Math 1220-006 (Calculus II), Fall 2016

Homework 12 Grading Key

10.6 # 1, 3, 5, 7, 9, 11, 17, 21, 23, 29
10.7 # 1, 3, 5, 7, 9, 11, 15, 19, 23, 25

Total: 20 problems

- Total score = completeness points + correctness points

Completeness (5 points)

- Points are awarded based on the following table:

Points	0	1	2	3	4	5
Percentage of problems attempted	<60%	60-69%	70-79%	80-89%	90-99%	100%

- In order for a problem to count as attempted, there must be some kind of work present. Simply writing down the problem doesn't count.

Correctness (5 points)

- Every week certain problems are selected for individual grading. Correct answers are only worth a small portion of the points. The majority of the points come from demonstrating conceptual knowledge and showing the calculations or reasoning that led to an answer.

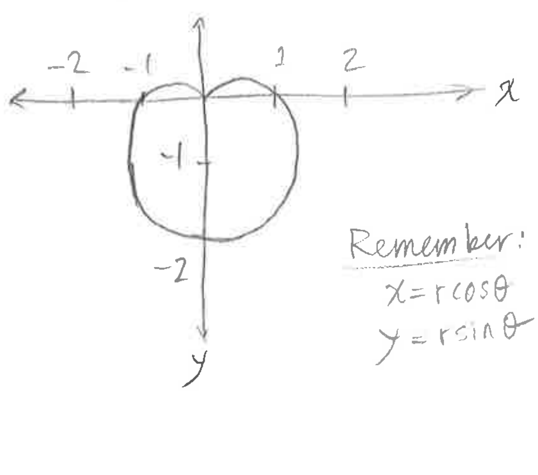
- This week, the following problems were selected for grading:

10.6 #11 (1 point)

Graph $r = 1 - \sin\theta$ and identify any symmetry.

We have an equation of the form $r = a \pm b\sin\theta$ with $a = b$, so we are looking at a cardioid. Since $\sin(\pi - \theta) = \sin\theta$ the cardioid must be symmetric about the y-axis (0.25 pt.).

Use a table of coordinates to determine the precise shape of the graph:

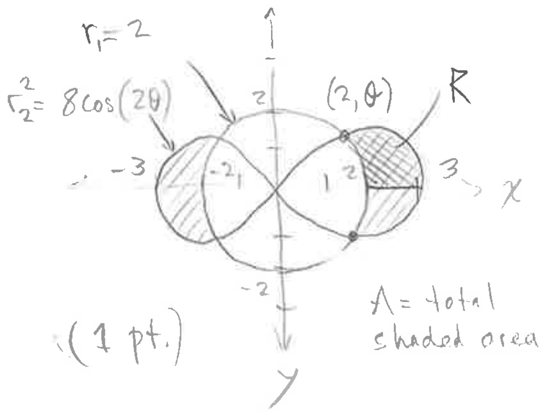


Remember:
$x = r\cos\theta$
$y = r\sin\theta$

θ	$r = 1 - \sin\theta$	x	y
0	1	1	0
$\pi/6$	0.5	0.433	0.25
$\pi/3$	$1 - \sqrt{3}/2 \approx 0.134$	0.067	0.116
$\pi/2$	0	0	0
$2\pi/3$	$1 - \sqrt{3}/2 \approx 0.134$	-0.067	0.116
$5\pi/6$	0.5	-0.433	0.25
π	1	-1	0
$7\pi/6$	1.5	-1.299	-0.75
$4\pi/3$	$1 + \sqrt{3}/2 \approx 1.866$	-0.933	-1.616
$3\pi/2$	2	0	-2
$\vdots$	$\vdots$	$\vdots$	$\vdots$

10.7 #19 (2 points)

Sketch the region outside the circle $r_1 = 2$ and inside the lemniscate $r_2^2 = 8\cos(2\theta)$, then compute this area.



(1 pt.)

Polar area formula: $A = \frac{1}{2}\int_\alpha^\beta [f(\theta)]^2 d\theta$

Solving for θ (0.5 pt.)

$(2)^2 = 8\cos(2\theta)$ $\quad \cos(2\theta) = \frac{1}{2}$

$2\theta = \frac{\pi}{3}$ $\quad \theta = \frac{\pi}{6}$

So $R = \frac{1}{2}\int_0^{\pi/6} [\underbrace{8\cos(2\theta)}_{r_2^2} - \underbrace{4}_{r_1^2}] d\theta$ (1 pt.)

$= \frac{1}{2}\left[4\sin(2\theta) - 4\theta\right]_0^{\pi/6} = \sqrt{3} - \frac{\pi}{3}$

By symmetry, $A = 4R = \boxed{4\sqrt{3} - \frac{4\pi}{3}}$

10.7 #25 (2 points)

Find all points on the limaçon $r = 1 - 2\sin\theta$ where the tangent line is horizontal.

(0.5 pt.) $f(\theta) = 1 - 2\sin\theta$

$f'(\theta) = -2\cos\theta$

$$m = \frac{f(\theta)\cos\theta + f'(\theta)\sin\theta}{-f(\theta)\sin\theta + f'(\theta)\cos\theta}$$

(0.5 pt.)

$$m = \frac{(1-2\sin\theta)\cos\theta + (-2\cos\theta)\sin\theta}{-(1-2\sin\theta)\sin\theta + (-2\cos\theta)\cos\theta} = \frac{\cos\theta(1-4\sin\theta)}{-\sin\theta + 2\sin^2\theta - 2\cos^2\theta}$$

$m = 0$ when $\cos\theta(1-4\sin\theta) = 0$, which occurs when:

$\theta = \frac{\pi}{2}$ $\quad f\left(\frac{\pi}{2}\right) = -1$

$\theta = \frac{3\pi}{2}$ $\quad f\left(\frac{3\pi}{2}\right) = 3$ (1 pt.)

$0 \le \theta \le 2\pi$ $\quad \theta = \sin^{-1}\left(\frac{1}{4}\right)$ $\quad f\left(\sin^{-1}\left(\frac{1}{4}\right)\right) = \frac{1}{2}$

$\theta = \pi - \sin^{-1}\left(\frac{1}{4}\right)$ $\quad f\left(\pi - \sin^{-1}\left(\frac{1}{4}\right)\right) = \frac{1}{2}$

Final answer: $\boxed{\left(-1, \frac{\pi}{2}\right), \left(3, \frac{3\pi}{2}\right), \left(\frac{1}{2}, \sin^{-1}\left(\frac{1}{4}\right)\right), \left(\frac{1}{2}, \pi - \sin^{-1}\left(\frac{1}{4}\right)\right)}$