VECTOR FIELDS

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administrivia . . .
- transfer function design assignment
- exams
- parallel coordinates assignment
- final project proposals and paper titles
  - due Nov 22
exam 1

parallel coordinates
last time . . .
Transfer Functions (TFs)

Simple (usual) case: Map data value $f$ to color and opacity

Shading, Compositing…

Human Tooth CT
Transfer function Unintuitive

\[ v = f(x) \]

“here’s the edge”
3D Transfer Function

\[ \alpha \left( f, |\nabla f|, D^2_{\nabla f} f \right) \]

\[ \text{RGB} \]
3D Transfer Function

enamel / background
dentin / background
dentin / enamel
dentin / pulp

1D: not possible
2D: specificity not as good
Multi-dimensional TFs

Higher dimensional TFs:

- Better flexibility, specificity
- Higher quality visualizations

- Even harder to specify
- Unintuitive relationship with boundaries
- Greater demands on user interface
today . . .
- vector fields
  - background (some math!)
  - sources
  - direct visualization methods
VECTOR FIELDS
What is a vector field?

**Scalar field**

\[ s : \mathbb{E}^n \rightarrow \mathbb{R} \]

**Vector field**

\[ v : \mathbb{E}^n \rightarrow \mathbb{R}^m \]

m will often be equal to n, but definitely not necessarily.
• Main application of vector field visualization is flow visualization
  – Motion of fluids (gas, liquids)
  – Geometric boundary conditions
  – Velocity (flow) field \( \mathbf{v}(x, t) \)
  – Pressure \( \rho \)
  – Temperature \( T \)
  – Vorticity \( \nabla \times \mathbf{v} \)
  – Density \( \rho \)
  – Conservation of mass, energy, and momentum
  – Navier-Stokes equations
  – CFD (Computational Fluid Dynamics)
• Flow data:
  • vector data on a 2D or 3D grid
  • in addition scalar data may be defined per grid point
    \( (E^V_2, E^V_3) \)

flow data; a) on regular grid; b) scattered flow data
Experimental Flow Visualization
Milestones in Flight History
Dryden Flight Research Center

L-1011
Airliner Wing Vortice Tests at Langley
Circa 1970s
Smoke angel

A C-17 Globemaster III from the 14th Airlift Squadron, Charleston Air Force Base, S.C. flies off after releasing flares over the Atlantic Ocean near Charleston, S.C., during a training mission on Tuesday, May 16, 2006. The "smoke angel" is caused by the vortex from the engines.

(U.S. Air Force photo/Tech. Sgt. Russell E. Cooley IV)
wool tufts
smoke injection

[NASA, J. Exp. Biol.]

http://autospeed.com/cms/A_108677/article.html

smoke nozzles

http://autospeed.com/cms/A_108677/article.html

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smoke nozzles
Smoke injection

Smoke injection

http://www-me.ccny.cuny.edu/research/aerolab/facilities/images/wt2.jpg
Clouds (satellite image)

Juan Fernandez Islands

Clouds (satellite image)

http://daac.gsfc.nasa.gov/gallery/frances/
Vector Fields in Engineering and Science

Automotive design [Chen et al. TVCG07, TVCG08]

Weather study [Bhatia and Chen et al. TVCG11]

Oil spill trajectories [Tao et al. EMI2010]

Aerodynamics around missiles [Kelly et al. Vis06]
Vector Field Design in Computer Graphics

- Texture Synthesis [Chen et al. TVCG11b]
- River simulation [Chenney SCA2004]
- Parameterization [Ray et al. TOG2006]
- Smoke simulation [Shi and Yu TOG2005]
- Painterly Rendering [Zhang et al. TOG2006]
- Shape Deformation [von Funck et al. 2006]
Why is It Challenging?

- to effectively visualize both *magnitude + direction*, often simultaneously
- large data sets
- time-dependent data

*magnitude only*  
*direction only*
Why is It Fun?

- Flow data appear in a variety of applications, i.e.
  - Simulation / computation of flow around cars, planes, ships
  - Atmospheric flow for weather forecast
  - Studying flow in bottling / filling devices

- There are different aims, such as:
  - Perceiving and understanding physical phenomena
  - Modeling of flow processes
  - Optimization in technical design
  - Search for damage reasons
VF Basics
- **Flow data** is converted to a **vector field** by **interpolation** of the vectors inside the cells.
scalar field  
\[ s : \mathbb{E}^n \to \mathbb{R} \]

vector field  
\[ \mathbf{v} : \mathbb{E}^n \to \mathbb{R}^m \]

tensor field  
\[ \mathbf{T} : \mathbb{E}^n \to \mathbb{R}^{m \times b} \]
scalar field

\[ s : \mathbb{R}^n \to \mathbb{R} \]

\[ s(x) \quad \text{with } x \in \mathbb{R}^n \]

vector field

\[ \mathbf{v} : \mathbb{R}^n \to \mathbb{R}^m \]

\[ \mathbf{v}(x) = \begin{pmatrix} c_1(x) \\ \vdots \\ c_m(x) \end{pmatrix} \quad \text{with } x \in \mathbb{R}^n \]

tensor field

\[ \mathbf{T} : \mathbb{R}^n \to \mathbb{R}^{m \times b} \]

\[ \mathbf{T}(x) = \begin{pmatrix} c_{11}(x) & \cdots & c_{1b}(x) \\ \vdots & \ddots & \vdots \\ c_{m1}(x) & \cdots & c_{mb}(x) \end{pmatrix} \quad \text{with } x \in \mathbb{R}^n \]
Scalar field

\[ s : \mathbb{R}^n \rightarrow \mathbb{R} \]

\[ s(x) \quad \text{with } x \in \mathbb{R}^n \]

Vector field

\[ \mathbf{v} : \mathbb{R}^n \rightarrow \mathbb{R}^m \]

\[ \mathbf{v}(x) = \begin{pmatrix} c_1(x) \\ \vdots \\ c_m(x) \end{pmatrix} \quad \text{with } x \in \mathbb{R}^n \]

2D vector field

\[ \mathbf{v}(x, y) = \begin{pmatrix} u(x, y) \\ v(x, y) \end{pmatrix} \]

Tensor field

\[ \mathbf{T} : \mathbb{R}^n \rightarrow \mathbb{R}^{m \times b} \]

\[ \mathbf{T}(x) = \begin{pmatrix} c_{11}(x) & \cdots & c_{1b}(x) \\ \vdots & \ddots & \vdots \\ c_{m1}(x) & \cdots & c_{mb}(x) \end{pmatrix} \quad \text{with } x \in \mathbb{R}^n \]
vector field

\[ \mathbf{v} : \mathbb{R}^n \rightarrow \mathbb{R}^m \]

tensor field

\[ \mathbf{T} : \mathbb{R}^n \rightarrow \mathbb{R}^{m \times b} \]

\[ \mathbf{v}(x, y) = \begin{pmatrix} u(x, y) \\ v(x, y) \end{pmatrix} \quad \nabla \mathbf{v}(x, y) = \mathbf{J}(x, y) = \begin{pmatrix} u_x & u_y \\ v_x & v_y \end{pmatrix} \]

2D vector field

Jacobian of 2D vector field
Supposed to be continuous and differentiable. This means we can compute its partial differentials and write them into the 2x2 matrix:

\[ \mathbf{v}(x, y) = \left( \begin{array}{c} u(x, y) \\ v(x, y) \end{array} \right) \]
\[ \nabla \mathbf{v}(x, y) = \mathbf{J}(x, y) = \left( \begin{array}{cc} u_x & u_y \\ v_x & v_y \end{array} \right) \]
vector field

\[ \mathbf{v} : \mathbb{R}^n \rightarrow \mathbb{R}^m \]

\[ \mathbf{v}(x, y) = \begin{pmatrix} u(x, y) \\ v(x, y) \end{pmatrix} \]
vector field

\[ \mathbf{v} : \mathbb{R}^n \rightarrow \mathbb{R}^m \]

\[ \mathbf{v}(x, y) = \begin{pmatrix} u(x, y) \\ v(x, y) \end{pmatrix} \]

steady vector field

parameter-independent
vector field

\[ \mathbf{v} : \mathbb{E}^n \rightarrow \mathbb{R}^m \]

\[ \mathbf{v}(x, y) = \begin{pmatrix} u(x, y) \\ v(x, y) \end{pmatrix} \]

**steady** vector field

\[ \mathbf{v} : \mathbb{E}^{n+1} \rightarrow \mathbb{R}^m \]

\[ \mathbf{v}(x, y, t) = \begin{pmatrix} u(x, y, t) \\ v(x, y, t) \end{pmatrix} \]

**unsteady** vector field
vector field

**steady** vector field

\[ \mathbf{v} : \mathbb{R}^n \rightarrow \mathbb{R}^m \]

\[ \mathbf{v}(x, y) = \begin{pmatrix} u(x, y) \\ v(x, y) \end{pmatrix} \]

**unsteady** vector field

\[ \mathbf{v} : \mathbb{R}^{n+1} \rightarrow \mathbb{R}^m \]

\[ \mathbf{v}(x, y, t) = \begin{pmatrix} u(x, y, t) \\ v(x, y, t) \end{pmatrix} \]

**two-parameter-dependent**

\[ \mathbf{v} : \mathbb{R}^{n+2} \rightarrow \mathbb{R}^m \]

\[ \mathbf{v}(x, y, s, t) = \begin{pmatrix} u(x, y, s, t) \\ v(x, y, s, t) \end{pmatrix} \]
Jacobian of a 3D vector field:
- Consists of all first-order information of $\mathbf{v}$

\[
\frac{\delta \mathbf{v}}{\delta x} = \mathbf{v}_x = \begin{pmatrix}
\frac{\delta u}{\delta x} \\
\frac{\delta v}{\delta x} \\
\frac{\delta w}{\delta x}
\end{pmatrix}, \quad \frac{\delta \mathbf{v}}{\delta y} = \mathbf{v}_y = \begin{pmatrix}
\frac{\delta u}{\delta y} \\
\frac{\delta v}{\delta y} \\
\frac{\delta w}{\delta y}
\end{pmatrix}, \quad \frac{\delta \mathbf{v}}{\delta z} = \mathbf{v}_z = \begin{pmatrix}
\frac{\delta u}{\delta z} \\
\frac{\delta v}{\delta z} \\
\frac{\delta w}{\delta z}
\end{pmatrix}
\]

First-order partial derivatives of $\mathbf{v}$

<table>
<thead>
<tr>
<th>Jacobian of $\mathbf{v}$</th>
</tr>
</thead>
</table>
| $\mathbf{J}_\mathbf{v}(\mathbf{x}) = (\mathbf{v}_x, \mathbf{v}_y, \mathbf{v}_z) = \begin{pmatrix}
\mathbf{v}_x & \mathbf{v}_y & \mathbf{v}_z \\
\mathbf{w}_x & \mathbf{w}_y & \mathbf{w}_z
\end{pmatrix}$ |
• Jacobian of an *unsteady* 3D vector field:

\[
J_v(x, y, z, t) = (v_x, v_y, v_z, v_t) = \begin{pmatrix}
u_x & u_y & u_z & u_t \\
v_x & v_y & v_z & v_t \\
w_x & w_y & w_z & w_t \\
\end{pmatrix}
\]
• Divergence of v:

• scalar field

• observe transport of a small ball around a point
  • expanding volume \(\Rightarrow\) positive divergence
  • contracting volume \(\Rightarrow\) negative divergence
  • constant volume \(\Rightarrow\) zero divergence

\[
\text{div } \mathbf{v} = \frac{\delta u}{\delta x} + \frac{\delta v}{\delta y} + \frac{\delta w}{\delta z} = u_x + v_y + w_z
\]

\[
\text{div } \mathbf{v} \equiv 0 \Leftrightarrow \mathbf{v} \text{ is incompressible}
\]
• Curl of $v$:

• vector field

• also called rotation (rot) or vorticity

• indication of how the field swirls at a point

\[
\text{curl } v = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
\frac{\delta}{\delta x} & \frac{\delta}{\delta y} & \frac{\delta}{\delta z} \\
u & v & w
\end{vmatrix} = \begin{pmatrix}
w_y - v_z \\
u_z - w_x \\
v_x - u_y
\end{pmatrix}
\]
● **Curl of v:**

● **paddle wheel model:**
  ● insert paddle wheel in a flow
  ● orient such that its rate of rotation is maximal
  ● \( \Rightarrow \) **curl** \( v \) is parallel to main rotation axis
  ● \( \Rightarrow \) \(|\text{curl } v|\) is corresponds to rate of rotation

● **golf ball model**
  ● consider golf ball in \( v \)
  ● is transported and rotates
  ● \( \Rightarrow \) **curl** \( v \) is parallel to main rotation axis
  ● \( \Rightarrow \) \(|\text{curl } v|\) is corresponds to rate of rotation
Flow Data

Data sources:

- **flow simulation:**
  - airplane- / ship- / car-design
  - weather simulation (air-, sea-flows)
  - medicine (blood flows, etc.)

- **flow measurement:**
  - wind tunnels, water channels
  - optical measurement techniques

- **flow models (analytic):**
  - differential equation systems (dynamic systems)
Flow Data

Simulation:
• flow: estimate (partial) differential equation systems
• set of samples (n-dims. of data), e.g., given on a curvilinear grid
• most important primitive: tetrahedron and hexahedron (cell)
• could be adaptive grids

Analytic:
• flow: analytic formula, differential equation systems \( \frac{dx}{dt} \) (dynamical system)
• evaluated where ever needed

Measurement:
• vectors: taken from instruments, often computed on a uniform grid
• optical methods + image recognition, e.g.: PIV (particle image velocimetry)
What is a vector field?
(from simulation)

- Our data is *samples* of a vector field, on some *mesh*

Comparison with Reality

Experiment

Simulation
Classification of Visualization Techniques

• **Direct:** overview of vector field, minimal computation, e.g. glyphs, color mapping

• **Texture-based:** covers domain with a convolved texture, e.g., Spot Noise, LIC, ISA, IBFV(S)

• **Geometric:** a discrete object(s) whose geometry reflects flow characteristics, e.g. streamlines

• **Feature-based:** both automatic and interactive feature-based techniques, e.g. flow topology
Direct Visualization
• **Elementary Methods:**

• Present the complete data set at low level of abstraction

• Mapping is direct, without complex conversions or extractions

• 3 main representations:
  • color coding
  • arrow plots
  • icons

• ➔ very frequently used
● **Color Coding:**

● Extract scalar field from vector field (i.e. magnitude, curvature, pressure, temperature)

● 2D: direct color coding to visualize them

● same for slices or boundary surfaces in 3D

● 3D → volume visualization (FlowVis → VolVis)

● → loss of information
Examples for Color Coding:

Vertical Velocity Distribution. The baseline condition illustrates the flow pattern expressing the vertical velocity components over the range front-10 to 15 m/s. The baseline case with only a single elevation of overfire air produces a high velocity flow channel attached to the front wall with an associated recirculation down the rear wall of the main combustor section. The optimized case includes a revised overfire air configured to centralize the vertical flow region. The peak vertical velocities and size of the recirculation...
Volume illustration for flow visualization [Svakine et al 05]

Figure 3: Volume illustrations of flow around the X38 spacecraft. (a) is an illustration of density flow and shock around the bow, while (b) highlights the vortices created above the fins of the spacecraft.

Figure 6: Use of two-dimensional transfer function with the Laplacian operator and other flow quantities. (a) shows heat inflow (red) and outflow (blue). (b) shows all values of the Laplacian of velocity magnitude in the tornado dataset. (c) visualizes the cloud TKE using the Laplacian to highlight boundaries (white) and velocity for silhouetting. (d) highlights emerging flow structures in the convection dataset using banding of the second derivative magnitude of the temperature field.
• Arrow plots:
  • also called hedgehog plots
  • represent velocity as arrows at regular locations, e.g., place arrows at grid points
  • ➔ overloading possible
  • arrows: (scaled) unit length or encode magnitude
  • well-established for 2D
• Arrows visualize
  – Direction of vector field
  – Orientation
  – Magnitude:
    • Length of arrows
    • Color coding
• [Kirby et al 99]: multiple values of 2d flow data by layering concept related to painting process of artists

Figure 1: Typical visualization methods for 2D flow past a cylinder at Reynolds number 100. On the left, we show only the velocity field. On the right, we simultaneously show velocity and vorticity. Vorticity represents the rotational component of the flow. Clockwise vorticity is blue, counterclockwise yellow.
Arrows in 2D

Scaled arrows vs. color-coded arrows
• 3D arrow plots:

• Occlusion problem → careful seeding

• Ambiguity problem

• Solutions:
  • 3D icons (cylinder + cone)
  • Highlight parts of error plot which points into a user defined direction
• Icons:

• Place icon at selected locations and encode different values of the flow

• Seeding strategy necessary (usually interactive)

• Example: probe for local flow visualization [de Leeuw, van Wijk 93]
• Advantages and disadvantages of glyphs and arrows:
  + Simple
  + 3D effects
  - Inherent occlusion effects
  - Poor results if magnitude of velocity changes rapidly
    (Use arrows of constant length and color code magnitude)
L22: Vector Fields 2

REQUIRED READING