Sample Spce: sets whose elemets are the outcomes we are interested in.

Discrete random variable: a func that maps a sample to a discrete number

pmf: $p(a) = P(X=a)$ pmf: $R \to [0,1]$

Cumulative distribution function

$F(a) = P(X \leq a)$

△ Bernoulli distribution

$p_X(1) = P(X=1) = p$

$p_X(0) = P(X=0) = 1-p$ $X \sim Ber(p)$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

△ Binomial Distribution

$p_X(k) = P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$

$Bin(n, p)$

Geometric Distribution

Example: Student look for interview at career fair booths

At each booth.

$P(\text{off-campus interview invitation}) = p$

X: the number of companies that student get the first invitation.

$P(X=k) = P(\text{No invitation at } k-1 \text{ booths}) \cdot P(\text{invitation at } k)$

Definition: $P(X=k) = (1-p)^{k-1} \cdot p$

$\text{Geo}(p)$

Exer: Show $P(X>k) = (1-p)^k$

$$\begin{array}{ccccc} \text{Sampling Space}: \{ & S, & FS, & FFS, & FFFS, \dots\} \\ & \downarrow & \downarrow & \downarrow & \downarrow \\ \text{R.V. (mapping)} & 1 & 2 & 3 & 4 \end{array}$$

A series of visiting company as a single experiments

Each visit cam be a event. but since we are interested in # visits needed for the 1st interview. we define experiments this way.

This is like the HW1 B/w card. we can define drawing a card as an experiments, but computation more difficult. So we define drawing + Looking at top as a single experiment.

Exer 4.5

Throw a die until the sum exceeds 6,

X : # of throws , F(1) , F(2) , F(7)

Remember $F(a) = p(X \le a)$

$F(1) = p(X \le 1) = p(X = 1) = 0$

$F(2) = p(X \le 2) = p(X=2) = \frac{21}{6 \times 6}$

$$\left.\begin{array}{ll} 1: & 6 \\ 2: & 5, 6 \\ 3: & 4, 5, 6 \\ 4: & 3. 4. 5. 6 \\ 5: & 2. 3. 4. 5. 6 \\ 6: & 1. 2. 3. 4. 5. 6 \end{array}\right\} \text{21 ways}$$

$F(7) = p(X \le 7) = 1 - p(X > 7) = 1$

Exec 4.6 : Draw three times from $\{1, 2, 3\}$

$\bar{X} = (X_1 + X_2 + X_3)/3$

$\bar{X} = \frac{3}{3}$, (1, 1, 1)

$\bar{X} = \frac{4}{3}$, (1, 1, 2) : 3 ways

$\bar{X} = \frac{5}{3}$: (1, 2, 2) : 3 ways

113 : 3 ways

$\bar{X} = \frac{6}{3}$: 123 : 6 ways

222 : 1 ways

$\bar{X} = \frac{7}{3}$: 223 : 3 ways

133 : 3 ways

$\bar{X} = \frac{8}{3}$: 233 : 3 ways

$\bar{X} = \frac{9}{3}$: 333 : 1 ways

② Two draws are exactly 1

$$p(A) = \frac{3+3}{3 \times 3 \times 3}$$

Example (3): Geometric Distribution

Student visit career fair booth for interview

P: a invitation in a company booth.

well dressd: $p = 0.8$

badly dressed: $p = 0.1$

① pmf of # of companies before getting an invitation

$$p_X(k) = P(X=k) = (1-p)^{k-1} \cdot p$$

② Well dressed stu get invitation in first 3 visits

$$\underbrace{P(X \leq 3) = F(3)}_{\text{cdf}} = \underbrace{P(X=1) + P(X=2) + P(X=3)}_{\text{disjoint}}$$

$P(X=1) = p = 0.8$

$P(X=2) = (1-p)\,p = 0.2 \times 0.8 = 0.16$

$P(X=3) = (1-p)^2 p = 0.2^2 \times 0.8 = 0.032$

$p(X \leq 3) = 0.8 + 0.16 + 0.032 = 0.992$

③ Well addressed Not got interview in first 3 visits

$$P(X>3) = p(X=4) + p(X=5) + \cdots$$
$$= \sum_{k} p(X=k)$$
$$= \sum_{k=4}^{\infty} p(1-p)^{k-1}$$

$$= (1-p)^3 \cdot p \cdot \sum_{k'=0}^{\infty} (1-p)^{k'}$$

geometric sequence $a_n = ar^{n-1}$

$a \quad ar \quad ar^2 \quad \cdots$
$1 \quad (1-p) \quad (1-p)^2 \quad \cdots$
$\left.\right\} \begin{array}{l} a = 1 \\ r = (1-p) \end{array}$

b.c. $\displaystyle\sum_{k=0}^{n-1} ar^k = \frac{a(1-r^n)}{1-r}$

$$\sum_{k'=0}^{n} (1-p)^{k'} = \frac{1-(1-p)^n}{1-(1-p)} \rightarrow \frac{1}{p}$$

$\therefore\ p(X>3) = (1-p)^3 \cdot p \cdot \frac{1}{p} = (1-p)^3$

Simpler solution, since first 3 visits fail

$p(X>3) = (1-p)^3$

④ if a student got a interview invitation from his 4th visit. What is his probability of well dressed? (Assume well/badly addressed have equal prob)

→ a r.v. that map "well dressed" to a disurete number.

$$p(d=1 \mid X=4) = \frac{P(d=1) \cdot p(X=4 \mid d=1)}{p(X=4)}$$

$$p(X=4) = p(d=1) \cdot p(X=4 \mid d=1) + p(d=0) \cdot p(X=4 \mid d=0)$$

$$= 0.5 \times (1-0.8)^3 \cdot 0.8 + 0.5 \times (1-0.1)^3 \cdot 0.1$$

$$= 0.03965$$

$$p(d=1 \mid X=4) = \frac{0.0032}{0.03965} \approx 0.08$$

if a stu got interview at 4th visit s/he must be dressed badly.

$\boxed{4.12}$

$p(\text{odd}) = p(X=1) + p(X=3) + \cdots$

$= (1-p)^0 p + (1-p)^2 p +$

$p(\text{even}) = p(X=2) + p(X=4) + \cdots$

$= (1-p)p + (1-p)^3 p + \cdots$

$= (1-p)\left[p + (1-p)^2 p + \cdots\right]$

$= (1-p) \cdot p(\text{odd}) < p(\text{odd})$