Math 1220-006 (Calculus II), Fall 2016

Homework 4 Grading Key

- Total score = completeness points + correctness points

Completeness (5 points)

- Points are awarded based on the following table:

Points	0	1	2	3	4	5
Percentage of problems attempted	<60%	60-69%	70-79%	80-89%	90-99%	100%

- In order for a problem to count as attempted, there must be some kind of work present. Simply writing down the problem doesn't count.

Correctness (5 points)

- Every week certain problems are selected for individual grading. Correct answers are only worth a small portion of the points. The majority of the points come from demonstrating conceptual knowledge and showing the calculations or reasoning that led to an answer.

- This week, the following problems were selected for grading:

  7.1 # 11
  7.2 # 15, 37
  7.3 # 13

7.1 #11 (1 point)

Compute $\int \frac{\tan z}{\cos^2 z}\,dz$

- Recognize that $\frac{1}{\cos^2 z} = \sec^2 z$ (0.25 pt.):

$$\int \frac{\tan z}{\cos^2 z}\,dz = \int \tan z \sec^2 z\,dz$$

- Use u-substitution with $u = \tan z$ (0.25 pt.):

$u = \tan z$
$du = \sec^2 z\,dz$

$$\int \tan z \sec^2 z\,dz = \int u\,du$$

- Take antiderivative: (0.5 pt.):

$$\int u\,du = \frac{1}{2}u^2 + C_1 = \frac{1}{2}\tan^2 z + C_1$$

Note:

If you use $u = \cos z$, $du = -\sin z\,dz$, you'll end up with:

$$\int \frac{\tan z}{\cos^2 z}\,dz = \int \frac{\sin z}{\cos^3 z}\,dz$$

$$= -\int \frac{du}{u^3} = \frac{1}{2u^2} + C_2$$

$$= \frac{1}{2\cos^2 z} + C_2$$

Notice that although both answers work, the integration constants aren't the same:

$C_2 = \frac{1}{2} + C_1$ since

$$\frac{1}{2\cos^2 z} + \underbrace{\frac{1}{2} + C_1}_{C_2} = \frac{1 + \sec^2 z}{2} + C_1 = \frac{\tan^2 z}{2} + C_1$$

1.25 points

7.2 #15 ~~(1 point)~~

Use integration by parts to compute $\int \frac{\ln x}{x^2}\,dx$

- Let $u = \ln x$, $dv = \frac{dx}{x^2}$, compute du and v (0.5 pt.):

$u = \ln x \qquad dv = \frac{dx}{x^2}$

$du = \frac{1}{x}dx \qquad v = -\frac{1}{x}$

- Use integration by parts formula (0.75 pt.):

$$\int \frac{\ln x}{x^2}\,dx = (\ln x)\left(-\frac{1}{x}\right) - \int\left(-\frac{1}{x}\right)\left(\frac{1}{x}\,dx\right) = -\frac{\ln x}{x} + \int \frac{dx}{x^2}$$

$$= -\frac{\ln x}{x} - \frac{1}{x} + C = \frac{-\ln x - 1}{x} + C$$

1.5 points

7.2 #37 ~~(1.25 points)~~

Use integration by parts twice to compute $\int x^2 e^x\,dx$

- Let $u = x^2$, $dv = e^x dx$, compute du and v (0.25 pt.):

$u = x^2 \qquad dv = e^x dx$

$du = 2x\,dx \qquad v = e^x$

- Use integration by parts formula (0.5 pt.):

$$\int x^2 e^x\,dx = (x^2)(e^x) - \int (e^x)(2x\,dx)$$

- Let $u = 2x$, $dv = e^x dx$, compute du and v (0.25 pt.):

$u = 2x \qquad dv = e^x dx$

$du = 2\,dx \qquad v = e^x$

- Use integration by parts formula again to get correct answer (0.5 pt.):

$$\int x^2 e^x\,dx = x^2e^x - (2x)(e^x) + \int (e^x)(2\,dx)$$

$$= y^2e^x - 2xe^x + 2e^x + C$$

7.3 #13 (1.25 points)

Compute $\int \sin 4y \cos 5y\,dy$

- Use a relevant trigometric identity to rewrite the integral in terms that are easier to integrate (0.75 pt.):

$$\int \sin 4y \cos 5y\,dy = \frac{1}{2}\left[\sin 9y + \sin(-y)\right]dy = \frac{1}{2}\int(\sin 9y - \sin y)\,dy$$

- Take antiderivative (0.75 pt.):

$$\frac{1}{2}\int(\sin 9y - \sin y)\,dy = \frac{1}{2}\left(-\frac{1}{9}\cos 9y + \cos y\right) + C = \frac{1}{2}\cos y - \frac{1}{18}\cos 9y + C$$