A variational approach for the correction of field-inhomogeneities in EPI sequences

Janine Olesch\textsuperscript{a,e}, Lars Ruthotto\textsuperscript{b}, Harald Kugel\textsuperscript{c}, Stefan Skare \textsuperscript{d}, Bernd Fischer\textsuperscript{a} and Carsten H. Wolters\textsuperscript{b}

\textsuperscript{a}Institute of Mathematics, University of Lübeck, Germany; \\
\textsuperscript{b}Institute for Biomagnetism and Biosignalanalysis, University of Münster, Germany; \\
\textsuperscript{c}Department of Clinical Radiology, University of Münster, Germany; \\
\textsuperscript{d}Lucas Center, Department of Radiology, Stanford University, 1201 Welch Road, Stanford, CA 94305-5488, USA; \\
\textsuperscript{e}Graduate School for Computing in Medicine and Life Sciences, University of Lübeck, Germany

ABSTRACT

A wide range of medical applications in clinic and research exploit images acquired by fast magnetic resonance imaging (MRI) sequences such as echo-planar imaging (EPI), e.g. functional MRI (fMRI) and diffusion tensor MRI (DT-MRI). Since the underlying assumption of homogeneous static fields fails to hold in practical applications, images acquired by those sequences suffer from distortions in both geometry and intensity. In the present paper we propose a new variational image registration approach to correct those EPI distortions. To this end we acquire two reference EPI images without diffusion sensitizing and with inverted phase encoding gradients in order to calculate a rectified image. The idea is to apply a specialized registration scheme which compensates for the characteristic direction dependent image distortions. In addition the proposed scheme automatically corrects for intensity distortions. This is done by evoking a problem dependent distance measure incorporated into a variational setting. We adjust not only the image volumes but also the phase encoding direction after correcting for patients head-movements between the acquisitions. Finally, we present first successful results of the new algorithm for the registration of DT-MRI datasets.

Keywords: Image registration, EPI distortion correction, MRI, DTI, fMRI

1. INTRODUCTION

EPI sequences that are sensitive to field inhomogeneities are widely used in medical research and in clinical applications, e.g., in DT-MRI and fMRI. All those applications require undistorted images with respect to both geometry and intensity. For example diffusion tensors computed from geometrical and intensity distorted images lack reliability in two ways. In our experiments, the locations of the calculated tensors differed by more than 10 mm and the intensity changes were found up to 100%. The high non-linearity of the distortions causes the registration of EPI to be a challenging task. However, a physical model for the occurring distortions exists which states that the field inhomogeneity deforms the image along the phase-encoding and slice-selection direction.\textsuperscript{1–3}

The deformation causes also a change in intensities which can be measured by the determinant of its Jacobian. As Chang and Fitzpatrick\textsuperscript{1} pointed out, the effects of the inhomogeneity when inverting the phase-encoding gradient are reversed along that direction. See Figure 1 for two axial and sagittal exemplary slices of two distorted EPI-scans $M_1$ and $M_2$, where $M_2$ was measured with reversed phase-encoding gradient.

One may distinguish between two different classes of correction approaches for EPI distortions. The first ones try to measure the field map itself and subsequently correct the acquired image.\textsuperscript{4,5} However, since the static field

Further author information:
(Send correspondence to the first two authors who equally contributed)
Janine Olesch: olesch@math.uni-luebeck.de
Lars Ruthotto: lars.ruthotto@uni-muenster.de
inhomogeneities are caused not only by imperfections in the magnet but also by spatial varying susceptibility of the object being imaged \footnote{6} the field maps vary from subject to subject. On the other hand there are image-based approaches based on two EPI-scans of the same object with inverted phase-encoding gradients. This approaches use the physical model and the CF-methodology that was developed and validated by Chang and Fitzpatrick in.\footnote{1} Weiskopf et. al\footnote{7} applied these methods to fMRI data and corrected them in real-time. As they consider each column of voxels individually the resulting deformation field is typically non-smooth. Moreover the accuracy relies heavily on the correct detection of edges.\footnote{7} To achieve smoothness, studies conducted by Skare and Andersson\footnote{3} and Tao et. al\footnote{6} modeled the deformation as a linear combination of B-splines or as a solution of a partial differential equation (PDE). Evaluating B-Spline basis functions however is computationally expensive and is not a guarantee for diffeomorphic displacement fields. In this work we follow a variational approach using a discretize-optimize strategy. The underlying idea is to combine the smoothness and regularity of the latter two studies\footnote{3, 6} with the speed of the first two.\footnote{1, 7} Thanks to the variational setting, a specification of basis functions is not necessary. That is, the deformation may freely move voxels along the phase-encoding direction. This not only increases the flexibility of the transformation but also drastically reduces computation time. Its smoothness is controlled by introducing an elastic regularization term. To further improve robustness against noise and to increase speed a multi-level registration technique is introduced and successfully applied.
2. METHODS

In the following we present a variational approach to solve the problem of field inhomogeneities in EPI sequences following the idea of the CF-method. For two EPI-scans $M_1, M_2$ with inverted phase encoding gradients, geometric and intensity deformations due to field inhomogeneities occur in both images along the phase encoding directions $v$ and its inverse $-v \in \mathbb{R}^3$. Figure 1 shows exemplary two corresponding slices. The inversion of the phase encoding gradient leads to interesting distortions in EPI-scans. While the distortions in Figure 1 (a) stretch the fronto-central area and the fronto-lateral areas are heavily edged, we observe the situation vice versa in (b) for the inverted phase encoding gradient. A usual registration method would register one moving image to a fixed image. In our special case we know, that both images are distorted in two inverse directions. Instead of having one moving and one fixed image, the idea is to find a deformation that is applied in opposite directions on both moving images to make them equal. Additionally we learn from Chang and Fitzpatrick and Skare and Andersson, that we can derive the intensity modulation of a voxel using the determinant of the Jacobian of the transformation. So, the directionally transformed and intensity modulated image $M_i$ can be written as

\[
\tilde{M}(d; M_i, v) := M_i(x + d(x) v)(1 + \langle \nabla d(x), v \rangle), \quad i = 1, 2.
\]

Note that $M_i$ is now evaluated in direction $v$ according to the deformation $d(x) \in \mathbb{R}$. The second part is the determinant of the Jacobian of the directional transformation after a simple calculation. To register both images, we need a distance measure to calculate their difference. Since both images are taken on the same device both moving images are comparable by the sum of squared differences measure (SSD)

\[
D(d; v) = \frac{1}{2} \int_{\Omega} \left( \tilde{M}(d; M_1, v) - \tilde{M}(d; M_2, -v) \right)^2 dx.
\]

In case of head-movements which are likely to happen during the acquisition process of $M_1$ and $M_2$, the assumption of the two opposite directions $v$ and $-v$ does no longer hold. In rigid correction was proposed to handle those artifacts simultaneously to the correction of the field inhomogeneities. We propose here the following extension to this approach. In a first step, in order to correct patients head movements, we register $M_1$ to $M_2$ rigidly and extract the rotational part $Q \in \mathbb{R}^{3 \times 3}$ of the resulting transformation matrix. This matrix stays fixed throughout the rest of the algorithm. The rotation of image $M_2$ by the orthogonal matrix $Q$ following this first step also affects the phase encoding direction in this image from $-v$ to $-Qv$. Although the rotation is expected to be minimal, even this effect should not be neglected since otherwise corrections along the true phase encoding direction $-Qv$ will be impossible. This extends the described distance measure (1) to

\[
D(d; v, Q) = \frac{1}{2} \int_{\Omega} \left( \tilde{M}(d; M_1, v) - \tilde{M}(d; M_2, -Qv) \right)^2 dx.
\]

To ensure smooth deformation fields we append an elastic regularizer to our registration problem. The elastic regularizer is given for our directional registration as

\[
S(d; v) = \frac{1}{2} \int_{\Omega} \sum_{\ell=1}^{3} \mu \| \nabla (d(x) v)_\ell \|^2 + (\mu + \lambda) \mathrm{div}^2 (d(x) v)) \, dx.
\]

Here $\lambda$ and $\mu \in \mathbb{R}^+$ are the so called Navier-Lamé constants, which control the elastic behavior of the deformation. The regularizer only needs to be evaluated for $d$ and $v$ but not $-Qv$ since both resulting deformation fields have identical smoothness. Adding the distance measure $D(d; v, Q)$ and the smoother $S(d; v)$ we yield the variational formulation of the registration setting. Find transformation parameters $d : \mathbb{R}^3 \rightarrow \mathbb{R}$ which solve the optimization problem

\[
\min_d \mathcal{J}(d; v, Q) = D(d; v, Q) + \alpha S(d; v)
\]

with a regularizing parameter $\alpha \in \mathbb{R}$.

When the optimal deformation $d$ is found we combine both corrected images $\tilde{M}(d; M_1, v)$ and $\tilde{M}(d; M_2, -Qv)$ by averaging both to the final result of the rectified image in order to improve signal-to-noise ratio.
The overall workflow is as follows. The first step is a rigid pre-registration based on the SSD-distance measure in order to find a suitable initial guess for the non-linear approach and to obtain the rotational component which is needed for the reorientation of $-v$. Next, we try to minimize the functional $J$. This is a tricky problem. To avoid local minima and to speed up the registration process we apply a multilevel-strategy. That is, we first calculate a solution at a low resolution level, then using this solution as a starting guess for the next finer level and so on. A proper solution on each individual level is obtained by invoking a discretize-optimize approach\textsuperscript{10} which means we discretize the functional $J$ and apply an optimization strategy. The discretization of the registration problem enables us to use the Gauss-Newton-method which is suitable for least squares problems.\textsuperscript{11} The search direction is calculated by applying the Conjugate Gradient (CG) method\textsuperscript{12} to the connected linear system. Finally, a strong Wolfe line search is applied to find a suitable step-length for the search-direction.

![Figure 2](image1.png)

(c) corrected $M_1$  
(d) corrected $M_2$

![Figure 2](image2.png)

(c) corrected $M_1$  
(d) corrected $M_2$

Figure 2. Axial and sagittal slices of the three step multi-level optimization and intensity modulation of initial $M_1$ and initial but rigidly reoriented $M_2$ (Figure 1).

3. RESULTS

We tested our algorithm on four EPI datasets of healthy subjects acquired on a Philips 3 Tesla scanner. The dimensions of the reconstructed matrix are $256 \times 256 \times 36$ and the intensities were scaled to a range between 0 and 255. The phase-encoding directions are initially parallel to the anterior-posterior direction. We performed the tests extending the FAIR (Flexible Algorithms for Image Registration) package\textsuperscript{13} for MATLAB R2008a on a Windows XP 32-bit machine with an Intel Core 2 duo P8400 (2 $\times$ 2.26 GHz) with 3 GB of RAM. We performed a three step coarse-to-fine multi-level optimization on grid-sizes of $32 \times 32 \times 5, 64 \times 64 \times 9$ and $128 \times 128 \times 18$. 
In the first step we rigidly corrected $M_1$ and $M_2$ for head-movement. The rigid correction resulted in a rotation of $[\phi_x, \phi_y, \phi_z] = [2.2e-3, 8.4e-3, -8.6e-3]$ which is small as expected. The other datasets were also hardly affected by head-movement. Choosing $\alpha = 80$, $\lambda = 0.1$, $\mu = 1.0$ the algorithm was able to correct geometric distortions of up to 1.6 cm without showing grid foldings, see Figure 4. The results of all intensity and geometrically corrected datasets are comparable to the one we show exemplary in Figure 2. Figure 3 shows the rectified result which is derived out of the averaged combination of $\tilde{M}(d; M_1, v)$ and $\tilde{M}(d; M_2, -Qv)$.

![Figure 3. Rectified result of images $M_1$ and $M_2$.](image)

(a) axial slice

(b) coronal slice
Figure 4. The left column shows the resulting deformation grids for $M_1$ and the right column shows the resulting deformation grids for $M_2$. (a) and (b) show exemplary two corresponding axial views of $M_1$ and $M_2$ resp. (c) and (d) show examples for corresponding sagittal views of the deformations grids.
The SSD was reduced by 71% and the algorithm was even able to reduce the highly non-linear distortions in the frontal area, where $M_2$ was stretched in the fronto-central but heavily edged in the fronto-lateral areas (see Figure 5). The overall computing time amounts to 270 seconds. In further tests using different tolerance levels we achieved much faster runtimes ($<70$ seconds) with a moderate loss of correction quality.

![Initial and final difference images](image)

*Figure 5. The absolute initial differences and the absolute differences after applying our proposed correction methods between $M_1$ and $M_2$.***

### 4. CONCLUSIONS

We have presented a new fast variational correction approach for EPI sequences based on image registration techniques equipped with a multi-level strategy. The algorithm corrects distortions caused by the field inhomogeneity with respect to geometry and intensity and is easily applicable to alternative tasks, like fMRT sequences. We corrected for head-movement in both the image volume and the phase-encoding direction. The approach was tested on unweighted DTI images on a standard PC and even there a fast and sound correction was achieved. Due to the use of a regularization term, a flexible deformation model and a fast optimization strategy EPI deformations can be corrected within very short time. The first very promising results highly encourage us to apply the algorithm on additional distortion-problems, to work on a detailed validation of the presented algorithm and a direct comparison to former approaches.

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### REFERENCES


