^2+4} - 3\ln\left|\sqrt{u^2+4}+u\right| + C$$
$$= 3\sqrt{x^2+2x+5} - 3\ln\left|\sqrt{x^2+2x+5}+x+1\right| + C$$

7.5 #3 (1.25 points)

Evaluate $\int \frac{3}{x^2-1}\,dx$

- Use partial fractions decomposition (0.75 pt.):

$$\frac{3}{x^2-1} = \frac{3}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1} \qquad 3 = A(x-1)+B(x+1) \qquad A = -\frac{3}{2} \quad B = \frac{3}{2}$$

- Rewrite integral, take antiderivative (0.5 pt.):

$$\int \frac{3}{x^2-1}\,dx = -\frac{3}{2}\int \frac{dx}{x+1} + \frac{3}{2}\int \frac{dx}{x-1} = -\frac{3}{2}\ln|x+1| + \frac{3}{2}\ln|x-1| + C$$

7.5 #21 (1.25 points)

Evaluate $\int \frac{x+1}{(x-3)^2}\,dx$

- Use partial fractions decomposition (0.75 pt.):

$$\frac{x+1}{(x-3)^2} = \frac{A}{x-3} + \frac{B}{(x-3)^2} \qquad x+1 = A(x-3)+B \qquad A = 1,\ B = 4$$

- Rewrite integral, take antiderivative (0.5 pt.):

$$\int \frac{x+1}{(x-3)^2}\,dx = \int \frac{dx}{x-3} + 4\int \frac{dx}{(x-3)^2} = \ln|x-3| - \frac{4}{x-3} + C$$