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The Quantum Zeno Effect of a Single System is Equivalent to the Indetermination of the Quantum State of a Single System

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Abstract. The quantum Zeno effect of a single system, the effect of a series of measurements on the free time evolution of the system and on the ability to monitor this time evolution using the measurement results, is shown to be equivalent to that of the indetermination of the quantum state of a single system, in which one considers the statistics of the results of a series of measurements performed on a single system, with no time evolution in between successive measurements. From this equivalence it is concluded that the quantum Zeno effect is a measurement effect, which originates in the generalized projection postulate.

Introduction.– The quantum Zeno effect was first introduced by Misra and Sudarshan [1] as the effect of a continuous and precise quantum measurement of an observable with a discrete eigenvalue spectrum on the free time evolution of a quantum system. Later their analysis was generalized to include the ensemble averaged effect of a series of approximate quantum measurements on the free time evolution of a quantum system. This effect, the quantum Zeno effect of an ensemble, was shown to be indistinguishable from the effect of dephasing (see, e.g., [2-5]), and therefore it was suggested that this effect may be due to the dynamics of the system, rather than a quantum measurement effect (see, e.g., [6-8]).

We consider the quantum Zeno effect of a single system. Specifically, we ask two questions: How would the free time evolution of a single quantum system change due to a series of quantum measurements? What information about the free time evolution of the single system could be obtained from the series of measurement results? In the frame of reference which evolves in time with the system, these questions regard the indetermination of the unknown quantum state of a single system [9,10]: How would the initial quantum state of a single system change due to a series of measurements? What information about the initial quantum state of the single system could be obtained from the series of measurement results? It was already suggested that unlike the ensemble averaged effect, the quantum Zeno effect of a single system cannot be thought of as a dephasing effect (see, e.g., [11]). We show that, indeed, the quantum Zeno effect of a single system and the indetermination of the quantum state of a single system are two different descriptions, in the Schrödinger picture and the Heisenberg picture respectively, of the same phenomenon, the "screening" of the quantum state of a single system by a series of measurements, as a consequence of the generalized projection postulate. Therefore, we prove that the quantum Zeno effect of a single system is a true measurement effect, and not a dephasing effect.

The quantum Zeno effect of an ensemble: The standard formulation.– In the standard formulation of the quantum Zeno effect [1-5] one considers the effect of a series of quantum measurements on the free time evolution of a single quantum system without refering to the series of measurement results. Therefore, in general, this is an ensemble averaged effect. Consider a series of n measurements of the observable $\hat{q}$ performed on a single system during its unitary time evolution in the time interval $t \in [0, T]$. The initial state of the system is described by the density operator $\hat{\rho}_0$, and the deterministic time evolution of the system in between the $(k-1)$-th and the k-th measurements, at $t_{k-1} = (k-1)T/n$ and $t_k = kT/n$ respectively, is described by the unitary operator $\hat{U}_k$, $\hat{\rho}_k = \hat{U}_k \, \hat{\rho}_{k-1} \, \hat{U}_k^\dagger$. The k-th measurement process at $t_k = kT/n$, i.e., the preparation of the k-th probe in the state $|\phi\rangle_{p,k}$, the interaction of this probe with the measured system $\hat{U}_M(\hat{q})$ and the result of the measurement $\tilde{q}_k$, which corresponds to the state of the probe after the measurement $|\tilde{q}_k\rangle_{p,k}$, is described by the probability-amplitude operator [2]

$$\hat{Y}_k \equiv \hat{Y}(\hat{q}, \tilde{q}_k) = {}_{p,k}\langle \tilde{q}_k | \hat{U}_M(\hat{q}) | \phi \rangle_{p,k} \quad . \tag{1}$$

The state of the system after the k-th measurement is $\hat{\rho}_{a,k} = P(\tilde{q}_k)^{-1} \, \hat{Y}_k \, \hat{\rho}_{b,k} \, \hat{Y}_k^\dagger$, where $\hat{\rho}_{b,k}$ is the state of the system before this measurement and where $P(\tilde{q}_k) = \mathrm{Tr}_s \, [\hat{Y}_k \, \hat{\rho}_{b,k} \, \hat{Y}_k^\dagger]$, with the trace over the operators of the measured system, is the probability to obtain the measurement result $\tilde{q}_k$. Note that after the measurement the single system is in a pure state, assuming that before the measurement both the system and the probe are in pure states.

The density operator which describes the single system at $t = T$, after the n-th measurement is

$$\hat{\rho}_S(\tilde{q}_1, \ldots, \tilde{q}_n) = P_S(\tilde{q}_1, \ldots, \tilde{q}_n)^{-1} \, \hat{Y}_n \, \hat{U}_n \ldots \hat{Y}_1 \, \hat{U}_1 \, \hat{\rho}_0 \, \hat{U}_1^\dagger \, \hat{Y}_1^\dagger \ldots \hat{U}_n^\dagger \, \hat{Y}_n^\dagger \quad , \tag{2}$$

where the probability density of obtaining the associated series of measurement results, $(\tilde{q}_1, \ldots, \tilde{q}_n)$, is

$$P_S(\tilde{q}_1, \ldots, \tilde{q}_n) = \mathrm{Tr}_s\,[\hat{Y}_n\,\hat{U}_n \ldots \hat{Y}_1\,\hat{U}_1\,\hat{\rho}_0\,\hat{U}_1^\dagger\,\hat{Y}_1^\dagger \ldots \hat{U}_n^\dagger\,\hat{Y}_n^\dagger] \ . \tag{3}$$

The ensemble averaged effect of the series of measurements on the single quantum system is described by

$$\hat{\rho}_S = \int \prod_{k=1}^{n} d\tilde{q}_k\, P_S(\tilde{q}_1, \ldots, \tilde{q}_n)\, \hat{\rho}_S(\tilde{q}_1, \ldots, \tilde{q}_n) \ . \tag{4}$$

Note that while the density operator $\hat{\rho}_S(\tilde{q}_1, \ldots, \tilde{q}_n)$ corresponds to a pure state, $\hat{\rho}_S$ corresponds to a mixture of states.

Misra and Sudarshan showed [1] that when the measured observable $\hat{q}$ has a discrete eigenvalue spectrum, with the initial state of the system $\hat{\rho}_0$ being one of the corresponding eigenstates, then in the limit of a continuous and precise measurement process, where the number of measurements goes to infinity, $n \to \infty$, and the time interval between each two measurements goes to zero, $T/n \to 0$, the time evolution of the system is frozen, i.e., $\hat{\rho}_S = \hat{\rho}_0$. It was later shown (see, e.g., [2-5]) that outside the limit considered by Misra and Sudarshan, i.e., in the case of a series of approximate measurements with a finite time interval between the measurements, $T/n \neq 0$, the ensemble averaged effect of the measurements on the time evolution of the system is equivalent to the effect of dephasing, where the coherences, i.e., the non-diagonal elements, of $\hat{\rho}_S$ decrease with an increase in the number of measurements or in the precision of the measurements. The freezing of the time evolution of the ensemble due to a continuous and precise measurement process is a special case of this dephasing phenomenon. It is, therefore, impossible to distinguish between the effect of a series of measuements and the effect of dephasing, when analyzing an ensemble of systems, where both effects give the same predictions regarding the time evolution of the ensemble (see, e.g., [6-8]). This is not the case when one is analyzing the effect of the process of a series of measurements on the time evolution of a single quantum system, i.e., the quantum Zeno effect of a single system.

The indetermination of the quantum state of a single quantum system.– Recently we showed that the quantum state which describes a single system cannot be determined from the results of a series of measurements performed on the single system [9,10]. Consider a series of n measurements of the observables $(\hat{q}_1, \ldots, \hat{q}_n)$ performed on a single quantum system, where the time evolution of the system in between successive measurements is neglected. The initial state of the system is described by the density operator $\hat{\rho}_0$, as before, and the k-th measurement of the observable $\hat{q}_k$ with the measurement result $\tilde{q}_k$ is described by the probability-amplitude operator [2]

$$\hat{Z}_k \equiv \hat{Z}(\hat{q}_k, \tilde{q}_k) = {}_{p,k}\langle \tilde{q}_k | \hat{U}_M(\hat{q}_k) | \phi \rangle_{p,k} \ . \tag{5}$$

The density operator which describes the system after the n-th measurement is

$$\hat{\rho}_H(\tilde{q}_1, ..., \tilde{q}_n) = P_H(\tilde{q}_1, ..., \tilde{q}_n)^{-1} \, \hat{Z}_n ... \hat{Z}_1 \, \hat{\rho}_0 \, \hat{Z}_1^\dagger ... \hat{Z}_n^\dagger \ , \tag{6}$$

where the probability density of obtaining the associated series of measurement results, $(\tilde{q}_1, \cdots, \tilde{q}_n)$, is

$$P_H(\tilde{q}_1, \cdots, \tilde{q}_n) = \mathrm{Tr}_s \, [\hat{Z}_n ... \hat{Z}_1 \, \hat{\rho}_0 \, \hat{Z}_1^\dagger ... \hat{Z}_n^\dagger] \ . \tag{7}$$

We showed that the statistics of the series of measurement results, $P_H(\tilde{q}_1, ..., \tilde{q}_n)$, could give estimates of the initial expectation values of the measured observables, but could not give estimates of the initial uncertainties associated with these observables. This is beacuse each time a measurement is performed, the quantum state of the system changes in accordance with the measurement result, as a consequence of the generalized projection postulate. Unlike the results of measurements performed each on a different system in an ensemble of identical systems, which are independent of each other, the result of a measurement performed on a single system would influence the statistics of the results of all future measurements. A series of measurements of a single quantum system corresponds to a measurement of the observables, and do not constitute a determination of the quantum state associated with the single system. We showed, therefore, that the determination of the a-priori unknown quantum state of a single system using a series of measurements is impossible due to the generalized projection postulate.

The quantum Zeno effect of a single system: The formulations in the Schrödinger and the Heisenberg pictures.– Let us now show that this effect of the indetermination of the unknown quantum state of a single system is equivalent to the quantum Zeno effect of a single system. The quantum Zeno effect of a single system is the effect of a series of quantum measurements on a single quantum system with reference to the corresponding series of measurement results. The effect of the series of measurements on the free time evolution of the single system is described by $\hat{\rho}_S(\tilde{q}_1, \cdots, \tilde{q}_n)$ of Eq. (2). The information about the free time evolution of the single system which can be obtained from the measurement results is described by $P_S(\tilde{q}_1, \ldots, \tilde{q}_n)$ of Eq. (3). Using the unitarity of the time evolution operators, $\hat{U}_k \hat{U}_k^\dagger = \hat{I}$, where $\hat{I}$ is the identity operator, with

$$\hat{Z}_k = \hat{U}_1^\dagger \ldots \hat{U}_k^\dagger \hat{Y}_k \hat{U}_k \ldots \hat{U}_1 \ , \tag{8}$$

the probability density $P_S(\tilde{q}_1, \ldots, \tilde{q}_n)$ can be rewritten as

$$\begin{aligned} P_S(\tilde{q}_1, \ldots, \tilde{q}_n) &= \mathrm{Tr}_s\, [\hat{Y}_n \hat{U}_n \ldots \hat{Y}_1 \hat{U}_1 \hat{\rho}_0 \hat{U}_1^\dagger \hat{Y}_1^\dagger \ldots \hat{U}_n^\dagger \hat{Y}_n^\dagger] \\ &= \mathrm{Tr}_s\, [\hat{Z}_n \ldots \hat{Z}_1 \hat{\rho}_0 \hat{Z}_1^\dagger \ldots \hat{Z}_n^\dagger] \\ &= P_H(\tilde{q}_1, \ldots, \tilde{q}_n) \ . \end{aligned} \tag{9}$$

Note that $P_H(\tilde{q}_1, \ldots, \tilde{q}_n)$ describes the probability density to obtain the series of results $(\tilde{q}_1, \ldots, \tilde{q}_n)$ in the series of measurements $(\hat{Z}_1, \ldots, \hat{Z}_n)$ of the single system, with no time evolution in between successive measurements. Since the statistics $P_S(\tilde{q}_1, \ldots, \tilde{q}_n)$ and $P_H(\tilde{q}_1, \ldots, \tilde{q}_n)$ are equal, the physical processes that they describe are equivalent.

While $\hat{\rho}_S$ is the density operator of the system at $t = T$ in the Schrödinger picture, where the time evolution is attributed to the state of the system, $\hat{\rho}_H$ is the density operator of this system in the Heisenberg picture, where the time evolution is attributed to the observables associated with the system, and therefore also to the probability-amplitude operators. In fact, $\hat{\rho}_H$ could be viewed as the state of the system at $t = T$ in the reference frame which evolves in time with the system. Indeed, in the specific case of back-action evading measurements, where $[\hat{Y}_k, \hat{q}] = [\hat{U}_M(\hat{q}), \hat{q}] = 0$, the probability-amplitude operators in the Heisenberg picture describe successive measurements of the time evolving observable

$$\hat{q}_k = \hat{U}_1^\dagger \ldots \hat{U}_k^\dagger \hat{q} \hat{U}_k \ldots \hat{U}_1 \ . \tag{10}$$

The Schrödinger picture and the Heisenberg picture describe equivalent physical phenomenon. In the Schrödinger picture: $\hat{\rho}_S$ describes the effect of a series of measurements of the same observable on the free time evolution of a single system, and $P_S(\tilde{q}_1, \ldots, \tilde{q}_n)$ describes the information about the free time evolution of the system which is contained in the measurement results. The Schrödinger picture, therefore, describes the quantum Zeno effect of a single system. In the Heisenberg picture: $\hat{\rho}_H$ describes the "inverse" quantum Zeno effect of a single system [12], i.e., the "time evolution" of a single quantum system due to a series of measurements of time varying observables, and $P_H(\tilde{q}_1, \ldots, \tilde{q}_n)$ describes the indetermination of the unknown quantum state of a single system. The quantum Zeno effect of a single system is equivalent to this effect of the indetermination of the unknown quantum state of a single system.

Presenting the quantum Zeno effect of a single system in the Heisenberg picture, it is obvious that the only changes in the state of the system arise from the measurement process, due to the generalized projection postulate. The changes in the state of the single system, or the measurement induced "time evolution" of the system, are the same changes which prohibit the determination of the quantum state of the single system using the measurement results. The quantum Zeno effect of a single system can be thought of as a "screening" effect: The series of measurements change the time evolution of the single system in such a way that the initial quantum state of the system could not be determined from the measurement results.

Conclusions.– We have shown that the quantum Zeno effect of a single system and the indetermination of the unknown quantum state of a single system are two descriptions, in the Schrödinger and the Heisenberg pictures respectively, of the same phenomenon: The effect of a series of measurements on the state of the single system, as a consequence of the generalized projection postulate. While in the Heisenberg picture the series of measurement results cannot give full information about the initial quantum state of the system, in the Schrödinger picture these results cannot give full information about the free time evolution of the single system. The quantum Zeno effect of a single system is, therefore, a true quantum measurement effect.

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