

# Math 1220-006 (Calculus II), Fall 2016

## Homework 3 Grading Key

- Total score = completeness points + correctness points

### Completeness (5 points)

- Points are awarded based on the following table:

Points	0	1	2	3	4	5
Percentage of problems attempted	<60%	60-69%	70-79%	80-89%	90-99%	100%

- In order for a problem to count as attempted, there must be some kind of work present. Simply writing down the problem doesn't count.

### Correctness (5 points)

- Every week certain problems are selected for individual grading. Correct answers are only worth a small portion of the points. The majority of the points come from demonstrating conceptual knowledge and showing the calculations or reasoning that led to an answer.
- This week, the following problems were selected for grading:

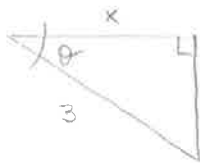
6.8 #25, #41, #59

6.9 #19, #41

#### 6.8 #25 (1 point)

Evaluate  $\cos\left[2\sin^{-1}\left(-\frac{2}{3}\right)\right]$ .

- Construct right triangle and/or use trigonometric identity (1 pt.):



$$\theta = \sin^{-1}\left(-\frac{2}{3}\right) \checkmark$$

$$x^2 + (-2)^2 = 3^2$$

$$x = \pm\sqrt{5}$$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 1 - 2\sin^2(\theta) = 2\cos^2(\theta) - 1$$

Using the third equivalence:

$$\cos(2\theta) = 2\cos^2\theta - 1 = 2\left(\frac{\sqrt{5}}{3}\right)^2 - 1 = \boxed{\frac{1}{9}}$$

#### 6.8 #41 (1 point)

Find  $\frac{dy}{dx}$  if  $y = \ln(\sec x + \tan x)$ .

- Find  $\frac{dy}{dx}$  by differentiating the natural logarithm and applying the Chain Rule (1 pt.)

$$\frac{dy}{dx} = \frac{1}{\sec x + \tan x} D_x(\sec x + \tan x) = \frac{1}{\sec x + \tan x} (D_x \sec x + D_x \tan x)$$

$$D_x \sec x = D_x (\cos x)^{-1} = \frac{-\sin x}{-\cos^2 x} = \sec x \tan x$$

$$D_x \tan x = D_x \frac{\sin x}{\cos x} = \frac{\cos^2 x - (-\sin^2 x)}{\cos^2 x}$$

$$\frac{dy}{dx} = \frac{1}{\sec x + \tan x} (\sec x (\tan x + \sec x)) = \boxed{\sec x}$$

$$= \sec^2 x$$

6.8 #59 (1 point)

Evaluate  $\int_0^1 e^{2x} \cos(e^{2x}) dx$ .

- Use u-substitution with  $u = e^{2x}$  (0.5 pt):

$$\begin{aligned} u &= e^{2x} \\ du &= 2e^{2x} dx \\ \frac{1}{2} du &= e^{2x} dx \end{aligned} \quad \int_0^1 e^{2x} \cos(e^{2x}) dx = \frac{1}{2} \int_0^1 \cos u du$$

- Find antiderivative, evaluate integral (0.5 pt):

$$\frac{1}{2} \int_0^1 \cos u du = \frac{1}{2} [\sin u]_0^1 = \frac{1}{2} [\sin(e^{2x})]_0^1 = \boxed{\frac{1}{2} [\sin(e^2) - \sin(1)]}$$

6.9 #19 (1 point)

Find  $D_x y$  of  $y = \ln(\sinh x)$ .

- Find  $D_x y$  by differentiating the natural logarithm and applying the Chain Rule (1 pt):

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{\sinh x} D_x(\sinh x) \\ &= \frac{1}{\sinh x} (\cosh x) = \boxed{\coth x} \end{aligned}$$

6.9 #41 (1 point)

Evaluate  $\int \frac{\sinh(2z^{\frac{1}{4}})}{4\sqrt[4]{z^3}} dz$ .

- Use u-substitution with  $u = 2z^{\frac{1}{4}}$  (0.5 pt):

$$\begin{aligned} u &= 2z^{\frac{1}{4}} \\ du &= \frac{1}{2} z^{-\frac{3}{4}} dz \\ 2du &= z^{-\frac{3}{4}} dz \end{aligned} \quad \int \frac{\sinh(2z^{\frac{1}{4}})}{4\sqrt[4]{z^3}} dz = 2 \int \sinh u du$$

- Find antiderivative (0.5 pt):

$$2 \int \sinh u du = 2 \cosh u + C = \boxed{2 \cosh(2z^{\frac{1}{4}}) + C}$$