Model Selection and Estimation of Multi-Compartment Models in Diffusion MRI with a Rician Noise Model

Xinghua Zhu, Yaniv Gur, Wenping Wang, and P. Thomas Fletcher

THE UNIVERSITY OF UTAH

www.sci.utah.edu

Introduction

- Existing multi-compartment model estimation methods generally assume Gaussian noise distribution

In this paper:
- Estimate ball-and-sticks model under Rician noise distribution.
- An automatic model selection scheme to select the number of fibers.

Proposed Method

The Ball-And-Sticks Diffusion Model

$$\nu_i(\Theta) = S_0 \left[\underbrace{w_0 \exp(-b\kappa_0)}_{\text{ball}} + \sum_{j=1}^{M} w_j \underbrace{\exp(-b\kappa_j (\mathbf{g}_i^T \mathbf{u_j})^2)}_{\text{sticks}} \right]$$

Unknown parameters

$$\Theta = \{\mathbf{u}_1, \ldots, \mathbf{u}_M; \quad \text{stick directions}$$
$$w_0, \ldots, w_M; \quad \text{compartment fractions}$$
$$\kappa_0, \ldots, \kappa_M\} \quad \text{diffusivities}$$

Rician Likelihood

$$p(S_i|\Theta) = \frac{S_i}{\sigma^2} \exp\left(-\frac{S_i^2 + \nu_i(\Theta)^2}{2\sigma^2}\right) I_0\left(\frac{S_i \nu_i(\Theta)}{\sigma^2}\right)$$

The Expectation Maximization Algorithm

- Raw data of MRI scans are contaminated by additive complex Gaussian noise, which becomes Rician after taking magnitude
- Hidden variable: the complex Gaussian affected signal of each compartment:

$$Y_{ij} = \nu_{ij}(\Theta) + \epsilon, \quad \epsilon \sim \mathcal{CN}\left(0, \frac{2\sigma^2}{M+1}\right)$$

- E-step:

$$Q(\Theta|\Theta^{(k)}) = E\left[l(\Theta|\mathbf{Y})|\mathbf{S}, \Theta^{(k)}\right]$$
$$= \sum_{i,j} 2\nu_{ij}(\Theta) \left[\frac{S_i}{M+1} A\left(\frac{S_i \nu_i^{(k)}}{\sigma^2}\right) - \frac{\nu_i^{(k)}}{M+1} + \nu_{ij}^{(k)}\right] - \nu_{ij}(\Theta)^2$$

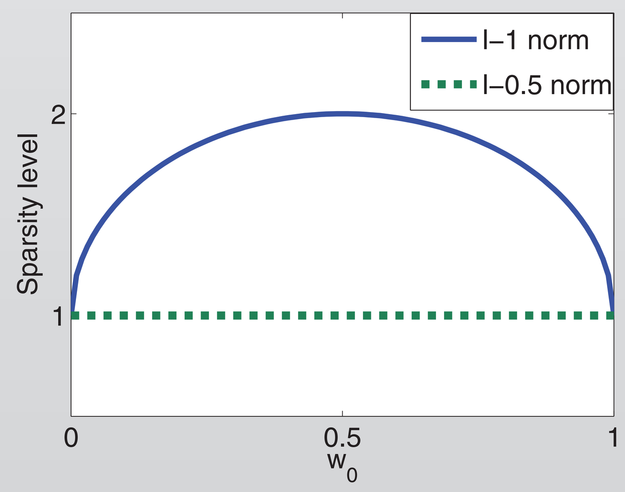
- M-step: gradient ascent of the Q-function
- Advantage of the EM algorithm:

The optimization parameters are no longer variables of the modified Bessel function. Therefore, maximizing the Q-function is more numerically stable and tractable.

Automatic Model Selection

- Sparsity Prior on Compartment Weights:

$$\mathcal{C}(w_0, w_1, \ldots, w_M) = \left(\sum_{j=0}^{M} w_j^{0.5}\right)^2$$

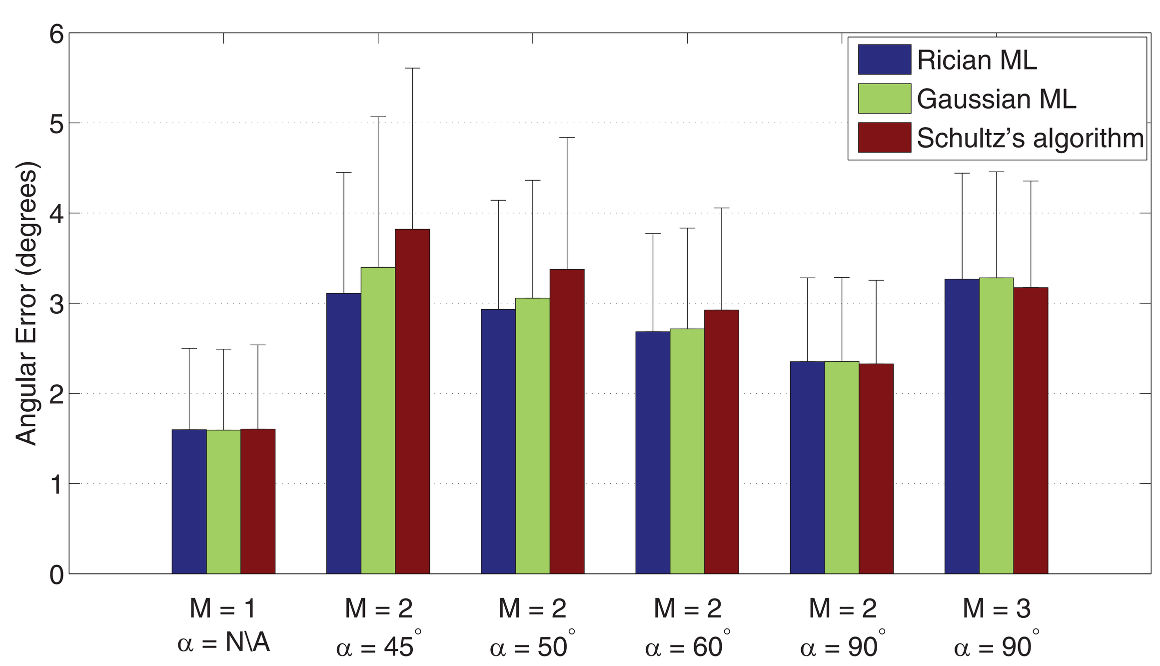


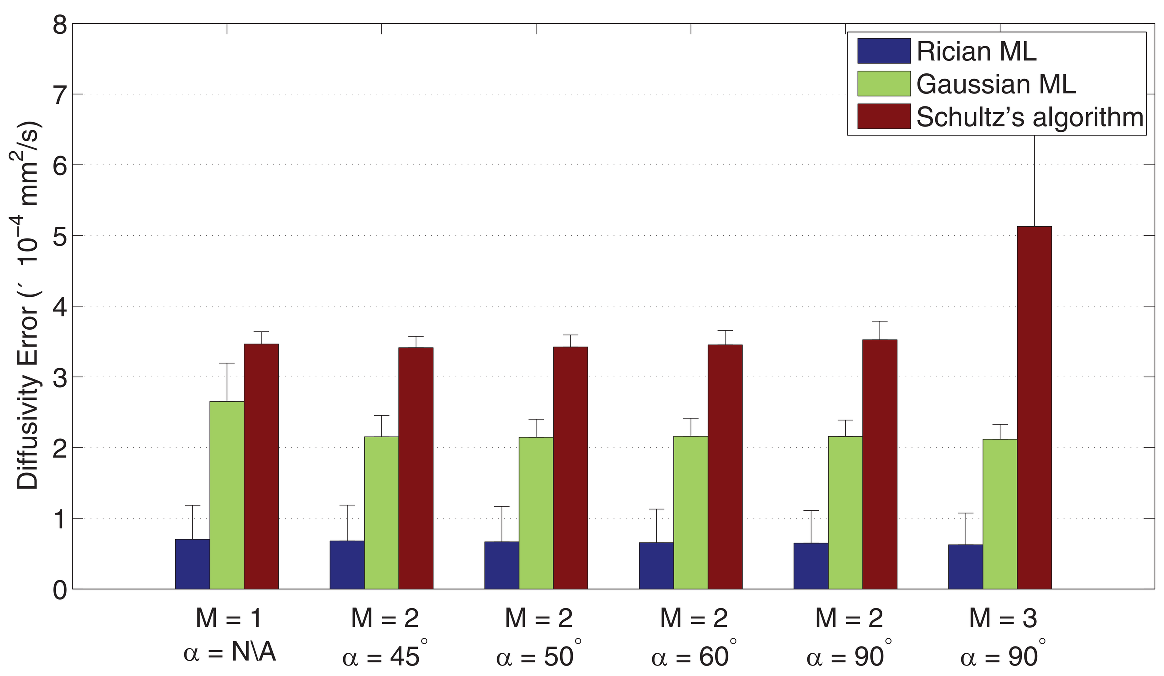
- The amended Q-function:

$$\hat{Q}(\Theta|\Theta^{(k)}) = Q(\Theta|\Theta^{(k)}) - \lambda_C \cdot \mathcal{C}(w_0, \ldots, w_M)$$

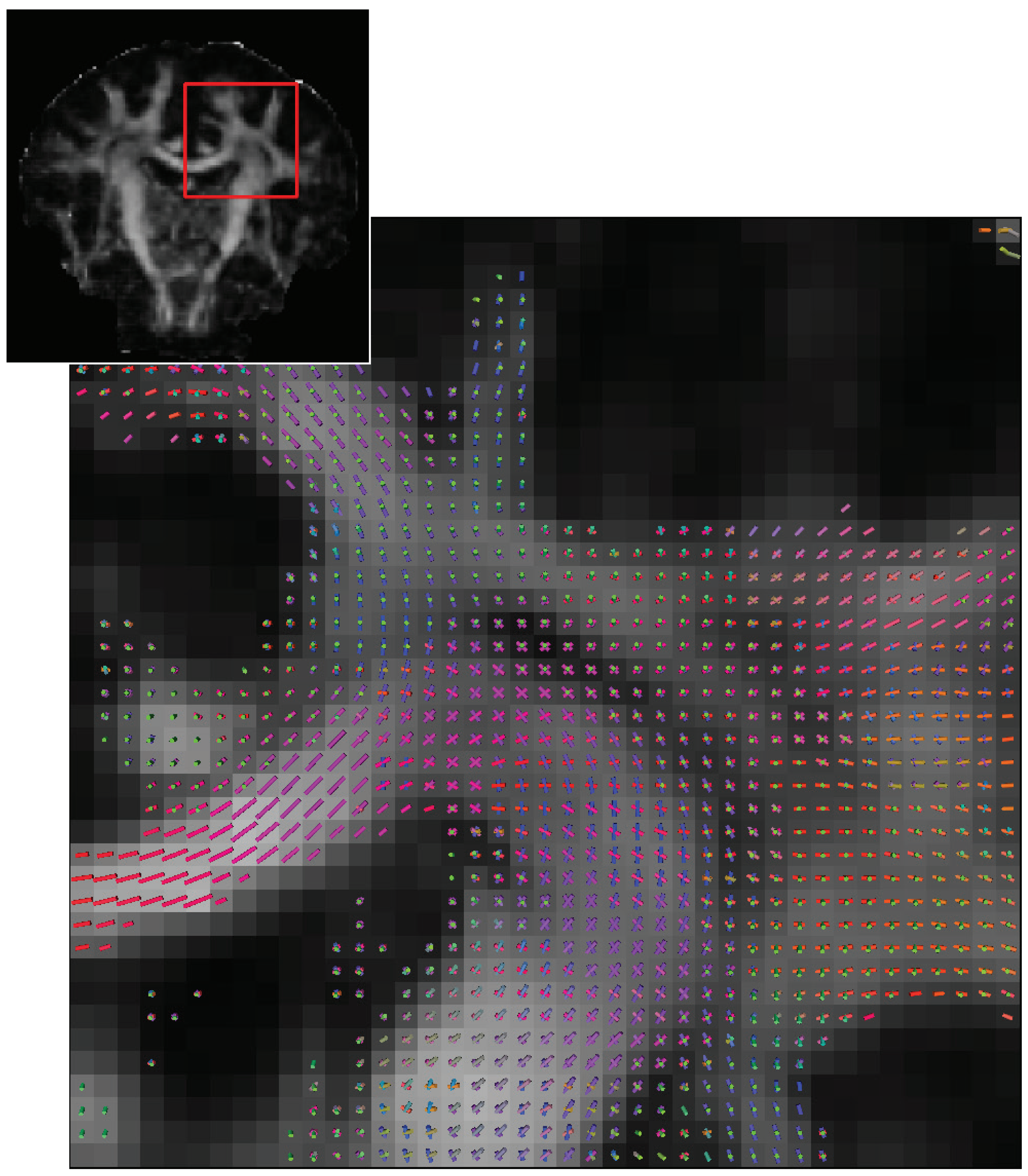
Results on Synthetic Data

b-value: 3000. SNR: 15

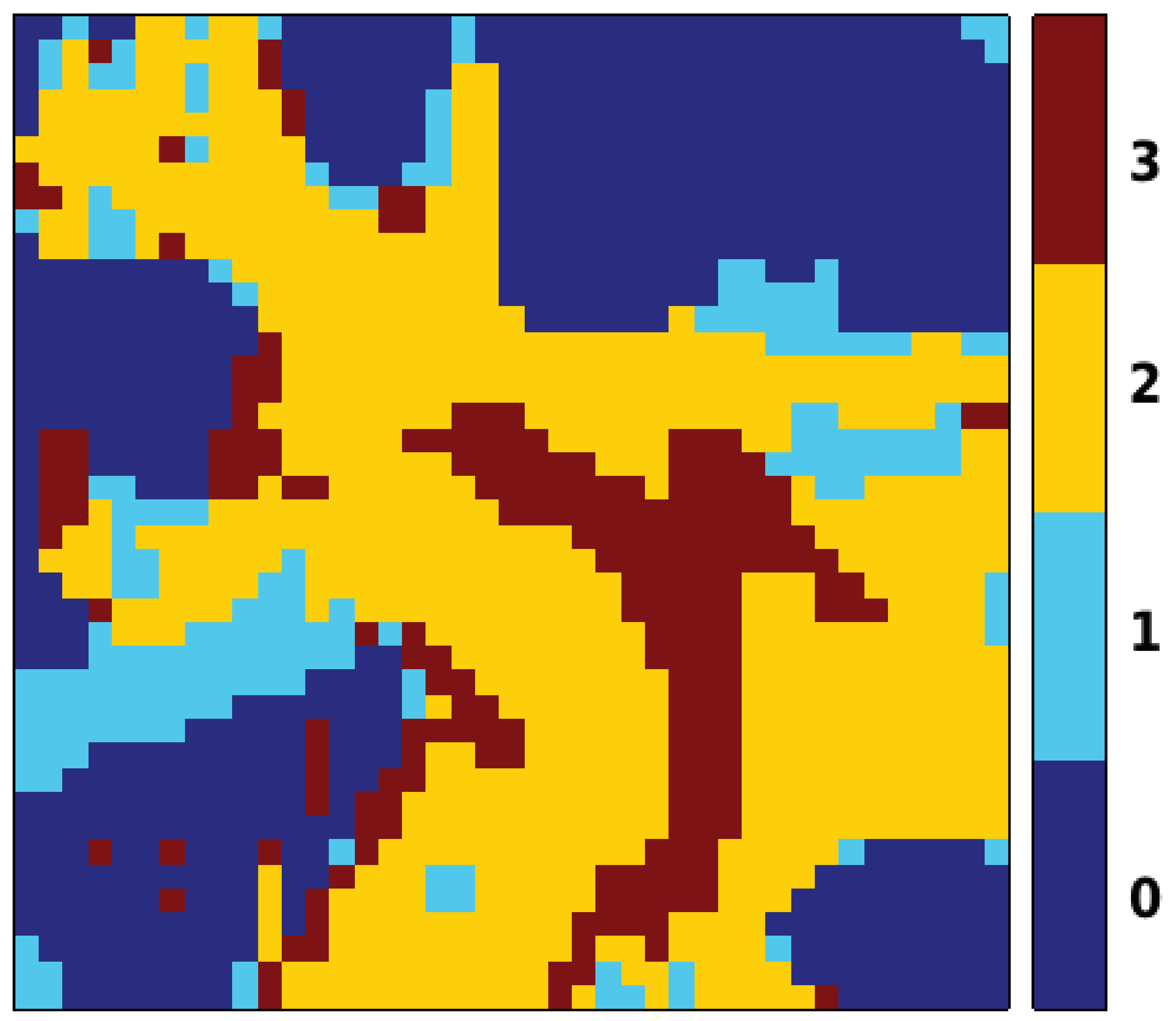




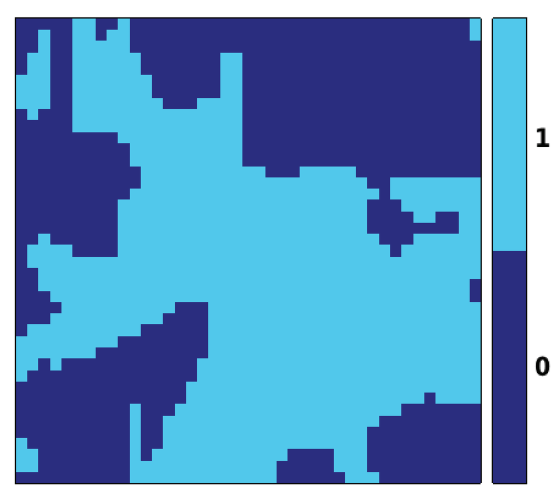
Results on Real Data



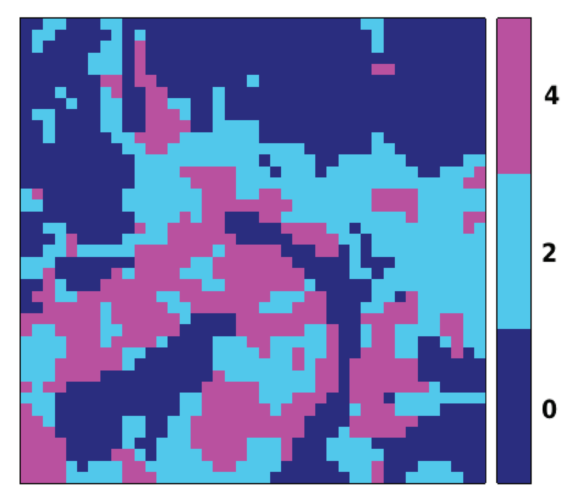
Model Selection Results:



Proposed method



AIC



F-test