The Unfitted Discontinuous Galerkin Method for Solving the EEG Forward Problem:

A Second Order Study

Andreas Nüßing[1,2,3], Carsten H. Wolters[1], Heinrich Brinck[2], Christian Engwer[3]

[1]Institute for Biomagnetism and Biosignal Analysis, University of Münster, Germany
[2]Westphalian University of Applied Sciences, Germany
[3]Institute for Computational and Applied Mathematics, University of Münster, Germany

Introduction

The unfitted discontinuous Galerkin finite element method (UDG-FEM) as a new method for solving the electroencephalography forward problem has been introduced in [1]. It showed an overall good accuracy when compared to competitive methods on conforming meshes, while providing a less complex simulation pipeline. In addition, it derived properties of the discontinuous Galerkin finite element method (DG-FEM), such as for example conservation properties on a discrete level. Locally, the discrete model employs polynomial basis functions. In [1], results where presented only for linear polynomials.
This study focusses on comparing the results for UDG-FEM with first order functions to the ones obtained by using quadratic polynomials. As UDG-FEM uses level set functions for the representation of the model geometry, a better geometric representation of the smooth surfaces can be obtained, which can be beneficial for a second order method.

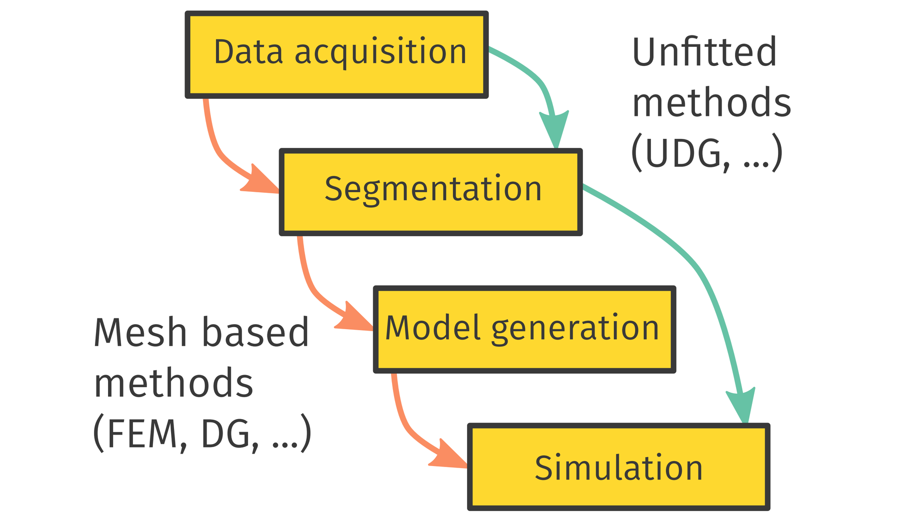

Figure 1: General pipeline of a patient specific simulation

Unfitted Discontinuous Galerkin

UDG-FEM is a method for discretizing partial differential equations, in this case the Poisson equation $\nabla \cdot \sigma \nabla u = f$

Unfitted

The computational mesh does not resolve the geometry. The latter is given as level sets. The elements of the mesh are restricted to the different domains. These restrictions are called cut cells.

Discontinuous Galerkin

Galerkin method similar to the finite element method [2]. Allow discontinuities of the potential between elements. Consider continuity in the weak formulation.

$$a(u,v) = \int_\Omega \nabla u \cdot \nabla v dx - \int_\Gamma ([\![u]\!] \cdot \{\nabla v\} + \{\nabla u\} \cdot [\![v]\!]) dx + \frac{\eta}{h} \int_\Gamma [\![u]\!] \cdot [\![v]\!] dx$$

The Unfitted Mesh

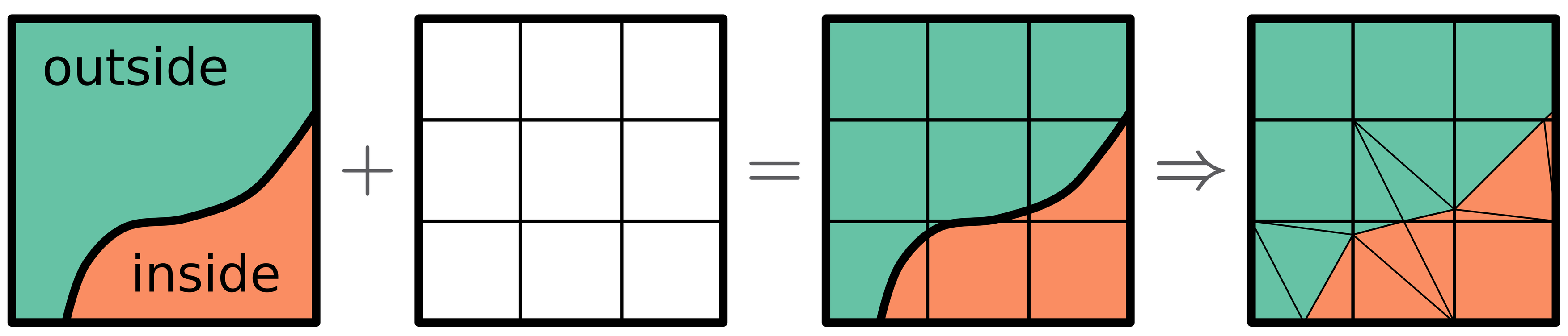

Figure 2: Construction of the unfitted mesh for a level set on a 2D grid

Local Polynomials

On each cut cell, the potential is approximated by polynomial functions of maximal degree $k \in \mathbb{N}$. By employing polynomials with higher degrees, a better approximation can be achieved. For first order and second order polynomials, the basis of the local polynomial space consists of 8 and 27 basis functions respectively.

Realistic Head Model

We test the first order UDG-FEM for the EEG forward problem with an auditory source in a 4 compartment isotropic head model. We use the same conductivities as for the sphere model on the right. The level sets are generated artificially from a surface based segmentation.

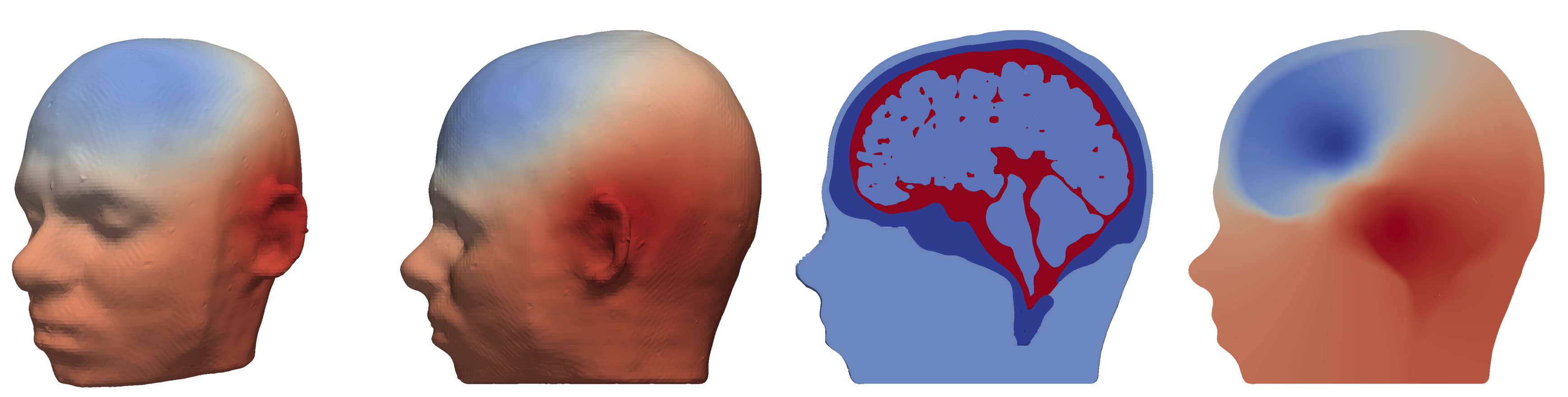

Figure 5: The potential u at the scalp surface (two left images) and the conductivities and the potential on a saggital slice (two right images)

DUNE Framework

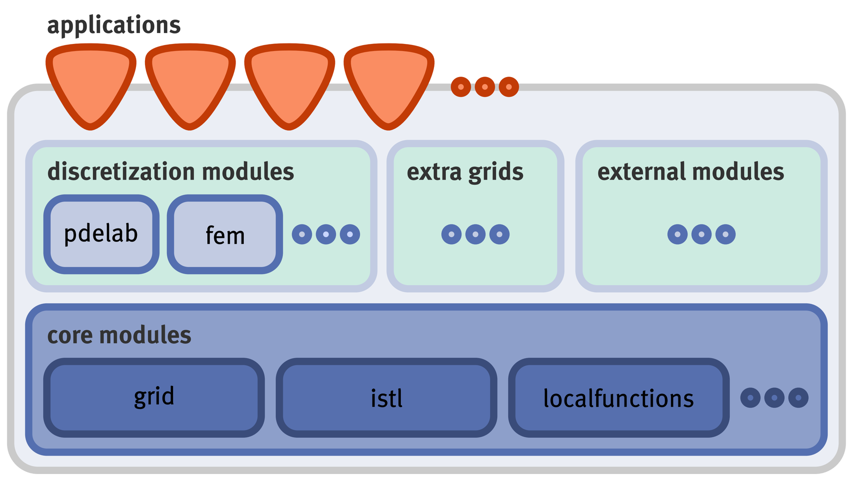

Figure 3: The modular structure of the DUNE library

DUNE = Distributed and Unified Numerics Environment
http://www.dune-project.org

- C++ open source library for the discretization and solution of partial differential equations
- modular structure, general interfaces

Multilayer Sphere Model

We evaluate first and second order UDG-FEM using a *partial integration* (PI) approach [3] for the EEG forward problem on a *multilayer sphere model*. We use 4 layers with conductivities from outer to inner compartment: 0.43, 0.01, 1.79 and 0.33 S/m. We generate 1000 radial dipoles and 1000 tangential dipoles on each of 10 eccentricities in the inner compartment and measure the potential at 200 surface electrodes. The potential is compared to the analytic solution and the error is measured as:

$$\text{RDM\%}(U_{num}, U_{ana}) = 50 \cdot \left\| \frac{U_{num}}{\|U_{num}\|} - \frac{U_{ana}}{\|U_{ana}\|} \right\| \in [0, 100] \qquad \text{MAG\%}(U_{num}, U_{ana}) = 100 \cdot \left(\frac{\|U_{num}\|}{\|U_{ana}\|} - 1\right) \in [-100, \infty)$$

Both measures have an optimal value of 0. We compare the UDG method with first order polynomials UDG(1) to the UDG method with second order polynomials UDG(2). The mesh size is scaled so that both method have approximately the same number of degrees of freedom (1009k DOFs for UDG(1) and 963k for UDG(1)).

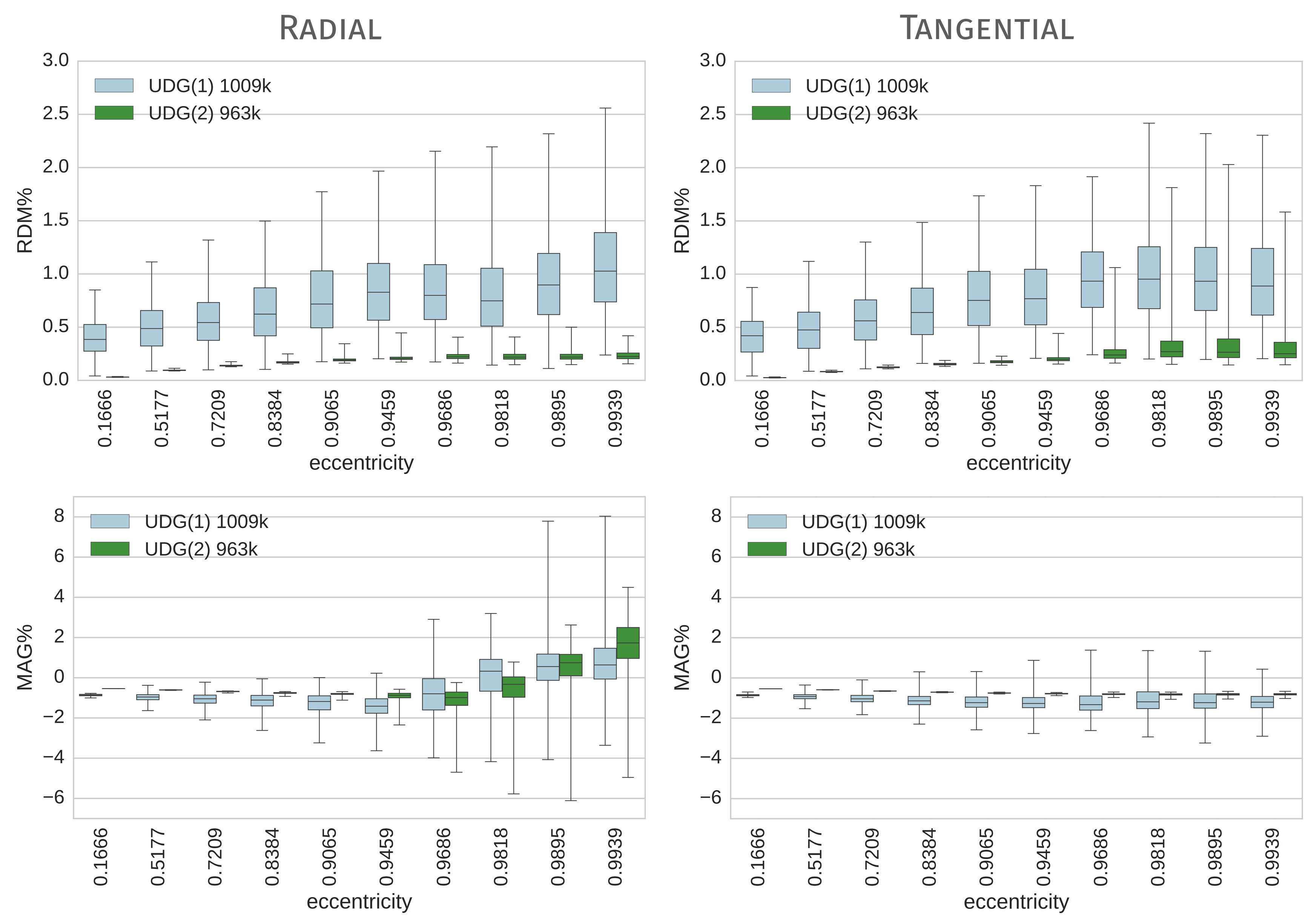

Figure 4: Comparison of the first (light blue) and second (green) order UDG-FEM model: RDM% (upper row) and MAG% (lower row) errors for radial (left column) and tangential (right column) sources.

Conclusion and Outlook

We presented a study of the unfitted discontinuous Galerkin method for solving the EEG forward problem with second order polynomials. Second order polynomials could achieve a better accuracy while using the same number of degrees of freedom compared to linear polynomials.

Contact

a.nuessing@wwu.de

References

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