Visualizing Robustness of Critical Points for Time-Varying Vector Fields

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Motivation

Analyzing critical points and their temporal evolutions plays a crucial role in extracting and tracking features of vector fields. A key challenge in understanding the behavior of a vector field is to quantify the stability of its features: more stable features may represent more important phenomena or vice versa.

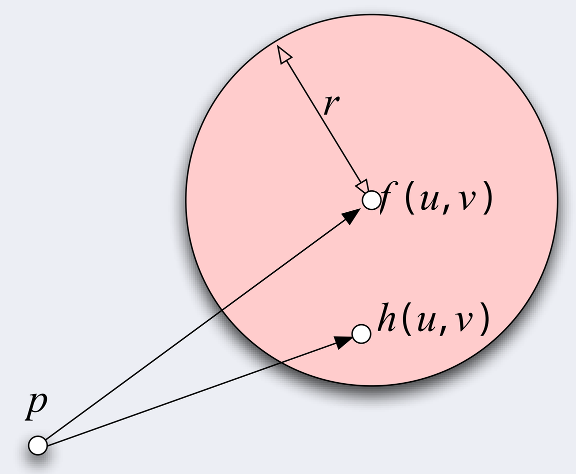
Contribution I

We introduce the topological notion of robustness, which allows us to quantify rigorously the stability of each critical point, with respect to perturbations of the vector field. Intuitively, the robustness of a critical point is the minimum amount of perturbation necessary to cancel it, measured under an appropriate metric.

Give two continous vectof field: $f, h : \mathbb{R}^2 \to \mathbb{R}^2$

A continous vector field h is a r **-purturbation** of f if

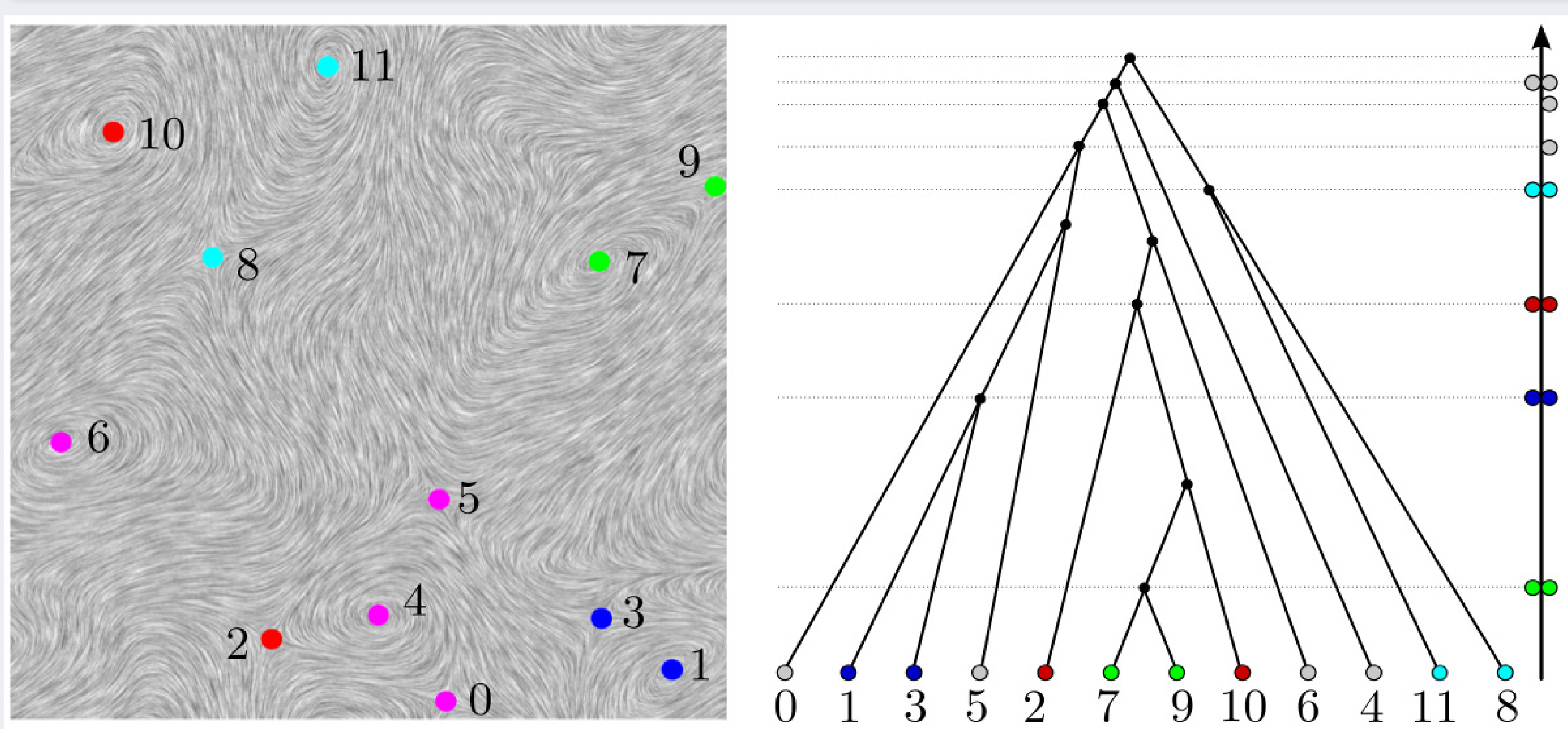
$$d(f, h) = \sup_{x \in \mathbb{R}^2} \|f(x) - h(x)\|_2 \le r$$



Given a continuous 2D vector field $f : \mathbb{R}^2 \to \mathbb{R}^2$

We can define its Euclidean norm: $f_0(x) = \|f(x)\|_2$

We compute the (augmented) merge tree of function f_0 while keeping track of the **degrees** (sum of their enclosed critical point indices) of the components in its sublevel sets. The **static robustness** of a critical point is the height of its lowest degree zero ancestor in the merge tree.

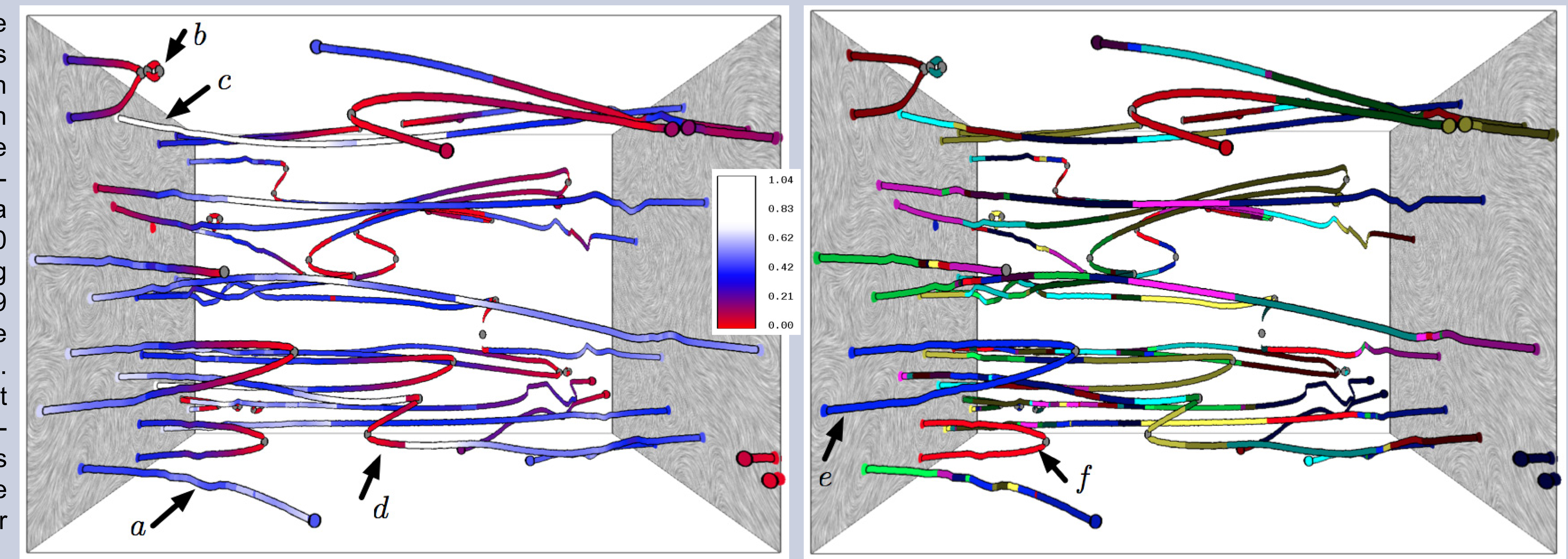


Key property (Critical Point Cancellation): Suppose a critical point x of f has static robustness r. Let C be the connected component of f_0 at level $(r + \delta)$ containing x, for an arbitrarily small $\delta > 0$ Then, there exists an $(r + \delta)$-perturbation h of f, such that $h^{-1}(0) \cap C = \emptyset$ and $h = f$ except possibly within the interior of C.

Contribution II

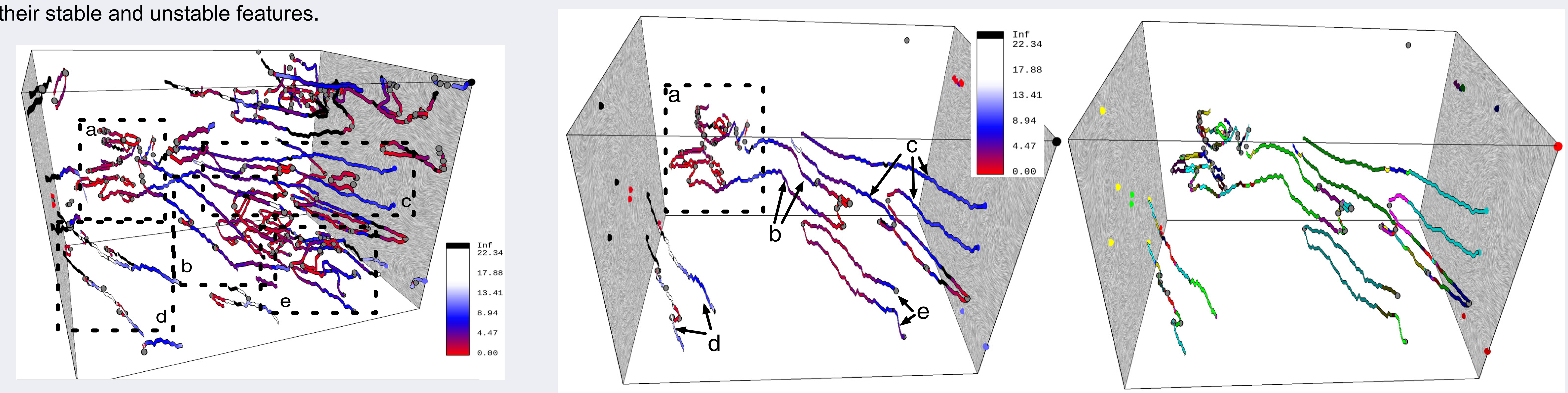
We design an algorithm to assign robustness to critical points for both stationary and time-varying vector fields. The computed assignments give us a structural description of the vector field. In particular, under the time-varying setting, they help us understand the temporal stability of critical points and how they migrate and interact with one another.

A dataset is taken from the simulation of homogenous charge compression ignition (HCCI) engine combustion [Hawkes et. al. 06]. The domain has periodic boundary, and is represented as a regular grid of resolution 640 × 640. The 2D time-varying vector field consists of 299 time-steps, each with a time interval of 10^(-5) seconds. Left: robustness assignment along critical point trajectories. Right: robustness partners colored by unique values showcasing partner switches.



Contribution III

We introduce a new visualization tool that enables interactive exploration of robustness of critical points for time-varying vector fields. To demonstrate the practicality of our theories and techniques, we use this tool on several real world datasets involving combustion and ocean eddy simulations. We obtain some key insights regarding their stable and unstable features.



We explore the simulation of global oceanic eddies [Maltrud et. al. 10] for 350 days (therefore 350 time slices) of the year 2002. We use the top layer of the 3D simulation which is represented as a 2D time-varying vector field of resolution 3600 × 2400. Here we show a tile extracted from this simulation data, representing the flow in the central Atlantic Ocean (60 × 60).