Math 1220-006 (Calculus II), Fall 2016

Homework 1 Grading Key

- Total score = completeness points + correctness points

Completeness (5 points)

- Points are awarded based on the following table:

Points	0	1	2	3	4	5
Percentage of problems attempted	<60%	60-69%	70-79%	80-89%	90-99%	100%

- In order for a problem to count as attempted, there must be some kind of work present. Simply writing down the problem doesn't count.

Correctness (5 points)

- Problems corrected for this week:

  6.1 #9, #33
  6.2 #13, #25

6.1 #9 (1 point)

Find $\frac{dz}{dx}$ if $z = x^2 \ln x^2 + (\ln x)^3$.

- Use Product Rule, Chain Rule, and Power Rule (0.75 pts.):

$$\frac{dz}{dx} = x^2\left(\frac{1}{x^2}\cdot 2x\right) + \ln x^2 (2x) + 3(\ln x)^2 \cdot \frac{1}{x}$$

- Simplify to get correct answer (0.25 pts.):

$$\frac{dz}{dx} = 2x + 4x\ln x + \frac{3}{x}(\ln x)^2$$

6.1 #33 (1.5 points)

If $y = \frac{\sqrt{x+13}}{(x+4)\sqrt[3]{2x+1}}$, find $\frac{dy}{dx}$ using logarithmic differentiation.

- Take natural logarithm of both sides and simplify using special properties about the natural logarithm (0.5 pts.):

$$\ln y = \ln\left(\frac{(x+13)^{\frac{1}{2}}}{(x+4)(2x+1)^{\frac{1}{3}}}\right) = \ln(x+13)^{\frac{1}{2}} - \ln\left((x+4)(2x+1)^{\frac{1}{3}}\right)$$

$$\ln y = \ln(x+13)^{\frac{1}{2}} - \left(\ln(x+4) + \ln(2x+1)^{\frac{1}{3}}\right) = \frac{1}{2}\ln(x+13) - \ln(x+4) - \frac{1}{3}\ln(2x+1)$$

- Take derivative of both sides with respect to x, solve for $\frac{dy}{dx}$ (1pt.):

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2(x+13)} - \frac{1}{x+4} - \frac{2}{3(2x+1)} \Rightarrow \frac{dy}{dx} = y\left(\frac{1}{2(x+13)} - \frac{1}{x+4} - \frac{2}{3(2x+1)}\right)$$

6.2 #13 (1 point)

Show that $f(x) = \int_0^x \sqrt{t^4+t^2+10}\,dt$ is strictly monotonic and hence that $f^{-1}(x)$ exists.

- Compute $f'(x)$ using Part 1 of the Fundamental Theorem of Calculus (0.5 pts.):

$$f'(x) = \frac{d}{dx}\int_0^x \sqrt{t^4+t^2+10}\,dt = \sqrt{x^4+x^2+10}$$

- State that $f'(x) > 0$ for all $x \in \mathbb{R}$ and thus that $f'(x)$ exists (0.5 pts.):

$\sqrt{x^4+x^2+10} > 0$ for all $x \in \mathbb{R}$, therefore $f^{-1}(x)$ exists.

6.2 #25 (1.5 points)

If $f(x) = \frac{x-1}{x+1}$, find $f^{-1}(x)$ and verify that $f^{-1}(f(x)) = f(f^{-1}(x)) = x$.

- Let $f(x) = y$, swap x and y, solve for $y = f^{-1}(x)$ (1 pt.):

$$y = \frac{x-1}{x+1} \rightarrow x = \frac{y-1}{y+1}$$

$$x(y+1) = y-1$$

$$xy + x = y-1$$

$$xy - y = -x-1$$

$$y(x-1) = -x-1$$

$$y = f^{-1}(x) = \frac{-x-1}{x-1} = \frac{1+x}{1-x}$$

- Verify $f^{-1}(f(x)) = f(f^{-1}(x)) = x$ (0.5 pts.):

$$f^{-1}(f(x)) = \frac{1+\left(\frac{x-1}{x+1}\right)}{1-\left(\frac{x-1}{x+1}\right)} = \frac{\frac{x+1+x-1}{\cancel{x+1}}}{\frac{x+1-x+1}{\cancel{x+1}}} = \frac{2x}{2} = x \ \checkmark$$

$$f(f^{-1}(x)) = \frac{\left(\frac{1+x}{1-x}\right)-1}{\left(\frac{1+x}{1-x}\right)+1} = \frac{\frac{1+x-1+x}{\cancel{1-x}}}{\frac{1+x+1-x}{\cancel{1-x}}} = \frac{2x}{2} = x \ \checkmark$$