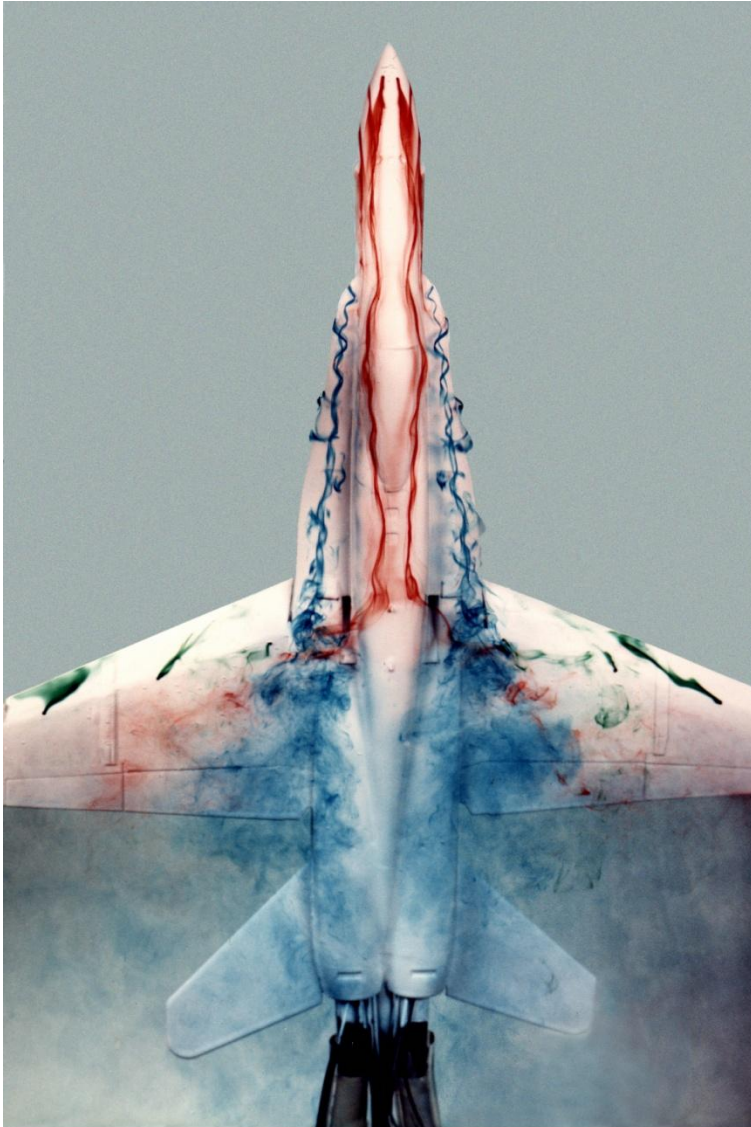


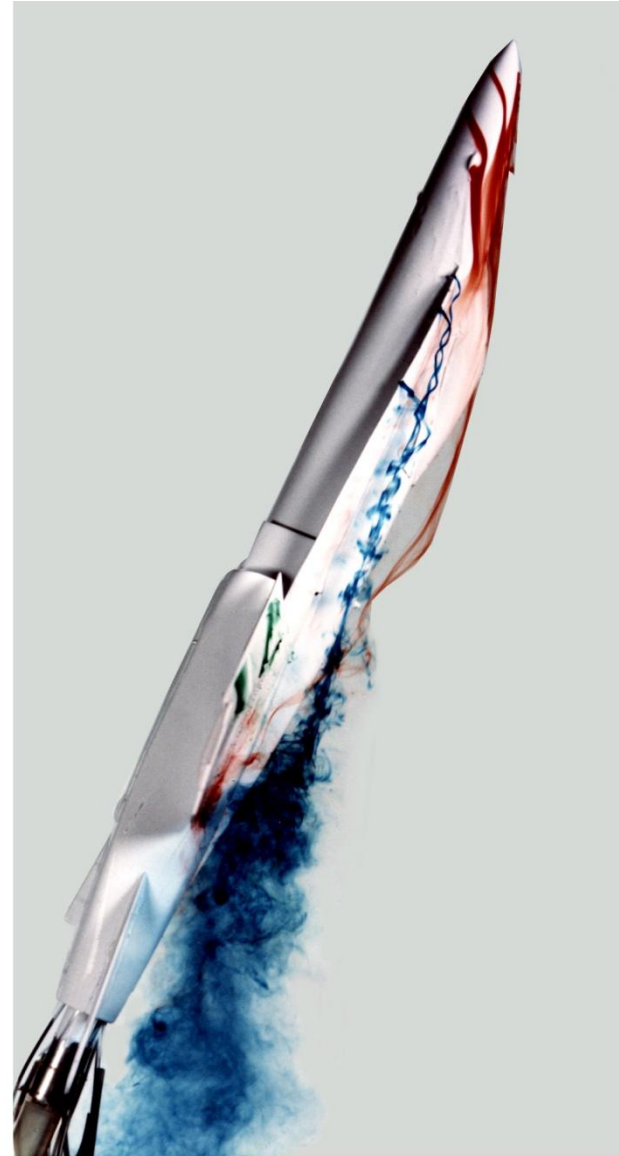
# **2D Vector Field Visualization**

# Experimental Flow Vis



NASA Dryden Flight Research Center Photo Collection  
<http://www.dfrc.nasa.gov/gallery/photo/index.html>  
NASA Photo: ECN-33298-03 Date: 1985

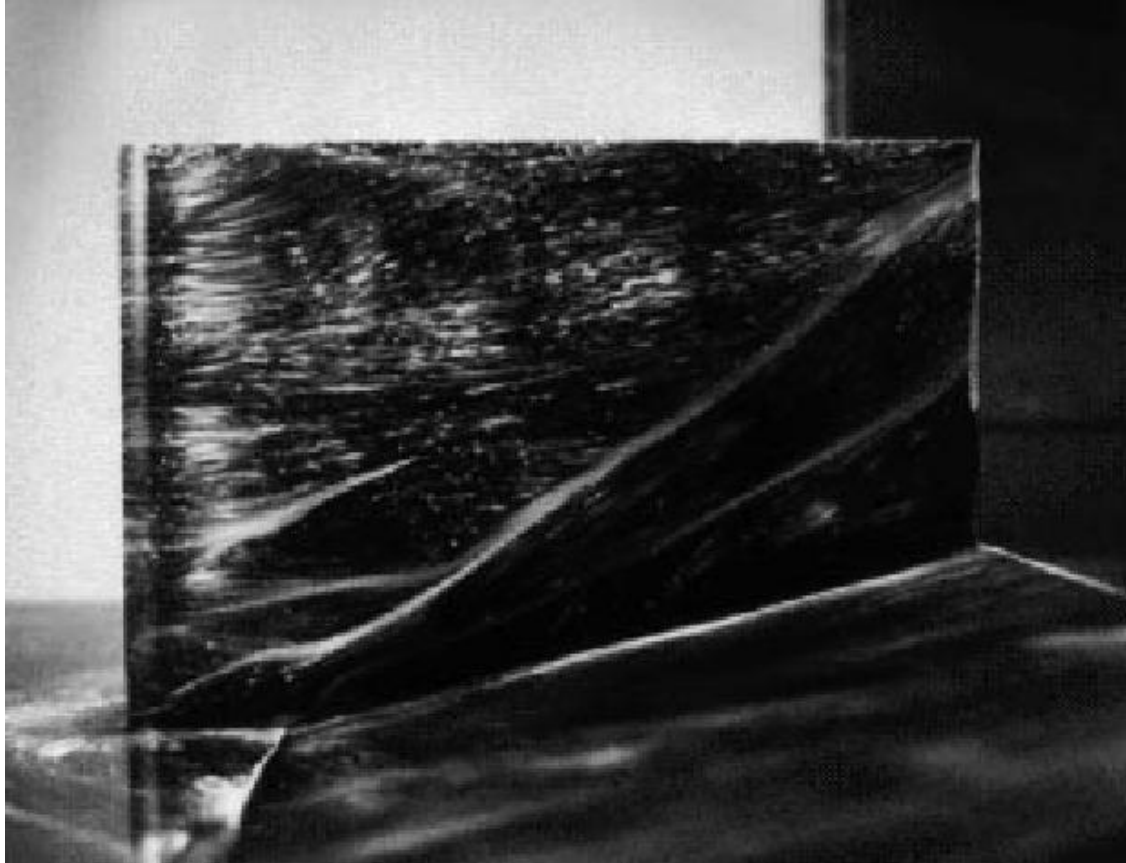
1/48-scale model of an F-18 aircraft in Flow Visualization Facility (FVF)



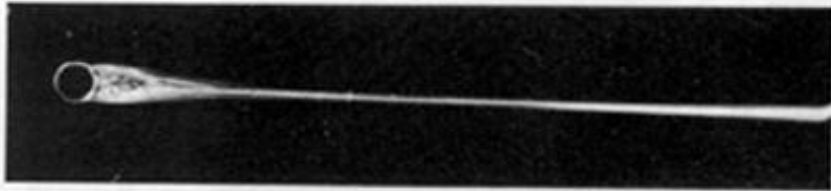
Dryden Flight Research Center ECN 33298-47 Photographed 1985  
F-18 water tunnel test in Flow Visualization Facility NASA/Dryden



# Experimental Flow Vis



# Experimental Flow Vis



$R = 32$



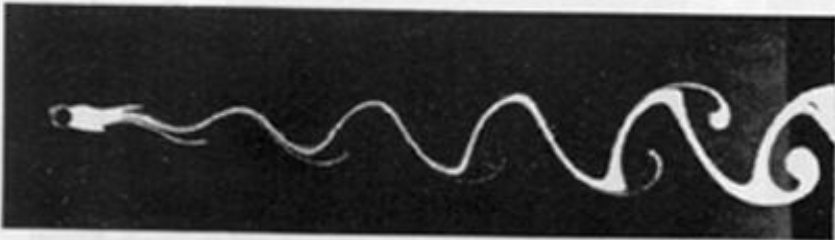
$R = 73$



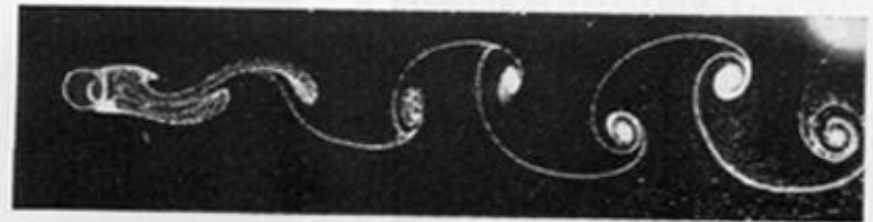
$R = 55$



$R = 102$



$R = 65$



$R = 161$

# Experimental Flow Vis



Why would we not  
stick with these?

# Vectors

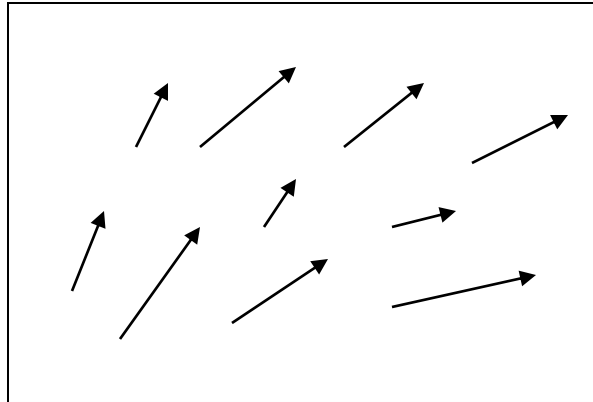
- Directional information
- Wind, mechanical forces (earthquakes)
- **Flows**
- Harder: more than one pixel per vector
  - Clutter

# Vector Field Visualization

-A vector field:  $F(U) = V$

U: field domain  $(x,y)$  in 2D

V: vector  $(u,v)$

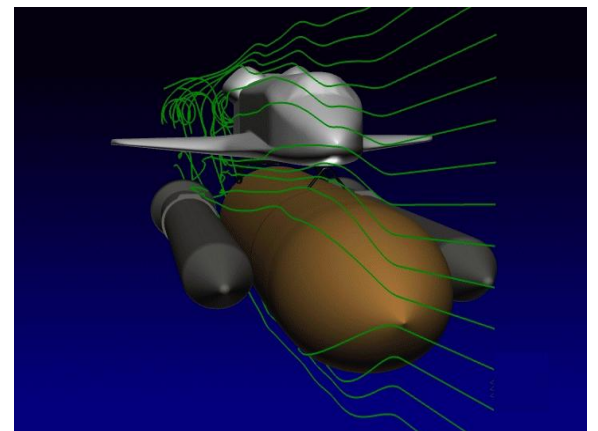
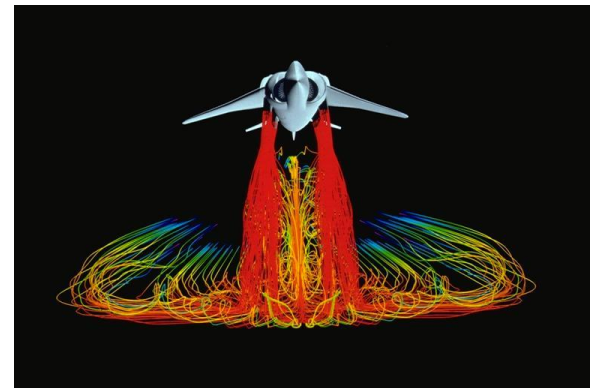


-Like scalar fields, vectors are defined at discrete points:  
-interpolation issues



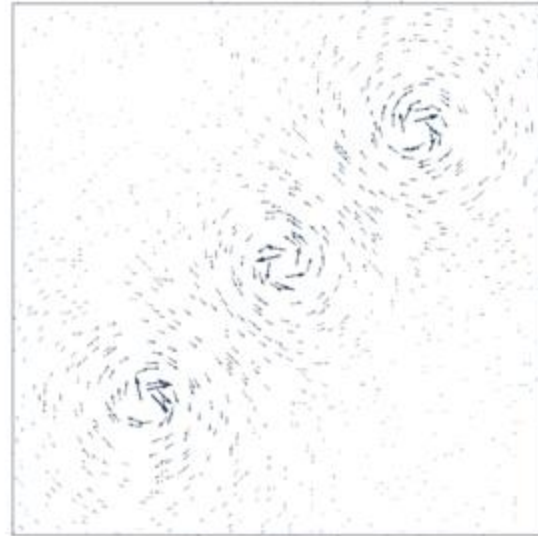
# Visualization techniques

- Geometry-based methods:  
rendering primitives built from  
particle trajectories
  - Glyphs
  - streamlines
  - pathlines
  - streaklines
  - topology
  - LIC
  - .....

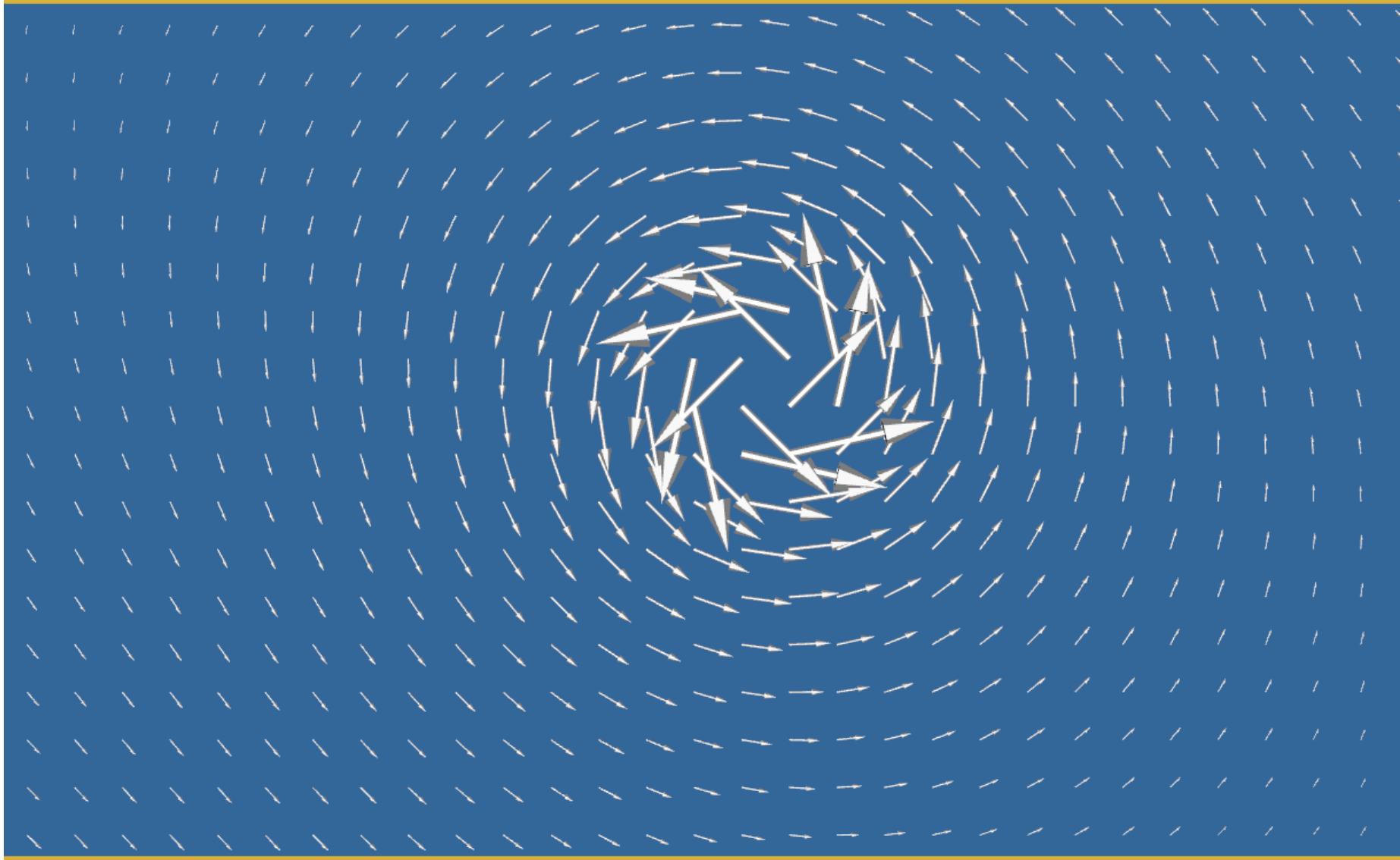


# Glyphs

- Place symbols over vector field
  - Regularly spaced
  - Randomly spaced
  - Scale
- Watch out for clutter

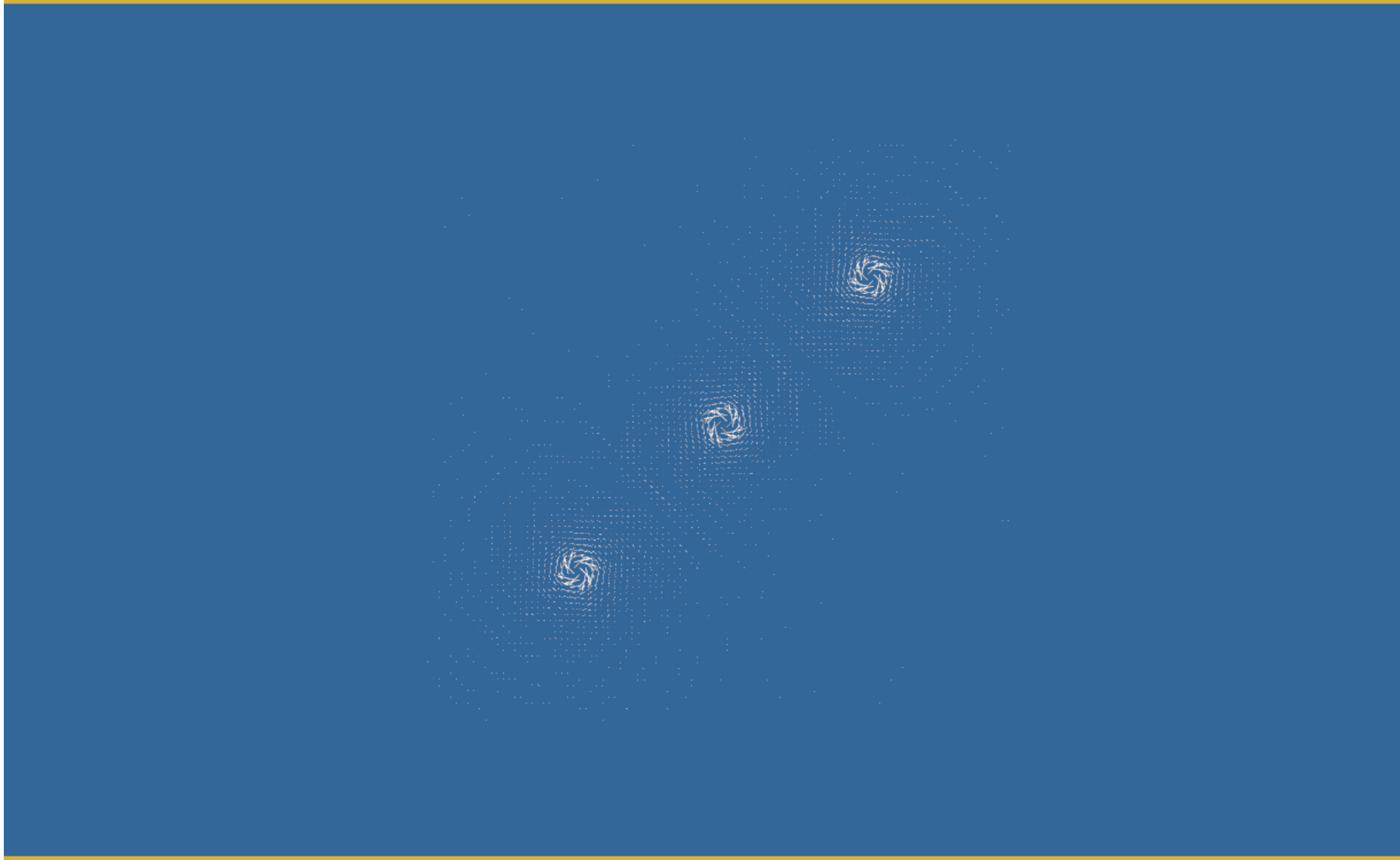


A

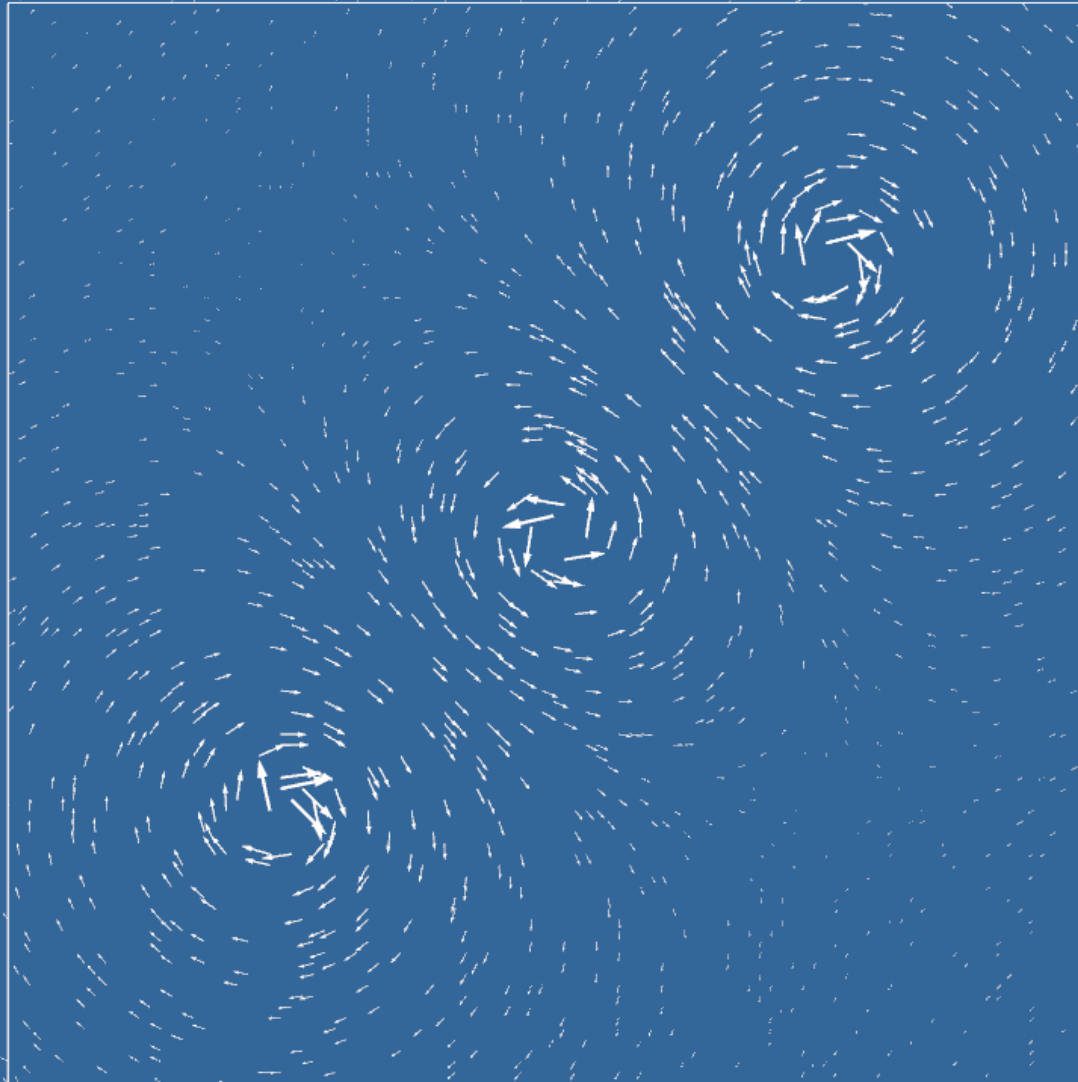


Export ▾ [Close] [Print] [Save] [Play] [Save Camera]

A

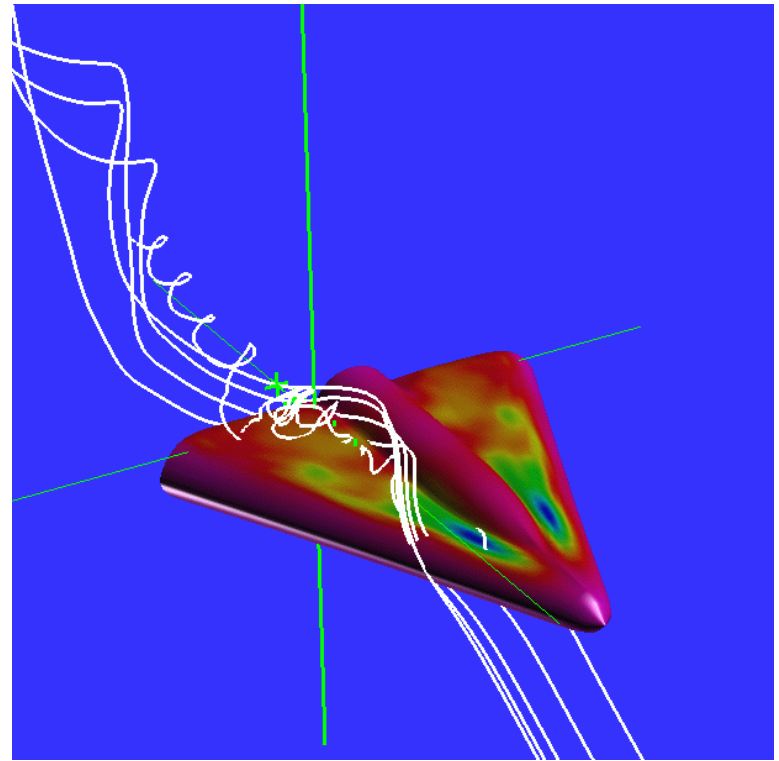
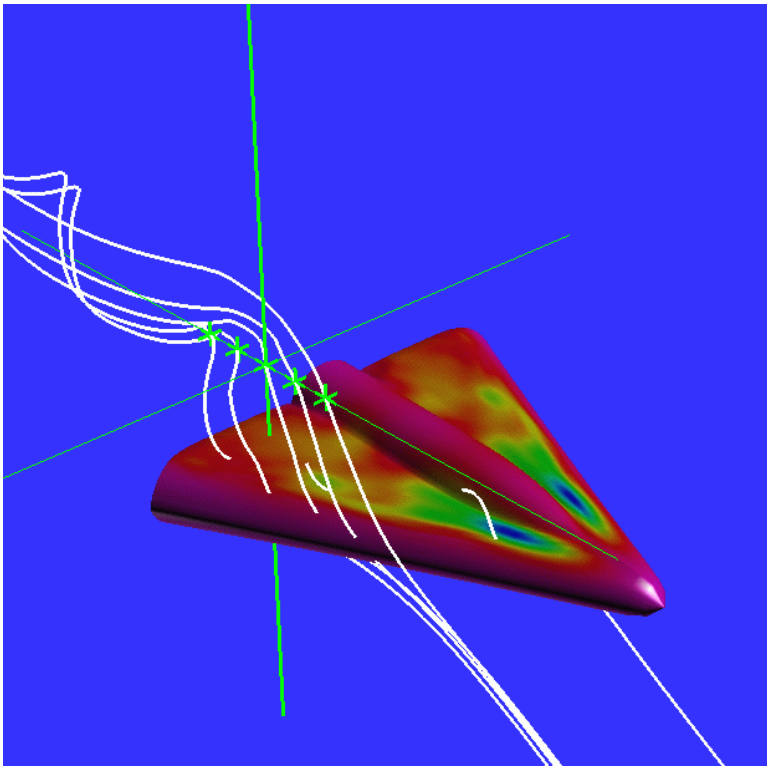


A

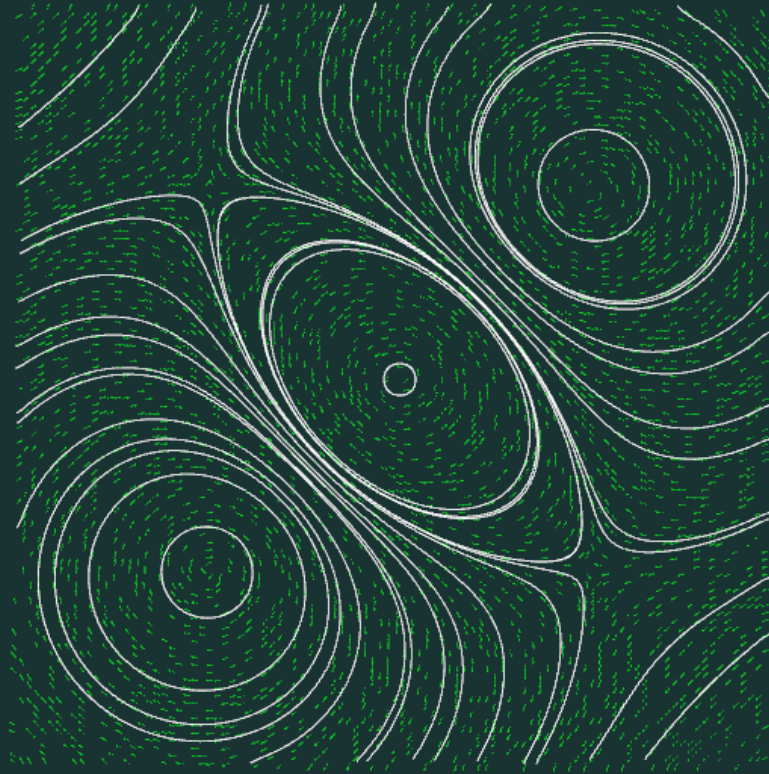


# Streamlines

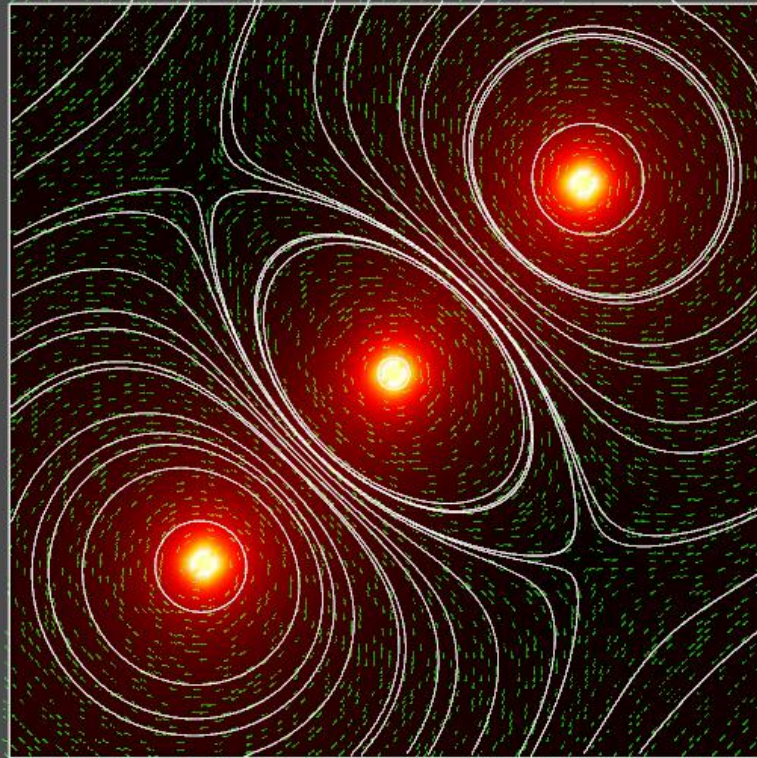
A curves that connect all the particle positions



A



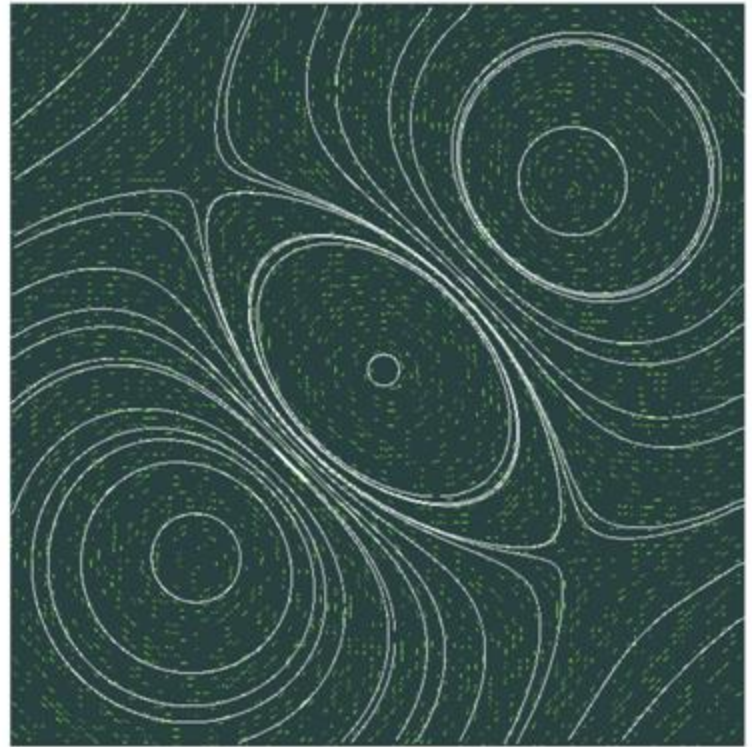
A





# Streamlines

- Lines that are everywhere tangent to the vector field
  - $f(0) = x_0, \dot{f}(x) = u(x)$
- That's a diff. eq.
- Solving for  $f(x)$  is an **initial value problem**



# Local technique - Particle Tracing

Visualizing the flow directions by releasing particles and calculating a series of particle positions based on the vector field

The motion of particle:  $dx/dt = v(x)$

$x$ : particle position (in 2D  $(x_1, x_2)$  position vector)

$v(x)$ : the vector (velocity) field

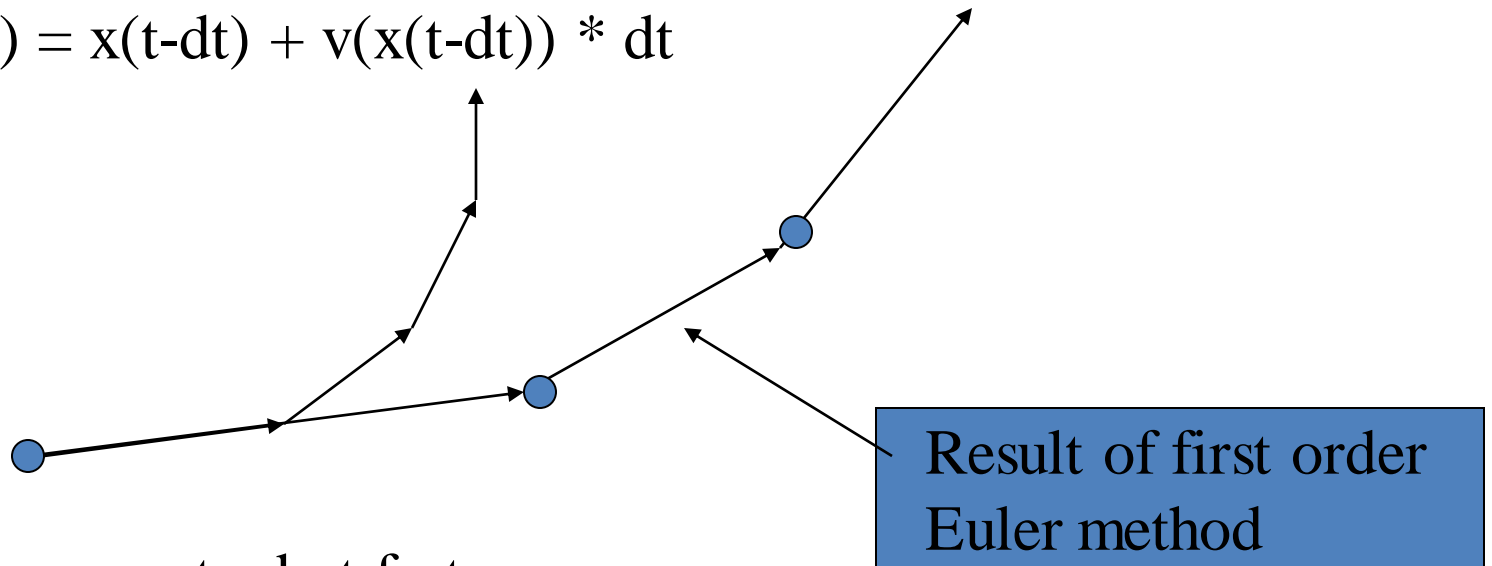
Use numerical integration to compute a new particle position

$$x(t) = x(t-dt) + \text{Integration}( v(x(t-dt)) dt )$$

# Numerical Integration

First Order Euler method:

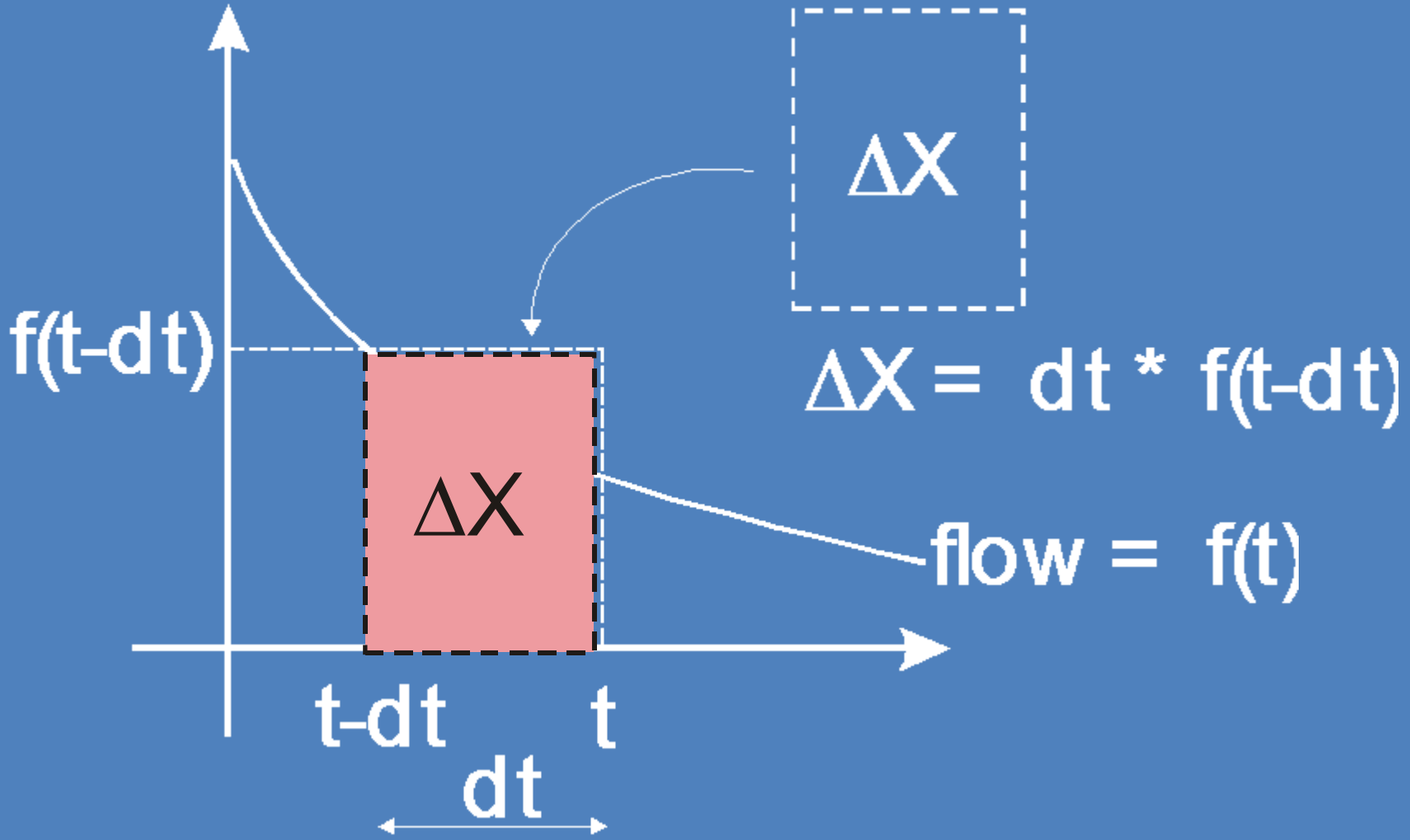
$$x(t) = x(t-dt) + v(x(t-dt)) * dt$$



- Not very accurate, but fast
- Other higher order methods are available: Runge-Kutta second and fourth order integration methods (more popular due to their accuracy)

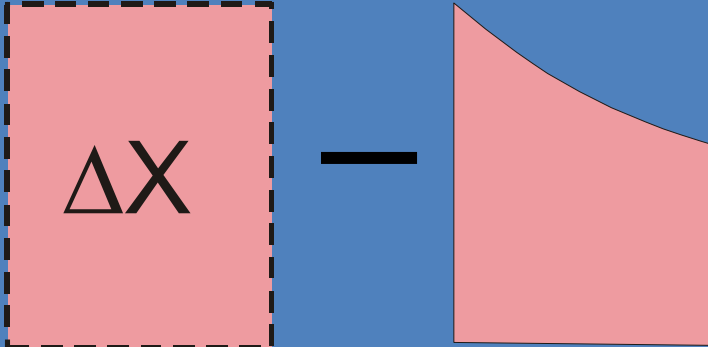
# Euler's Method

Assume flow =  $f(t)$



# Euler Integration Error

- Error =  $\Delta X$  - area under flow curve

Error = 

The diagram illustrates the Euler integration error. It shows the equation: Error =  $\Delta X$  - area under flow curve. The  $\Delta X$  term is represented by a red square with a dashed border. The area under the flow curve is represented by a red area under a downward-sloping curve.

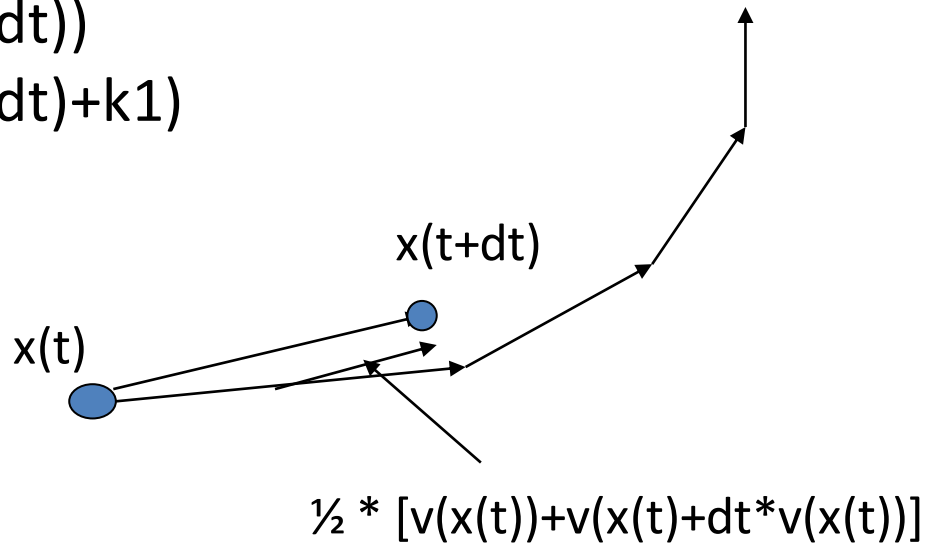
# Numerical Integration (2)

## Second Runge-Kutta Method

$$x(t) = x(t-dt) + \frac{1}{2} * (K1 + K2)$$

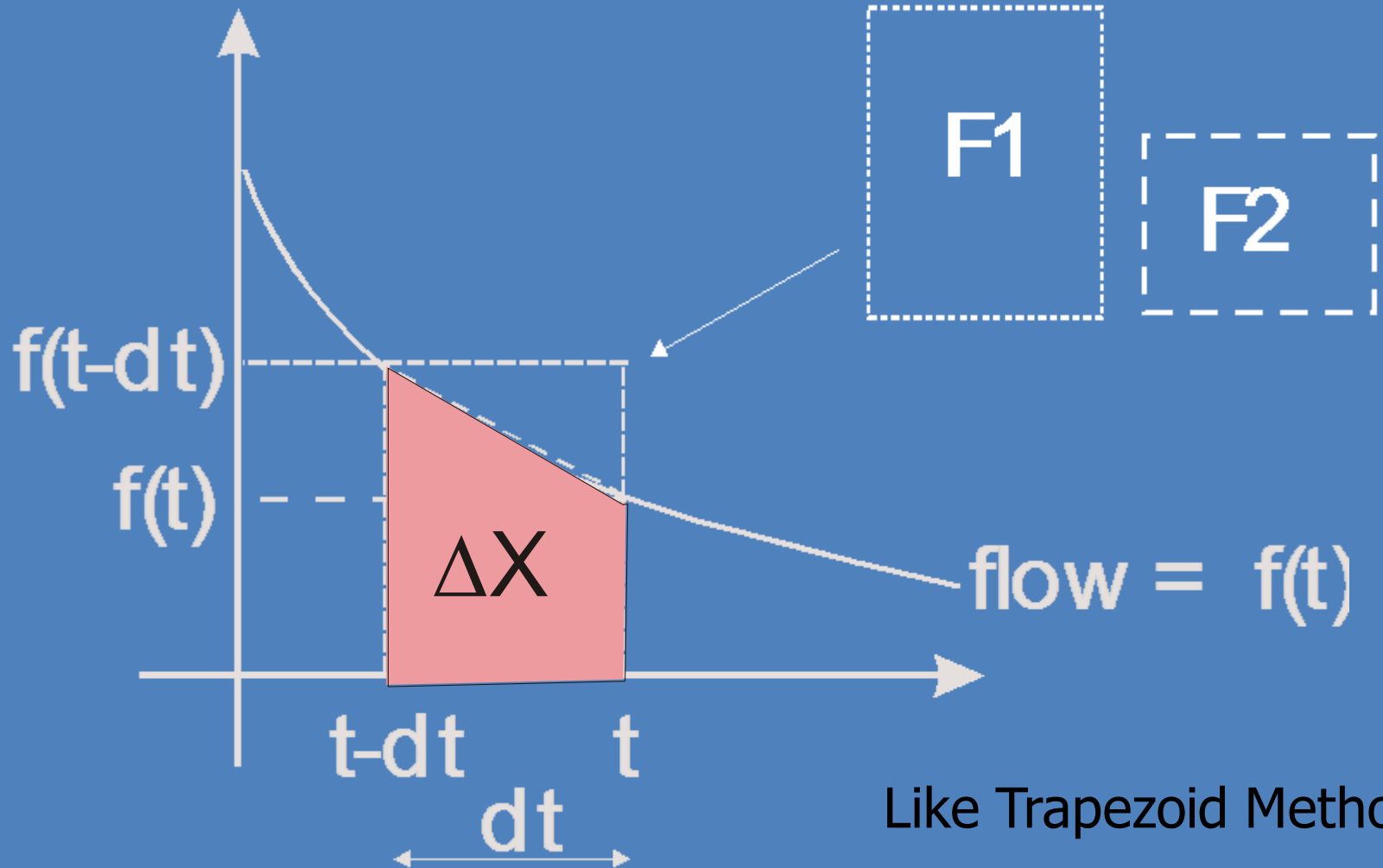
$$k1 = dt * v(x(t-dt))$$

$$k2 = dt * v(x(t-dt)+k1)$$



# Runge-Kutta 2

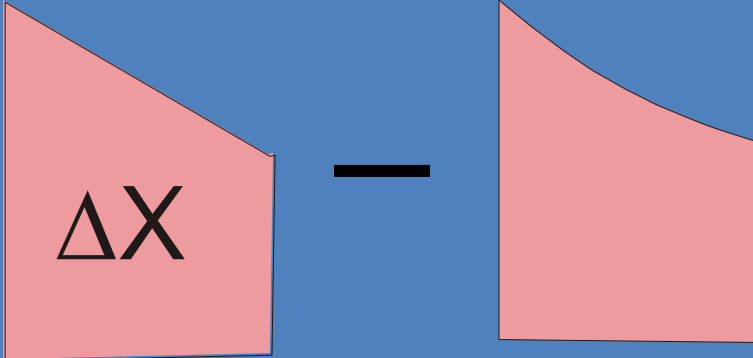
Assume flow =  $f(t)$



Like Trapezoid Method.

# RK2 Integration Error

- Error =  $\Delta X$  - area under flow curve

Error = 

The diagram illustrates the RK2 integration error. It shows the difference between a rectangular area labeled  $\Delta X$  and the area under a curved flow curve. The rectangular area is on the left, and the area under the curve is on the right, with a minus sign between them.



# Numerical Integration (3)

Standard Method: Runge-Kutta fourth order

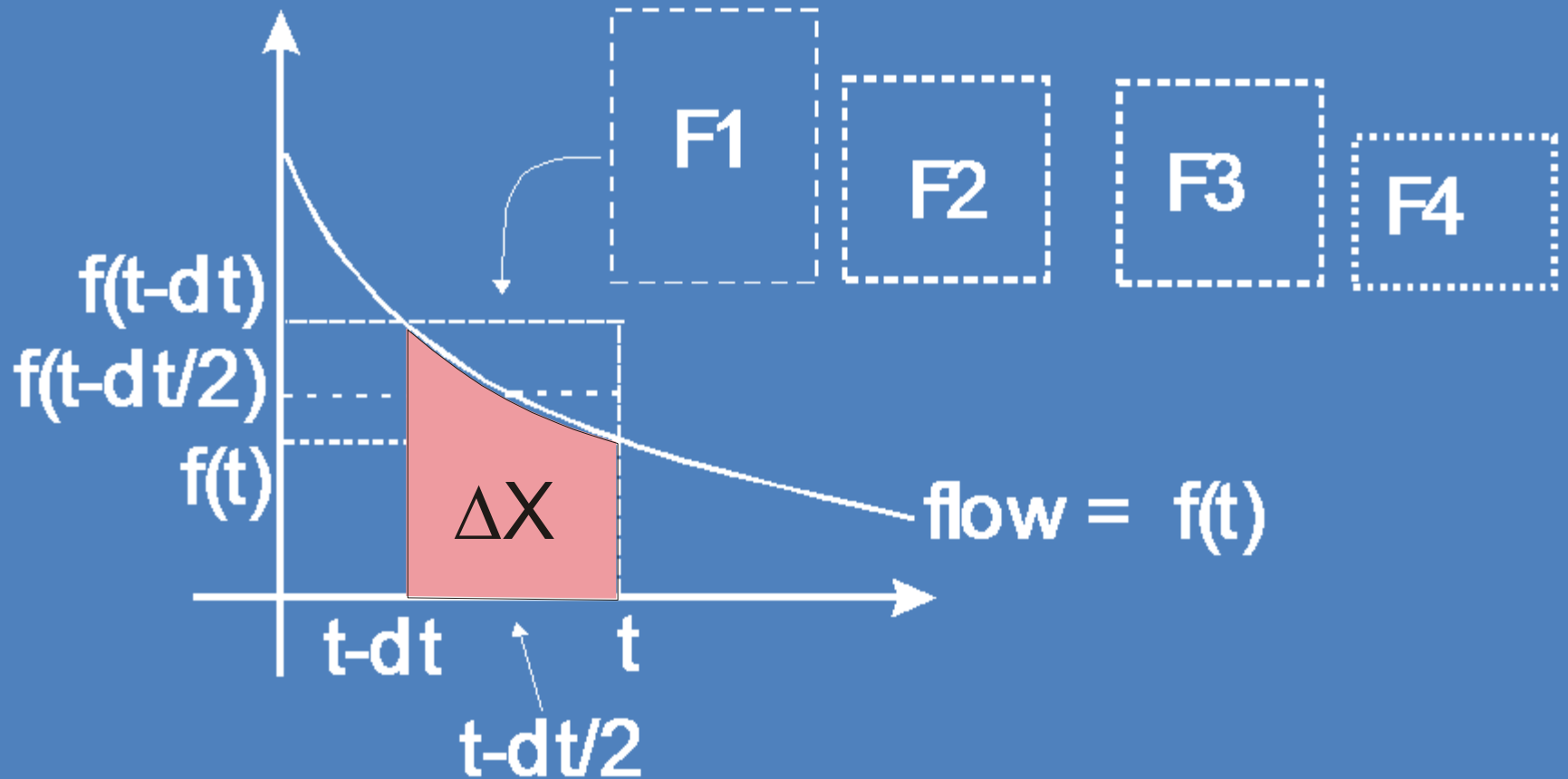
$$x(t) = x(t-dt) + 1/6 (k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = dt * v(t-dt); k_2 = dt * v(x(t-dt) + k_1/2)$$

$$k_3 = dt * v(x(t-dt) + k_2/2); k_4 = dt * v(x(t-dt) + k_3)$$

# Runge-Kutta 4

Assume flow =  $f(t)$

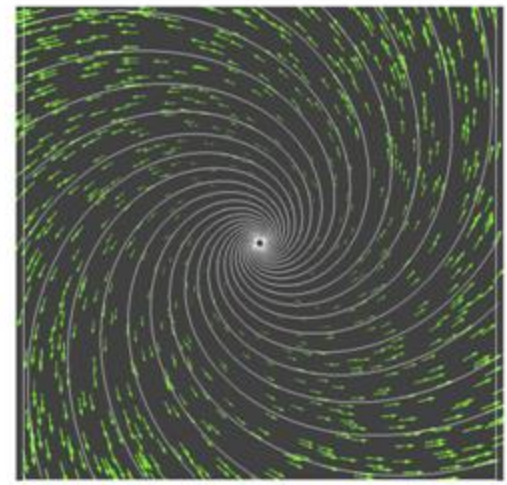
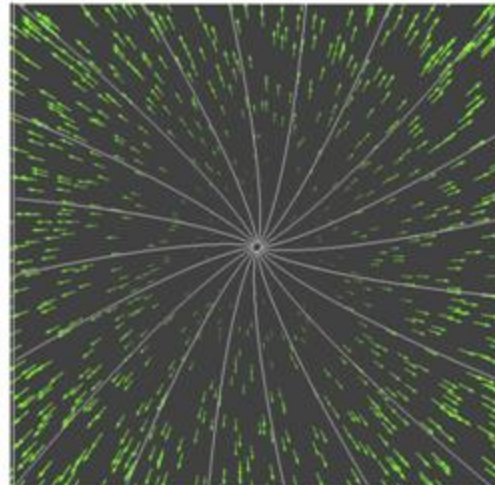
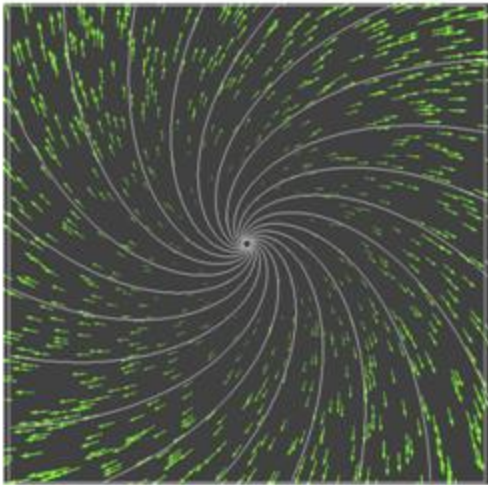


# What Method to Use?

- RK2 and RK4 are more accurate for same  $dt$  than Euler
- Euler works poorly for oscillatory systems
- RK2 and RK4 work well for continuous systems
- RK2 and RK4 work poorly with discrete systems

# Steady vs. unsteady

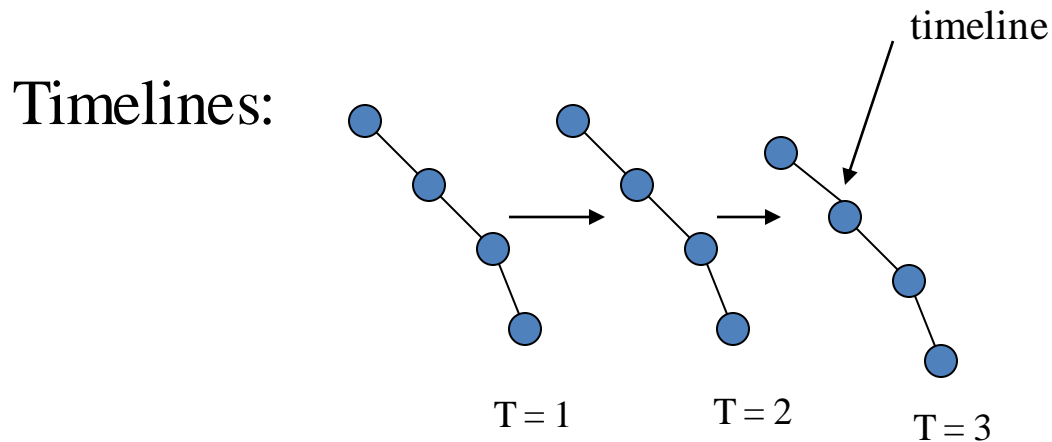
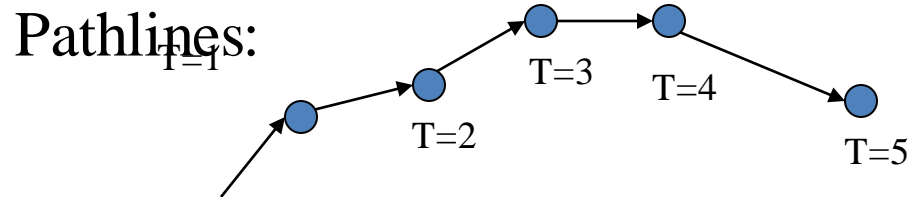
- Flows change with time
- For every timestep, a different vector



- But, what about streamlines, then?

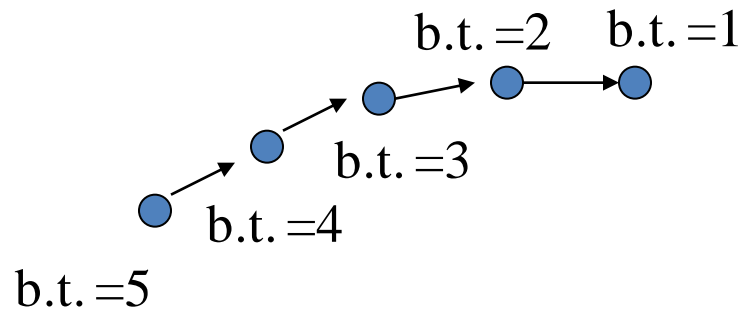
# Pathlines, Timelines, and Streaklines

-Extension of streamlines for time-varying data

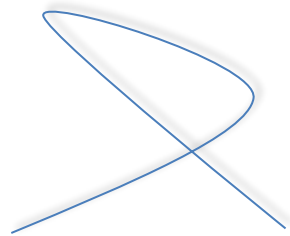


# Streaklines

- Continuously injecting a new particle at each time step, advecting all the existing particles and connect them together into a *streakline*



# When should we expect self-intersections?



- Streamlines
- Pathlines
- Streaklines

# Pathlines and Streaklines

- Streamlines do not cross
- Streaklines still never cross
- Pathlines do cross



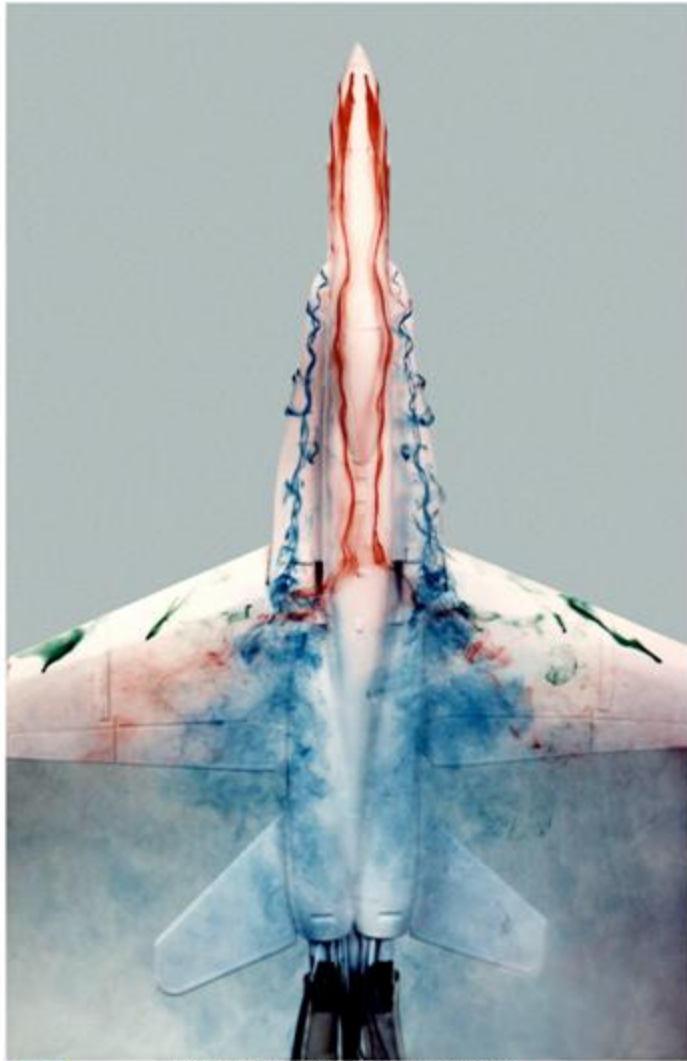
# Pathlines and Streaklines

- Streamlines **should** not cross
- Streaklines still **seldom** cross
- Pathlines do cross

# Seed Placement

- The placement of seeds directly determines the visualization quality
  - Too many: scene cluttering
  - Too little: no pattern formed
- It has to be the right number at the right places!!!

# Streaklines in real life



NASA Dryden Flight Research Center Photo Collection  
<http://www.dfrc.nasa.gov/gallery/photo/index.html>  
NASA Photo: ECN-33298-03 Date: 1985

1/48-scale model of an F-18 aircraft in Flow Visualization Facility (VVF)



Dryden Flight Research Center ECN 33298-47 Photographed 1985  
F-18 water tunnel test in Flow Visualization Facility NASA/Dryden



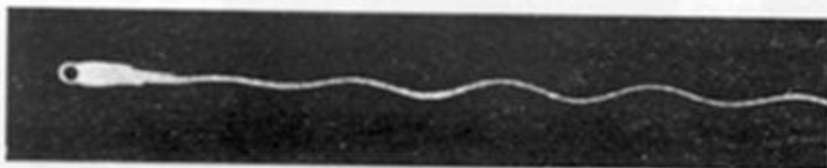
# Streaklines in real life



$R = 32$



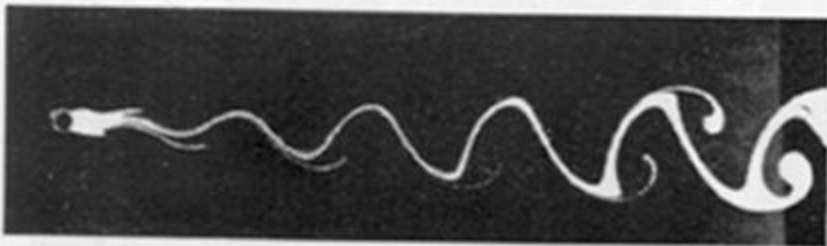
$R = 73$



$R = 55$



$R = 102$



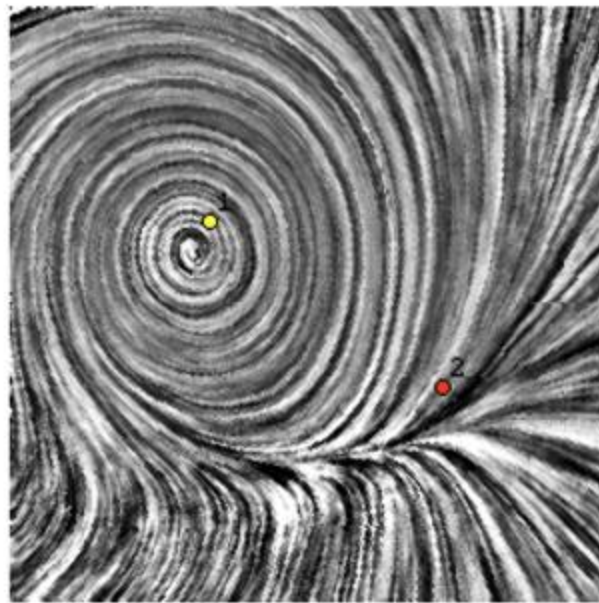
$R = 65$



$R = 161$

# Line Integral Convolution

- Basic idea: Integrate noise along streamlines
- demo: <http://www.javaview.de/demo/PaLIC.html>



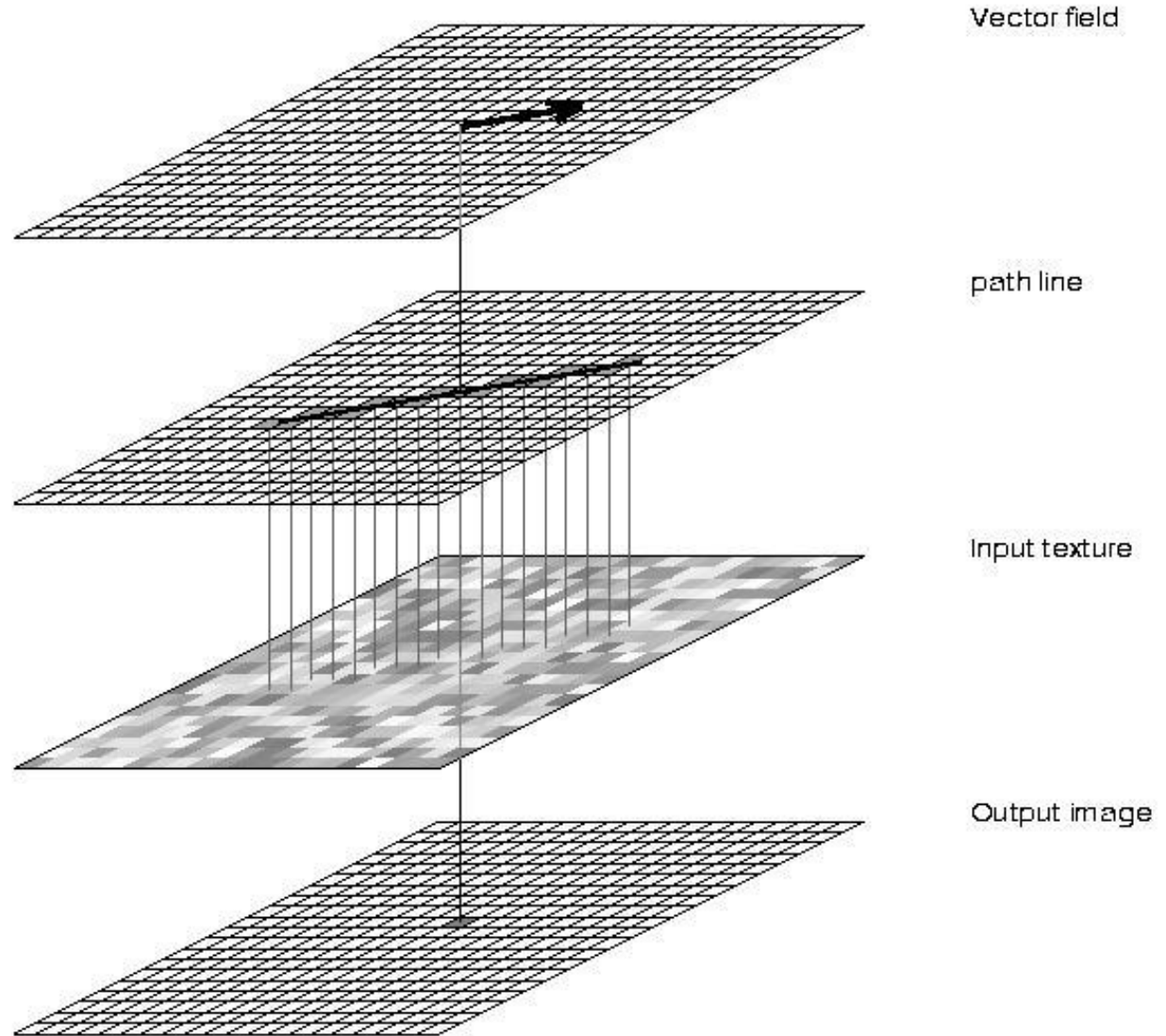
# Rendering - LIC

- embed a noise texture under the vector field
- integrates along a streamline



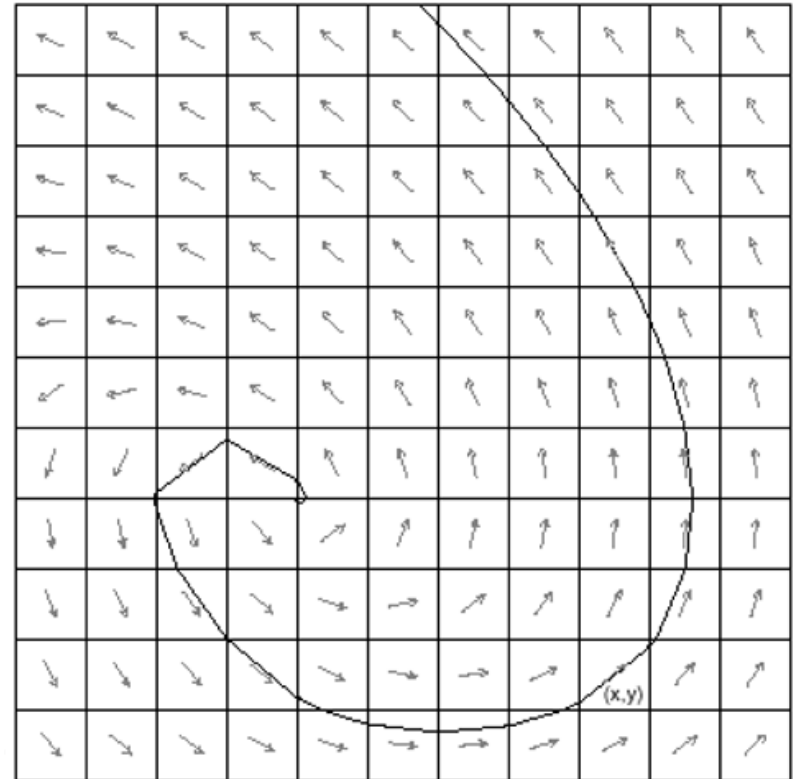
# Line Integral Convolution (LIC)

- LIC:
- convolve a
- random texture
- along the
- streamlines



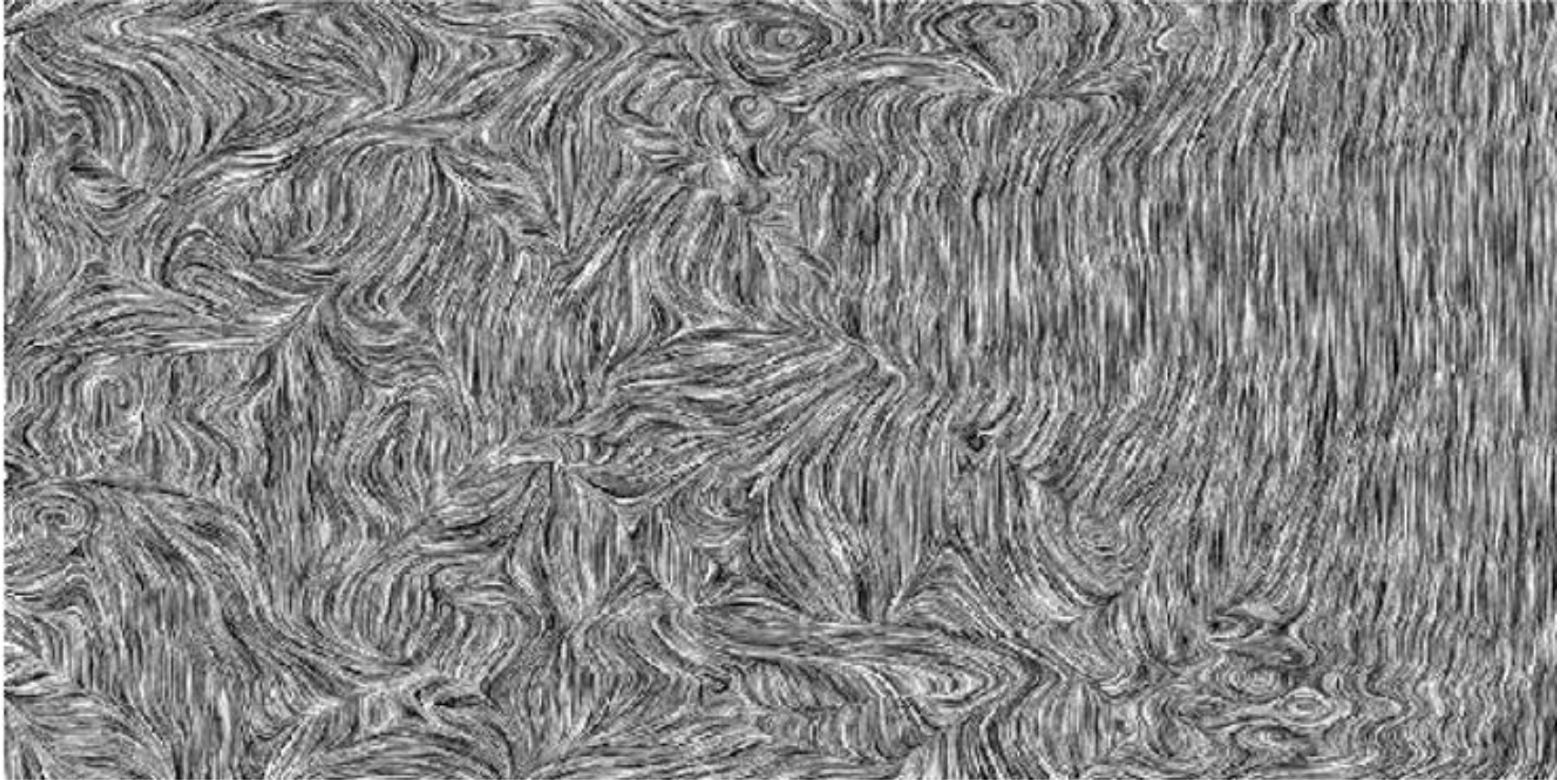
# Line Integral Convolution (LIC)

- Assume input texture, vector and output images are all the same resolution.
- For each output pixel/voxel, generate a streamline both forwards and backwards of a fixed length.
- Integrate the intensity that the streamline passes through

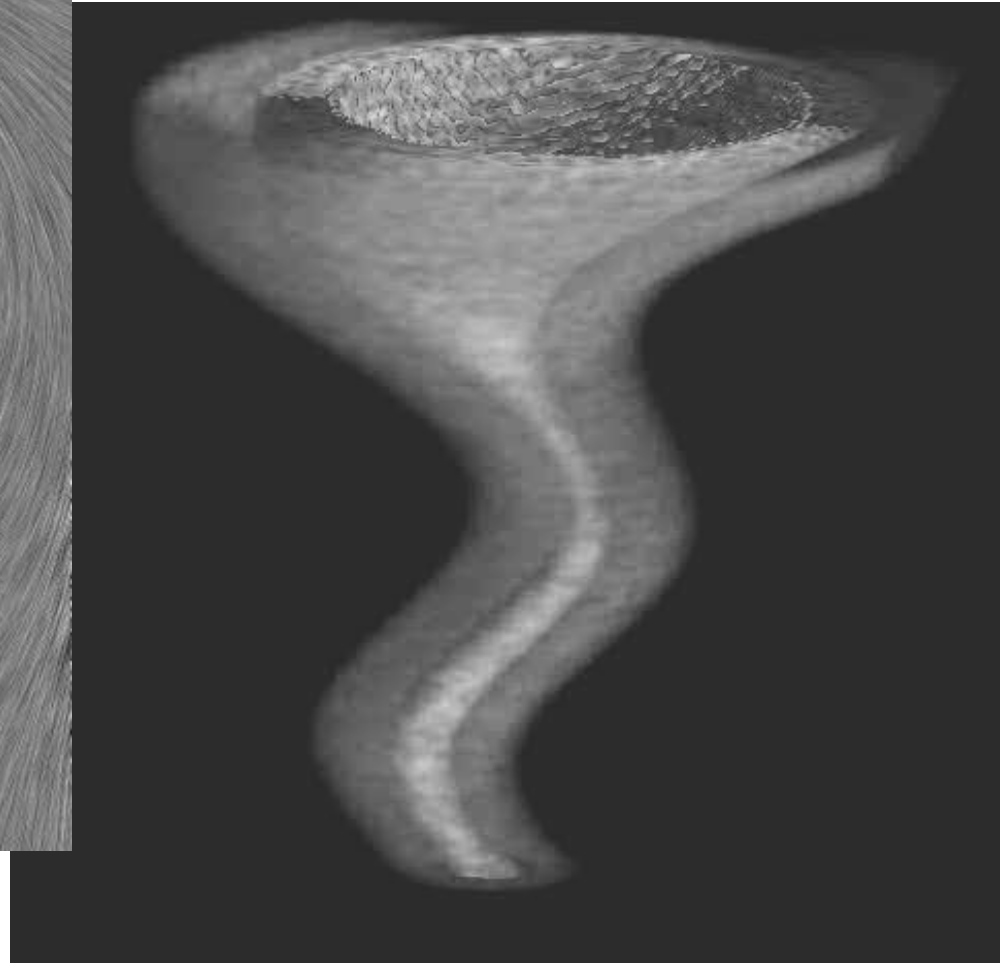
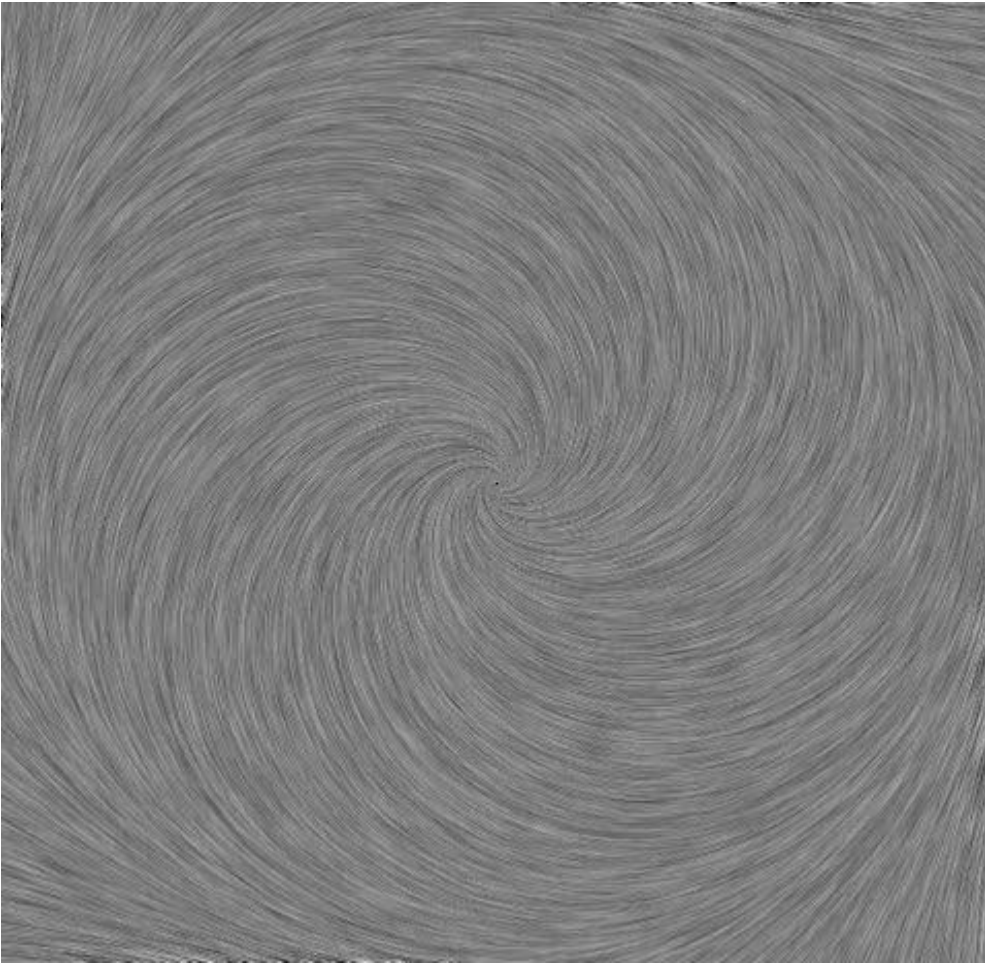




# Line Integral Convolution (LIC)



# Line Integral Convolution (LIC)



# Comparison (LIC and Streamlines)

