Math 1220-006 (Calculus II), Fall 2016

Homework 10 Grading Key

9.8 # 15, 17, 19, 21, 23, 27
9.9 # 1, 3, 5, 7, 9, 11, 13, 15, 19, 21, 23, 25, 27
Total: 19 problems

- Total score = completeness points + correctness points

Completeness (5 points)

- Points are awarded based on the following table:

Points	0	1	2	3	4	5
Percentage of problems attempted	<60%	60-69%	70-79%	80-89%	90-99%	100%

- In order for a problem to count as attempted, there must be some kind of work present. Simply writing down the problem doesn't count.

Correctness (5 points)

- Every week certain problems are selected for individual grading. Correct answers are only worth a small portion of the points. The majority of the points come from demonstrating conceptual knowledge and showing the calculations or reasoning that led to an answer.

- This week, the following problems were selected for grading:

9.8 # 19 (1.5 points)

Find the third-order Taylor series for $f(x)=e^x$, $a=1$.

- Calculate three derivatives of $f(x)$ and evaluate at $x=1$ (0.75 pt.):

$$f(a)=f'(a)=f''(a)=f'''(a)=e$$

- Construct the corresponding Taylor series (0.75 pt.):

$$\boxed{f(x)\approx e+e(x-1)+\frac{e}{2}(x-1)^2+\frac{e}{6}(x-1)^3}$$

9.9 # 5 (1.5 points)

Find the fourth-order Maclaurin series for $f(x)=\ln(1+x)$ and use it to estimate $f(0.12)$.

- Calculate four derivatives of $f(x)$ and evaluate them at $x=0$ (0.75 pt.):

$$f(x)=\ln(1+x) \quad f'(x)=\frac{1}{1+x} \quad f''(x)=\frac{-1}{(1+x)^2} \quad f'''(x)=\frac{2}{(1+x)^3} \quad f^{(4)}(x)=\frac{-6}{(1+x)^4}$$

$$f(0)=\ln(1)=0 \quad f'(0)=\frac{1}{1+0}=1 \quad f''(0)=-1 \quad f'''(0)=2 \quad f^{(4)}(0)=-6$$

- Construct the corresponding Maclaurin series (0.5 pt.), evaluate at $x=0.12$ (0.25 pt.):

$$f(x)=\ln(1+x)\approx\boxed{x-\frac{x^2}{2!}+2\frac{x^3}{3!}-6\frac{x^4}{4!}}$$

$$f(0.12)\approx 0.12-\frac{(0.12)^2}{2!}+2\cdot\frac{(0.12)^3}{3!}-6\cdot\frac{(0.12)^4}{4!}\approx\boxed{0.1133}$$

9.9 #15 (2 points)

Find the order 3 Taylor series for $f(x) = x^3 - 2x^2 + 3x + 5$ at $a = 1$,
Show that this Taylor series represents $f(x)$ exactly.

- Calculate three derivatives of $f(x)$, evaluate them at $x = 1$ (0.75 pt.):

$f(x) = x^3 - 2x^2 + 3x + 5$ $\qquad f(1) = 7$

$f'(x) = 3x^2 - 4x + 3$ $\qquad f'(1) = 2$

$f''(x) = 6x - 4$ $\qquad f''(1) = 2$

$f'''(x) = 6$ $\qquad f'''(1) = 6$

- Construct the corresponding Taylor series (0.75 pt.):

$$f(x) = 7 + 2(x-1) + \frac{\cancel{2}}{\cancel{2!}}(x-1)^2 + \frac{\cancel{6}}{\cancel{3!}}(x-1)^3$$

- Multiply the Taylor series out to show that it is exactly equal to what we started with (0.5 pt.)

$$7 + 2(x-1) + (x-1)^2 + (x-1)^3 = 7 + (2x - 2) + (x^2 - 2x + 1) + (x^3 - x^2 - 2x^2 + 2x + x - 1)$$

$$= x^3 - 2x^2 + 3x + 5 = f(x) \checkmark$$