Part Chapter 2: Cameras “Lenses”

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(slides modified from Marc Pollefeys, UNC Chapel Hill)
Snell’s law

$$n_1 \sin \alpha_1 = n_2 \sin \alpha_2$$
Paraxial (or first-order) optics

Snell’s law: \( n_1 \sin \alpha_1 = n_2 \sin \alpha_2 \)

Small angles: \( n_1 \alpha_1 \approx n_2 \alpha_2 \)

\[ \alpha_1 = \beta_1 + \gamma \approx \frac{h}{d_1} + \frac{h}{R} \]

\[ \alpha_2 = \gamma - \beta_2 \approx \frac{h}{R} - \frac{h}{d_2} \]
Thin Lenses

spherical lens surfaces; incoming light ± parallel to axis; thickness << radii; same refractive index on both sides

\[ \frac{n_1}{d_1} + \frac{n_2}{d_2} = \frac{n_2 - n_1}{R} \]

\[ \frac{1}{Z} + \frac{n}{Z^*} = \frac{n-1}{R} \]

\[ \frac{n}{Z^*} + \frac{1}{Z'} = \frac{1-n}{R} \]

\[ \frac{n-1}{R} - \frac{1-n}{Z} = \frac{1}{Z'} \]

\[ \frac{1}{z'} - \frac{1}{z} = \frac{1}{f} \]

and

\[ f = \frac{R}{2(n-1)} \]
Thin Lenses

\[
\begin{align*}
x' &= \frac{z'}{z} x \\
y' &= \frac{z'}{z} y
\end{align*}
\]

where

\[
\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}
\]

and

\[
f = \frac{R}{2(n-1)}
\]

http://www.phy.ntnu.edu.tw/java/Lens/lens_e.html
Thick Lens
The depth-of-field
The depth-of-field yields

\[ Z_o^- = f \left( \frac{|Z_i^-|}{|Z_i^-| - f} \right) \]

\[ Z_o^- = f \frac{d Z_o}{b Z_o + f (d - b)} \]

\[ \Delta Z_o^- = Z_o - Z_o^- = \frac{Z_o (Z_o - f)}{Z_o + f d / b - f} \]

Similar formula for \( \Delta Z_o^+ = Z_o^+ - Z_o \)
The depth-of-field decreases with $d$, increases with $Z_0$

strike a balance between incoming light and sharp depth range
Deviations from the lens model

3 assumptions:

1. all rays from a point are focused onto 1 image point
2. all image points in a single plane
3. magnification is constant

deviations from this ideal are *aberrations*
Aberrations

2 types:

1. geometrical
2. chromatic

**geometrical**: small for paraxial rays

study through 3\textsuperscript{rd} order optics \(\sin(\theta) \approx \theta - \frac{\theta^3}{6}\)

**chromatic**: refractive index function of wavelength
Geometrical aberrations

- spherical aberration
- astigmatism
- distortion
- coma

aberrations are reduced by combining lenses
Spherical aberration

rays parallel to the axis do not converge

outer portions of the lens yield smaller focal lengths
Astigmatism

Different focal length for inclined rays
Distortion

magnification/focal length different for different angles of inclination

pincushion (tele-photo)

barrel (wide-angle)

Can be corrected! (if parameters are know)
Coma

point off the axis depicted as comet shaped blob
Chromatic aberration

rays of different wavelengths focused in different planes

cannot be removed completely

sometimes *achromatization* is achieved for more than 2 wavelengths
Lens materials

reference wavelengths:

\[ \lambda_F = 486.13 \text{nm} \]
\[ \lambda_d = 587.56 \text{nm} \]
\[ \lambda_C = 656.28 \text{nm} \]

lens characteristics:

1. refractive index \( n_d \)
2. Abbe number \( V_d = (n_d - 1) / (n_F - n_C) \)

typically, both should be high
allows small components with sufficient refraction

notation: e.g. glass BK7(517642)
\( n_d = 1.517 \) and \( V_d = 64.2 \)
Lens materials

additional considerations:
humidity and temperature resistance, weight, price,...
Vignetting