Math 1220-006 (Calculus II), Fall 2016

Homework 2 Grading Key

- Total score = completeness points + correctness points

Completeness (5 points)

- Points are awarded based on the following table:

Points	0	1	2	3	4	5
Percentage of problems attempted	<60%	60-69%	70-79%	80-89%	90-99%	100%

- In order for a problem to count as attempted, there must be some kind of work present. Simply writing down the problem doesn't count.

Correctness (5 points)

- Every week certain problems are selected for individual grading. Correct answers are only worth a small portion of the points. The majority of the points come from demonstrating conceptual knowledge and showing the calculations or reasoning that led to an answer.

- This week, the following problems were selected for grading:

  6.3 #19
  6.4 #25
  6.5 #5
  6.6 #7

6.3 #19 (1.25 points)

Find $D_x y$ if $y = \sqrt{e^{x^2}} + e^{\sqrt{x^2}}$

- Compute $D_x y$ using Power Rule and Chain Rule (1 pt.):

$$D_x y = \frac{1}{2}\left(e^{x^2}\right)^{-\frac{1}{2}} D_x\left(e^{x^2}\right) + e^{\sqrt{x^2}} \cdot D_x\left(\sqrt{x^2}\right)$$

$$= \frac{1}{2}\left(e^{x^2}\right)^{-\frac{1}{2}}\left(e^{x^2}\right) D_x\left(x^2\right) + e^{\sqrt{x^2}}\left(\frac{1}{2}\left(x^2\right)^{-\frac{1}{2}}\right) D_x\left(x^2\right)$$

$$= \frac{1}{2}\left(e^{x^2}\right)^{-\frac{1}{2}}\left(e^{x^2}\right)(2x) + e^{\sqrt{x^2}}\left(\frac{1}{2}\left(x^2\right)^{-\frac{1}{2}}\right)(2x)$$

- Simplify to get correct answer (0.25 pt.):

$$D_x y = x\left(e^{x^2}\right)^{-\frac{1}{2}+1} + e^{\sqrt{x^2}}\left(\frac{x}{|x|}\right) = x\left(\sqrt{e^{x^2}} + \frac{e^{\sqrt{x^2}}}{|x|}\right)$$

6.4 #25 (1.25 points)

Find $\int_1^4 \frac{5^{\sqrt{x}}}{\sqrt{x}}\,dx$

- Use u-substitution with $u = \sqrt{x}$ (0.5 pt.):

$u = \sqrt{x}$
$du = \frac{1}{2}x^{-\frac{1}{2}}dx$ $\int_1^4 \frac{5^{\sqrt{x}}}{\sqrt{x}}\,dx = \int_1^4 \left(\frac{1}{2}\right)(2)\frac{5^{\sqrt{x}}}{\sqrt{x}}\,dx = 2\int_1^4 5^u\,du$

- Compute integral (0.75 pts.):

$$2\int_1^4 5^u\,du = 2\left[\frac{5^u}{\ln 5}\right]_1^4 = 2\left[\frac{5^{\sqrt{x}}}{\ln 5}\right]_1^4 = 2\left(\frac{5^{\sqrt{4}}}{\ln 5} - \frac{5^{\sqrt{1}}}{\ln 5}\right) = \frac{40}{\ln 5}$$

6.5 #5 (1.25 points)

A bacterial population grows at a rate that is proportional to its size. Initially it is 10,000, and after 10 days it is 20,000. What is the population after 25 days?

- Recognize that bacteria population grows exponentially (0.25 pt.):

$\frac{dP}{dt} = kP \qquad \frac{dP}{P} = k\,dt \qquad \int\frac{dP}{P} = \int k\,dt \qquad \ln P = kt + C \qquad P(t) = e^{kt+C}$

- Solve for C using initial conditions (0.25 pt.):

$y(0) = 10{,}000 \qquad 10{,}000 = e^{k(0)+C} \qquad 10{,}000 = e^C \qquad C = \ln 10{,}000$

$P(t) = e^{kt + \ln 10{,}000} = e^{kt}e^{\ln(10{,}000)} = 10{,}000\,e^{kt}$

- Solve for k using other given data (0.5 pt.):

$y(10) = 20{,}000 \qquad 20{,}000 = 10{,}000\,e^{k(10)} \qquad 2 = e^{10k} \qquad 10k = \ln 2 \qquad k = \frac{\ln 2}{10}$

$P(t) = 10{,}000\,e^{\left(\frac{\ln 2}{10}\right)t} = 10{,}000\left(2^{t/10}\right)$

- Find bacteria population after 25 days (0.25 pt.):

$P(25) = 10{,}000\left(2^{2.5/10}\right) \approx 56{,}568$

6.6 #7 (1.25 points)

Solve the differential equation $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x}$

- Find integrating factor (0.5 pt.):

$P(x) = \frac{1}{x} \qquad I.F. = e^{\int P(x)dx} = e^{\int \frac{1}{x}dx} = e^{\ln x} = x$

- Multiply equation by integrating factor and simplify (0.25 pt.):

$x\left(\frac{dy}{dx}\right) + x\left(\frac{y}{x}\right) = x\left(\frac{1}{x}\right) \qquad x\frac{dy}{dx} + y = 1 \qquad \frac{d}{dx}(xy) = 1$

- Integrate equation, solve for y (0.5 pt.):

$\int \frac{d}{dx}(yx) = \int 1 \qquad yx = x + C \qquad y = 1 + \frac{C}{x}$