Introduction

Patient specific simulation is a useful tool in different areas of brain research. A main method in use for such simulations is the finite element method (FEM) [1]. It uses a volume tessellation and can treat models with anisotropic conductivities or complex domains with fine structures or holes. While hexahedral meshes reduce the complexity of a simulation pipeline, tetrahedral meshes offer a better geometric approximation. However, the automatic construction of a high quality triangulation from quasi-non-invasive magnetic resonance imaging (MRI) is a difficult task.

The unfitted discontinuous Galerkin method (UDG) [2] avoids these problems by using a structured mesh that does not resolve the geometry. It works directly on a level set segmentation and includes the geometry in its mathematical formulation. In addition it maintains conservations laws on a discrete level.

Unfitted Discontinuous Galerkin

The UDG method is a method for discretizing partial differential equations, in this case the Poisson equation \(\nabla \cdot \sigma \nabla u = f\).

**Unfitted**

The computational mesh does not resolve the geometry. The latter is given as level sets. The elements of the mesh are restricted to the different domains.

**Discontinuous Galerkin**

Galerkin method similar to the finite element method. Allow discontinuities of the potential between elements. Consider continuity in the weak formulation.

\[
\alpha(u, v) = \int_{\Omega} \nabla u \cdot \nabla v \, dx - \int_{\Omega} (|u| + |v|) \cdot (|u| + |v|) \, dx + \frac{\eta}{\beta} \int_{\Gamma} |u| \cdot |v| \, ds
\]

EEG Forward Problem

We evaluate the UDG method using a partial integration (PI) approach for the EEG forward problem on a multilayer sphere model. We use 4 layers with conductivity from outer to inner compartment: 0.33, 0.0042, 1.79 and 0.33 S/m. We generate 500 random dipoles on each of 15 eccentricities in the inner compartment and measure the potential at 200 surface electrodes. The potential is compared to the analytic solution and the error is measured as:

\[
\text{RDM} \% \left( \text{U}_{\text{num}}, \text{U}_{\text{ana}} \right) = 50 \left( \frac{\left\| \text{U}_{\text{num}} - \text{U}_{\text{ana}} \right\|}{\left\| \text{U}_{\text{ana}} \right\|} \right) \in [0, 100]
\]

Both measures have an optimal value of 0. The method is evaluated on grids with different element diameter and compared to the DG method on a conforming mesh of similar size.

Conclusion and Outlook

We presented first promising results of the application of the UDG method in EEG and tES modeling. It shows proper convergence behavior when reducing the element size. We can observe higher (RDM) or at least comparable (MAG) accuracy to a DG method on a conforming mesh with a similar number of unknowns.

Besides the PI approach, we are deriving more accurate source models for the UDG method. For the application to realistic head models, we are currently investigating a smoothing procedure based on constrained mean curvature (FEM) [1]. It uses a volume tessellation and can treat models with anisotropic conductivities or complex domains with fine structures or holes. While hexahedral meshes reduce the complexity of a simulation pipeline, tetrahedral meshes offer a better geometric approximation. However, the automatic construction of a high quality triangulation from quasi-non-invasive magnetic resonance imaging (MRI) is a difficult task.

The modular structure of the DUNE library

**DUNE Framework**

DUNE = Distributed and Unified Numerics Environment

**References**

