A Modified POCSENSE Technique for Accelerated Iterative Reconstruction from Sensitivity Encoded MRI Data

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Synopsis

Recently, we have proposed a projection onto convex sets (POCS) based method for reconstruction from sensitivity-encoded data (POCSENSE). The POCS formulation of image reconstruction offers a straightforward and computationally efficient way to incorporate non-linear constraints into a reconstruction and improve the image quality in cases of ill-posed and underdetermined problems. However, POCSENSE demonstrates slow convergence in cases of high reduction factors. This limits the practical utility of the original technique. In this work, we present a novel method for reconstruction from sensitivity-encoded data based on extrapolated iterations of parallel POCS that overcome the limitation of the original POCSENSE technique.

Theory and Methods

The extrapolated parallel projection method (EPPM) [1] uses a convex combination of projections onto available convex sets to obtain a current estimate of the solution via relaxation. Convergence of POCS and EPPM algorithms crucially depend on the relaxation parameter. In the conventional POCS algorithms, the relaxation parameter is static (iteration-independent) and strictly defined on the interval (0, 2]. In EPPM, the relaxation parameter is adaptively adjusted after each iteration and can extend far beyond the traditional POCS range, resulting in highly efficient convergence. The description of the modification of the original POCSENSE method [2] based on the EPPM approach is given below.

<u>*Notations:*</u> $\|\cdot\|$ is *L2* norm; "*" denotes complex conjugation; $\boldsymbol{r}$, $\boldsymbol{k}$ are image space and *k*-space coordinates, respectively; N_c is the number of coil elements; $s_i(\boldsymbol{r})$, $S_i(\boldsymbol{k})$ are *i*-th coil sensitivity profile and its Fourier transform (i=1…N_c); $f(\boldsymbol{r})$,$F(\boldsymbol{k})$ are the image function and its Fourier transform; $\tilde{K}$ is the *k*-space sampling pattern; and $m_i(\boldsymbol{k})$ are data acquired by the *i*-th coil ($\boldsymbol{k} \in \tilde{K}$).

<u>*Algorithm:*</u> Starting with some initial guess $f^{(0)}$, iterate until the convergence criteria is met, updating the image estimate as follows:

$$f^{(n+1)} = f^{(n)} + \lambda_n \left(\frac{\sum_{i=1}^{N_C} P_M P^i_{data}(s_i f^{(n)}) \cdot s_i^*}{\sum_{i=1}^{N_C} |s_i|^2} - f^{(n)} \right),\ \varepsilon \le \lambda_n \le (2-\varepsilon) L_n,\ 0 \le \varepsilon \le 1 \ \text{, where } L_n = \frac{\sum_{i=1}^{N_C} \left\| P_M P^i_{data}(s_i f^{(n)}) \cdot s_i^* - f^{(n)} |s_i|^2 \right\|^2}{\left\| \sum_{i=1}^{N_C} P_M P^i_{data}(s_i f^{(n)}) \cdot s_i^* - f^{(n)} \sum_{i=1}^{N_C} |s_i|^2 \right\|^2}.$$

On each iteration, the current image estimate is projected in parallel onto subgroups of the convex sets defined in Table 1 and results are then combined using known coil sensitivity profiles. The extrapolation coefficient L_n and the relaxation parameter λ_n are then found as proposed and used to update the image estimate.

Results

The algorithm validation was accomplished using phantom and in vivo data obtained on a 1.5T GE SIGNA (GE Medical Systems, Milwaukee, WI) system using 4-coil phased arrays. Typical results of the proposed method for reconstruction from data acquired with the maximal reduction factor are shown in Figs. 1 and 2. The convergence of the modified POCSENSE was compared with the convergence of the original POCSENSE (Fig. 1). The results indicate that the convergence rate of the new method is significantly higher (approximately an order of magnitude faster) than that of the original POCSENSE algorithm. Non-monotonic behavior of the error curve (Fig. 1) for the extrapolated projections method is a well-known phenomenon and may be resolved by periodic recentering of the extrapolation parameter [1].

Discussion

In this work, we demonstrated that slow convergence of the original POCSENSE method can be overcame by using an extrapolated parallel projection onto convex sets. The presented method provides significant acceleration of the POCS-based reconstruction in comparison with the POCSENSE method, especially for ill-conditioned cases that are typical at high and/or maximal reduction factors and for non-optimal coil geometries. The flexibility of the proposed technique provided by POCS formalism allows for improved image reconstruction when additional reconstruction constraints are available [3].

Acknowledgements

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References

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Convex sets	***Corresponding projection operators***
Ω^i_{data}, set of images with *k*-space values $m_i(\boldsymbol{k})$ at $\boldsymbol{k} \in \tilde{K}$	$P^i_{data} f(\boldsymbol{r}) \Leftrightarrow \begin{cases} m_i(\boldsymbol{k}), & \boldsymbol{k} \in \tilde{K} \\ F(\boldsymbol{k}) \otimes S_i(\boldsymbol{k}), & \boldsymbol{k} \notin \tilde{K} \end{cases}$
Ω_M, set of images with spatial support $\boldsymbol{M}$	$P_M f(\boldsymbol{r}) = \begin{cases} f(\boldsymbol{r}), & \boldsymbol{r} \in \boldsymbol{M} \\ 0, & otherwise \end{cases}$

Table 1. The definition of convex sets and corresponding projections.

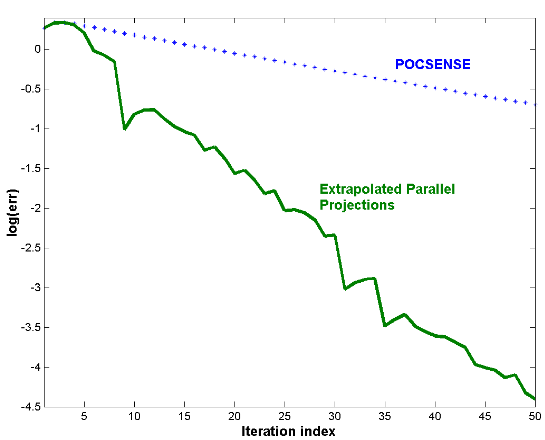

Figure 1. The comparison of convergence of POCS-based methods. The error after *n* iterations was estimated as $err^{(n)} = \|f^{(n)} - f_0\| / \|f_0\|$, where f_0 is the solution obtained by direct inversion of the corresponding matrix equation.

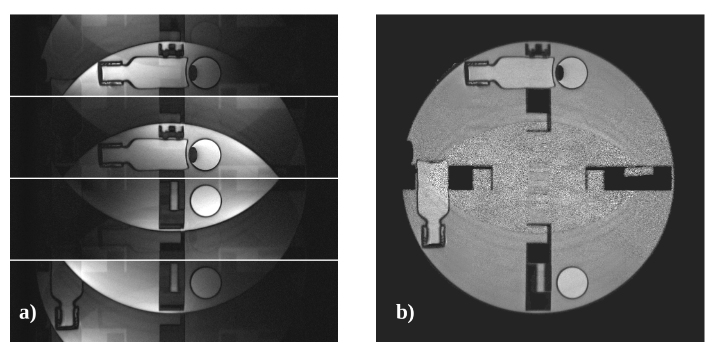

Figure 2. Initial images (a) were reconstructed from data acquired with reduction factor *R*=4 by 4-coil phase array (N_c=4); (b) is the resulting image (50 iterations of the proposed method with λ_n=1.5L$_n$).