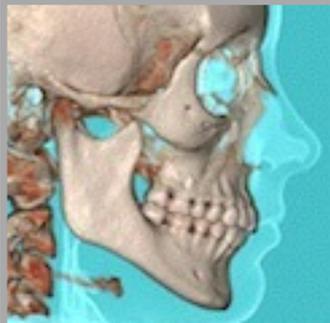
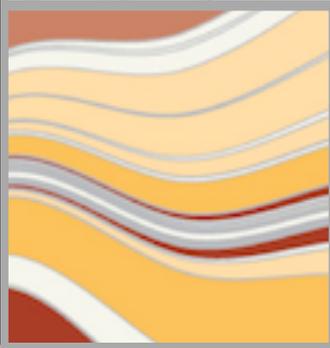
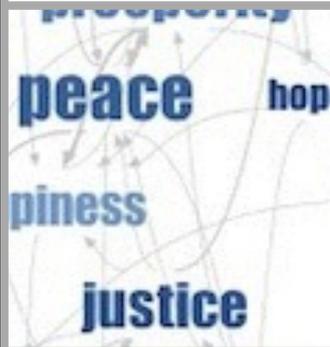
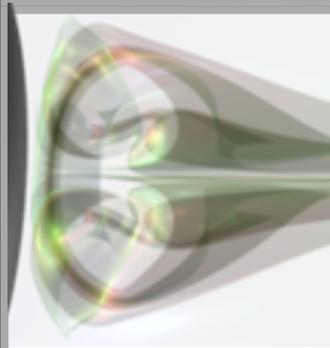
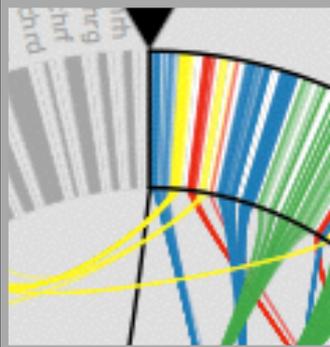
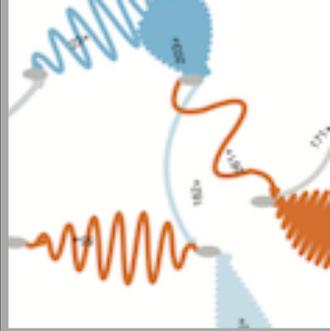


MAPS

Miriah Meyer
University of Utah



administrivia . . .

-parallel coordinates assignment due Thursday

last time . . .

Tables

Items

Attributes

Networks & Trees

Items (nodes)

Links

Attributes

Fields

Grids

Positions

Attributes

Geometry

Items

Positions

Clusters, Sets, Lists

Items

Tables

Items

Attributes

Networks &
Trees

Items (nodes)

Links

Attributes

Fields

Grids

Positions

Attributes

Geometry

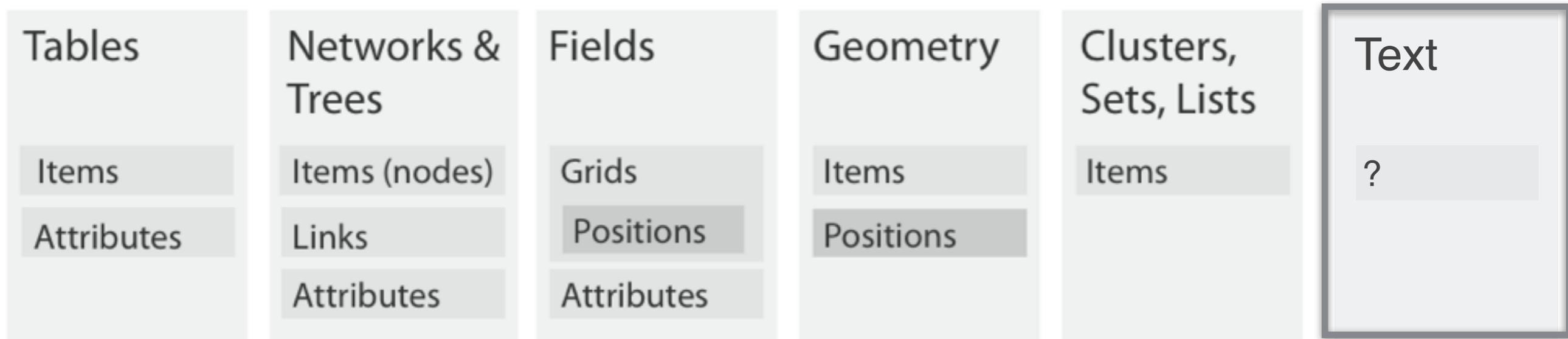
Items

Positions

Clusters,
Sets, Lists

Items

Text



WHAT DOES IT MEAN TO BE AN "ITEM"?

single document

Tag Clouds / Word Clouds

abstract accepted analogue applications applying attuned bar burgeoning challenging
chapters chart collections combine communicate conducted convert data date difficult
discussed earlier effectively end evaluation evocative familiar field focus focused form
general goal graph highly human hundreds ideas images improve
information innovative insight kinds line makes means
meta-analysis nature new numbers order ost perceive perceptual points positive
problems providing purpose range rapidly read reading reasons representations results
retrieval robust search shortciten{chen2000esi} shortcite{larkin1987dsw} shown space
studies successful system table task tasks text textual time translate underlying
usability vibrant visual visualization visually web wide widely

<http://www.tagcrowd.com>



<http://www.wordle.com>

Visualizations : definitions of visualization word tree

Uploaded by: mhalle

Created at: Wednesday May 21 2008, 11:37 PM

Tags: text

Search Start End

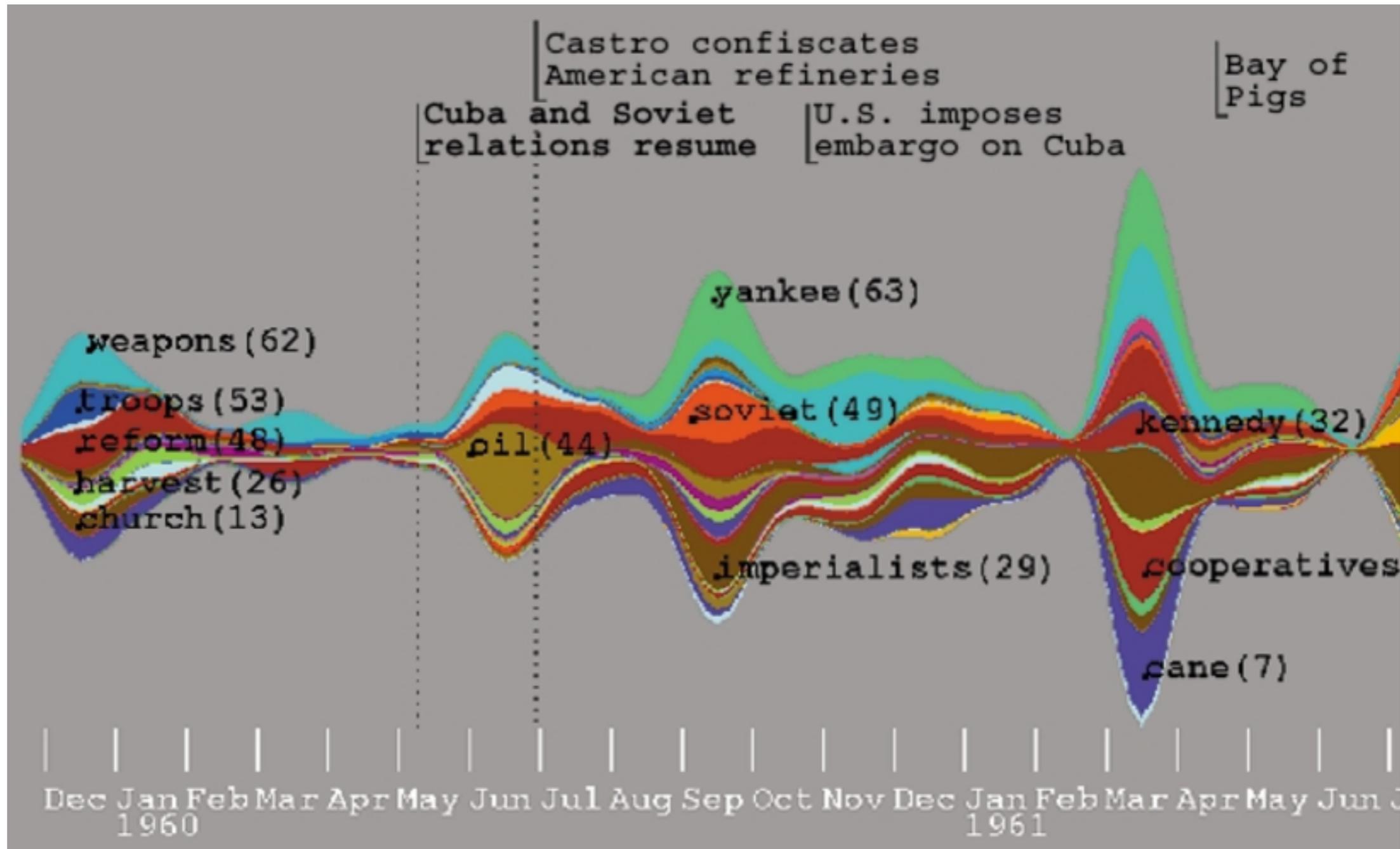
63 hits

visualization



collection of documents

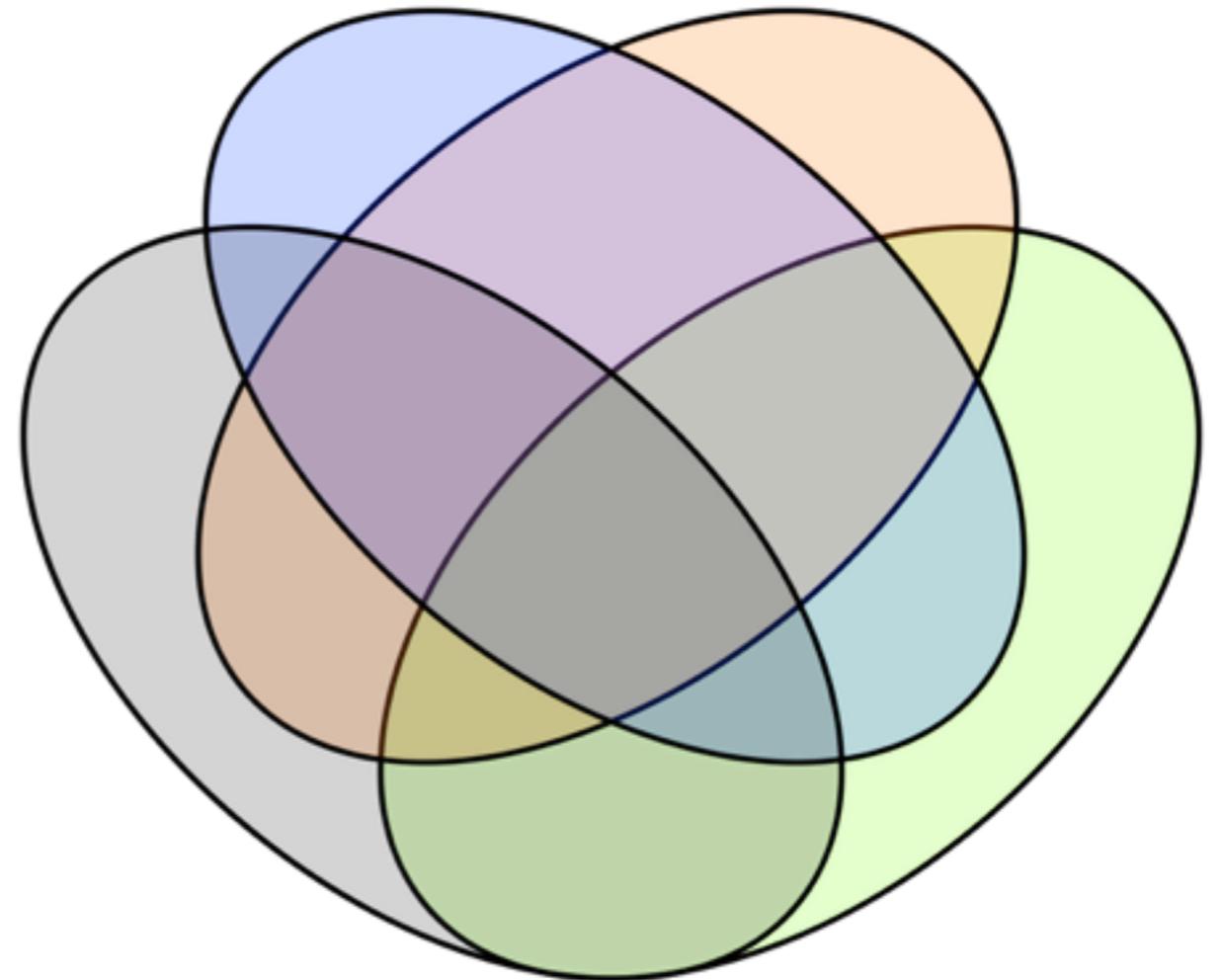
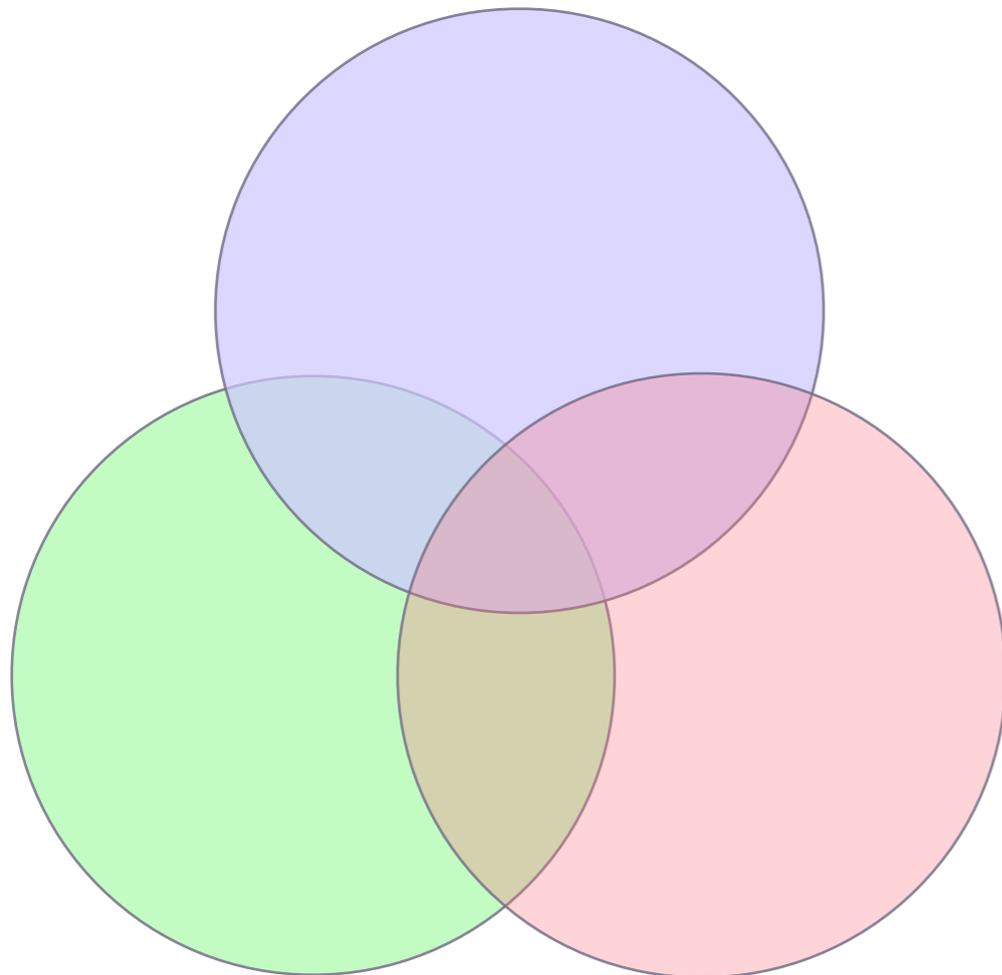
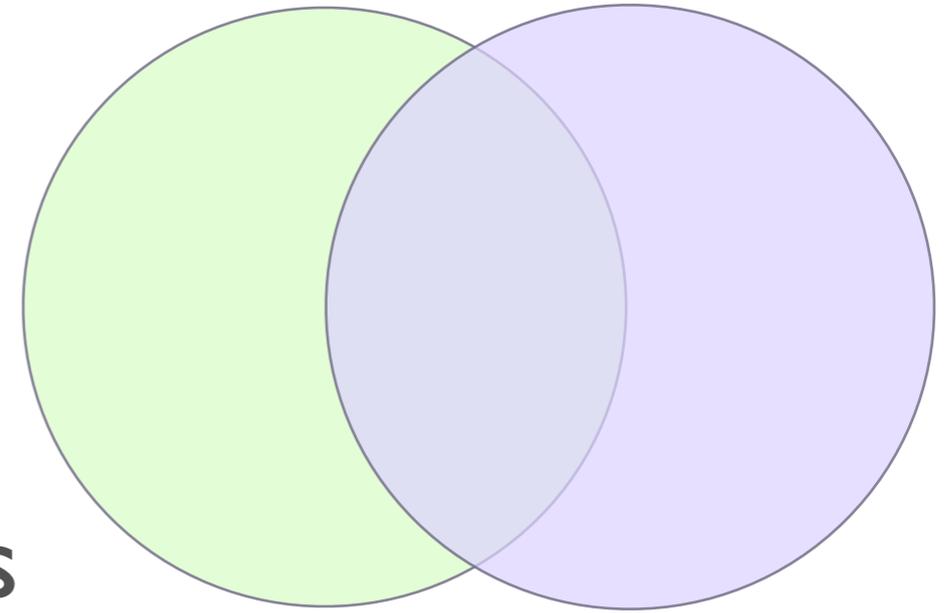
Showing Temporal Relationships: ThemeRiver (Stream Graph)



SETS

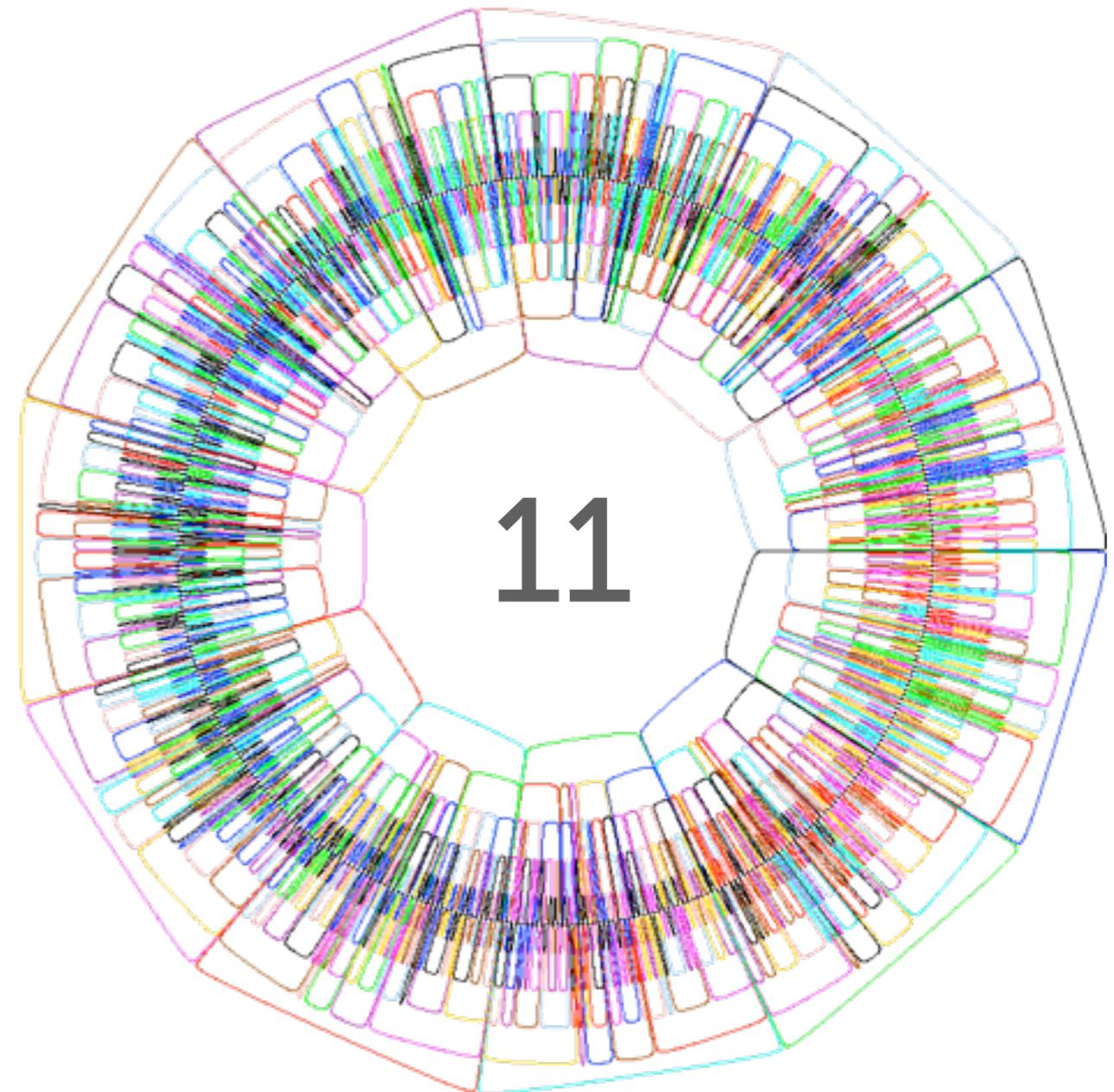
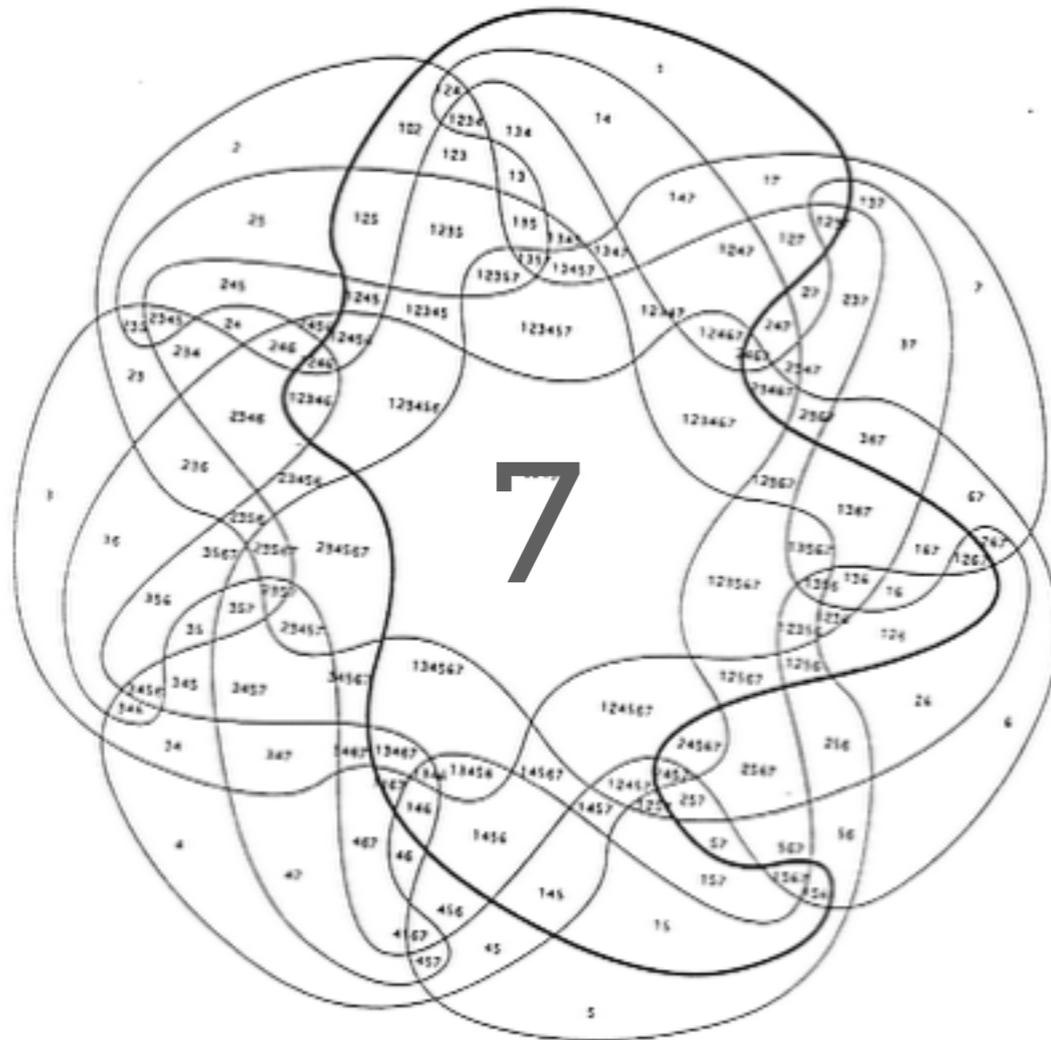
venn diagrams

-show all possible relationships



venn diagrams

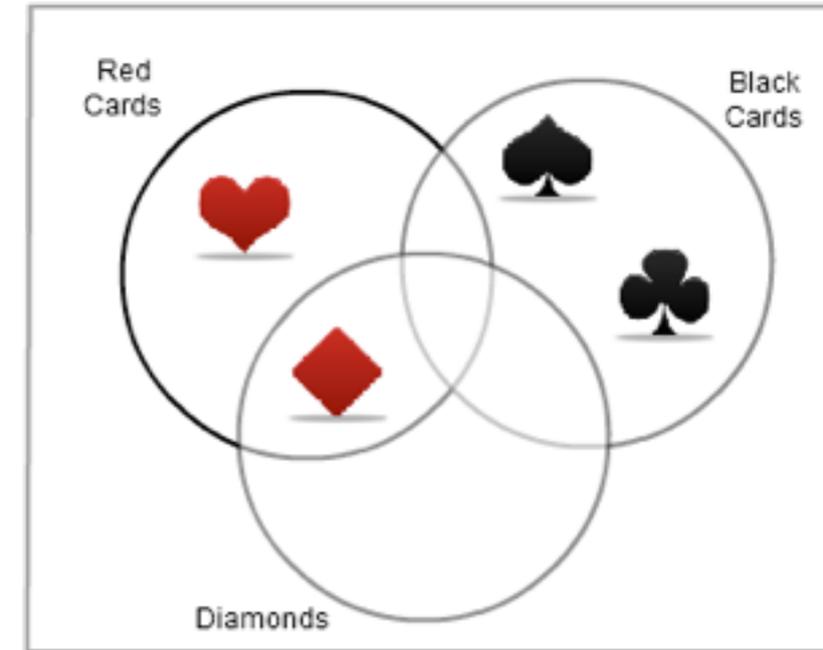
-get messy fast



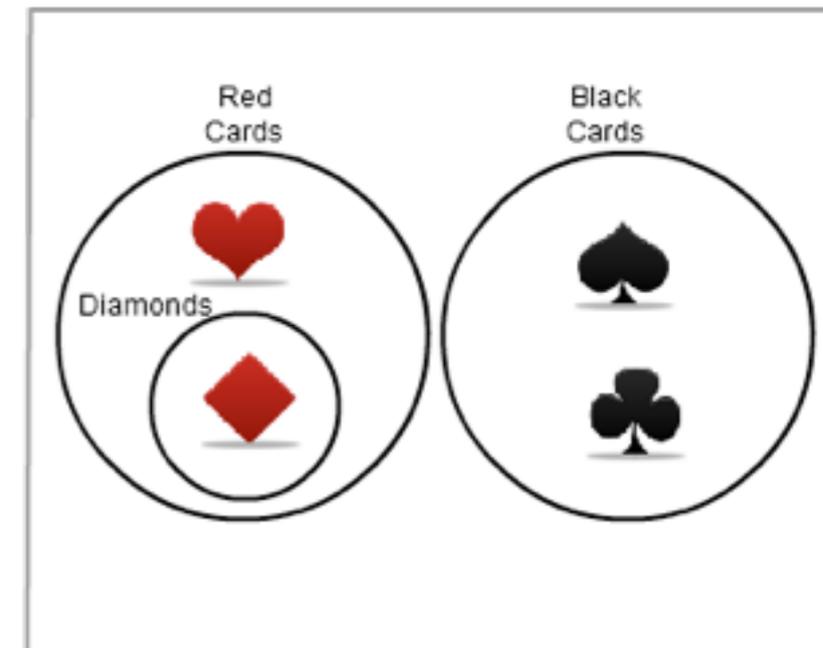
euler diagrams

-show only existing relationships

V
E
N
N

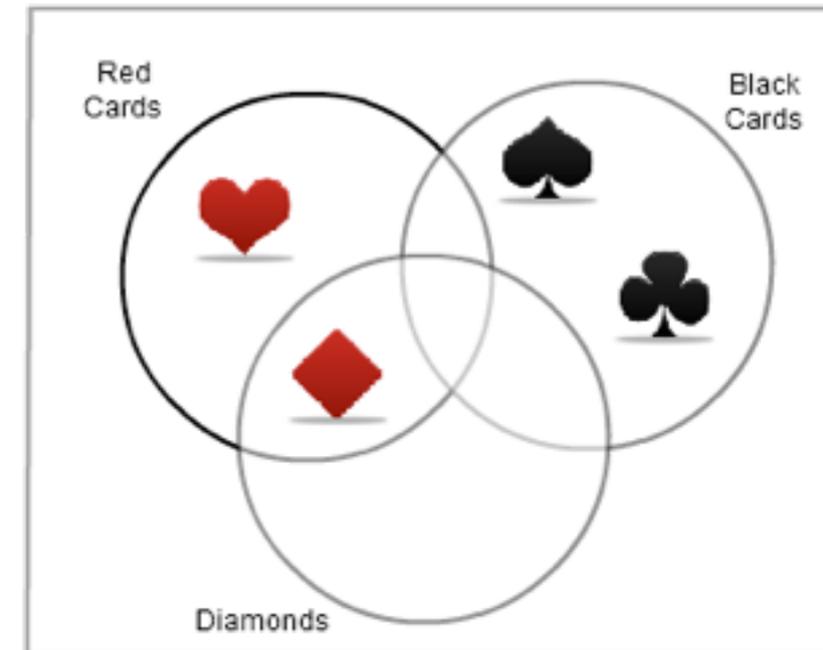
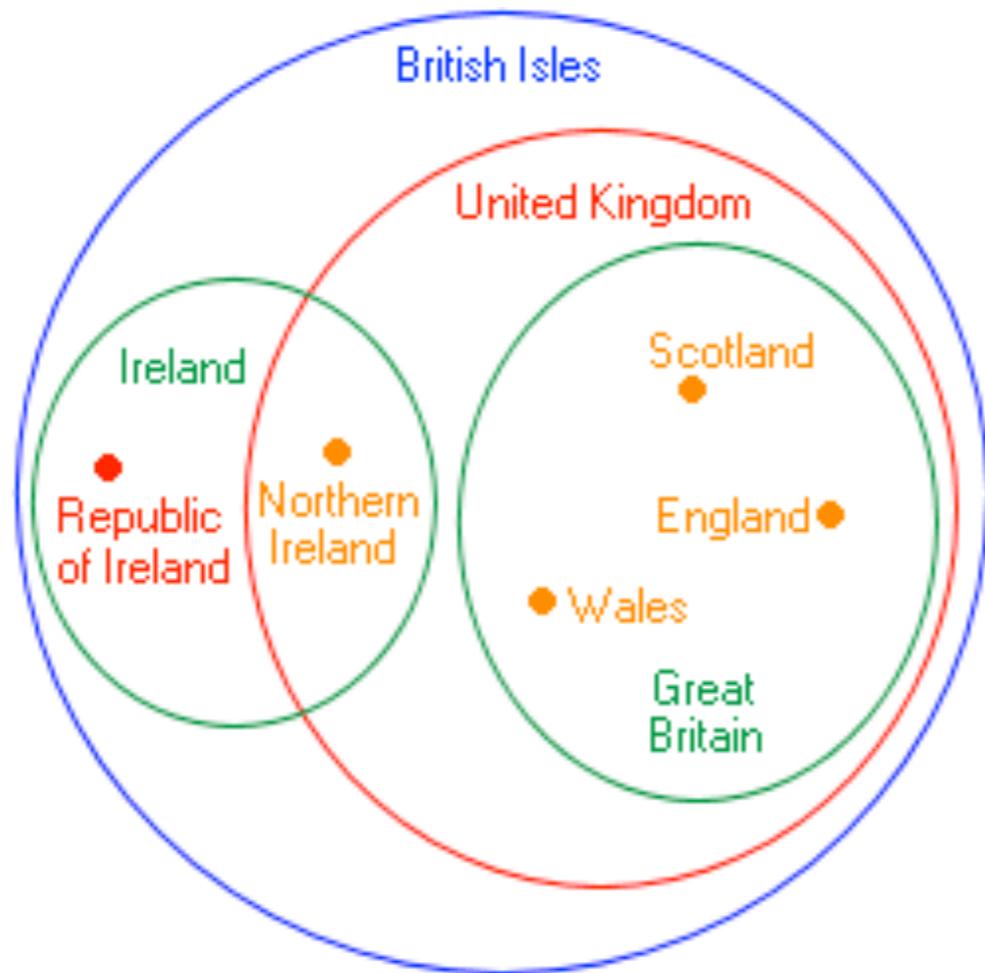


E
U
L
E
R

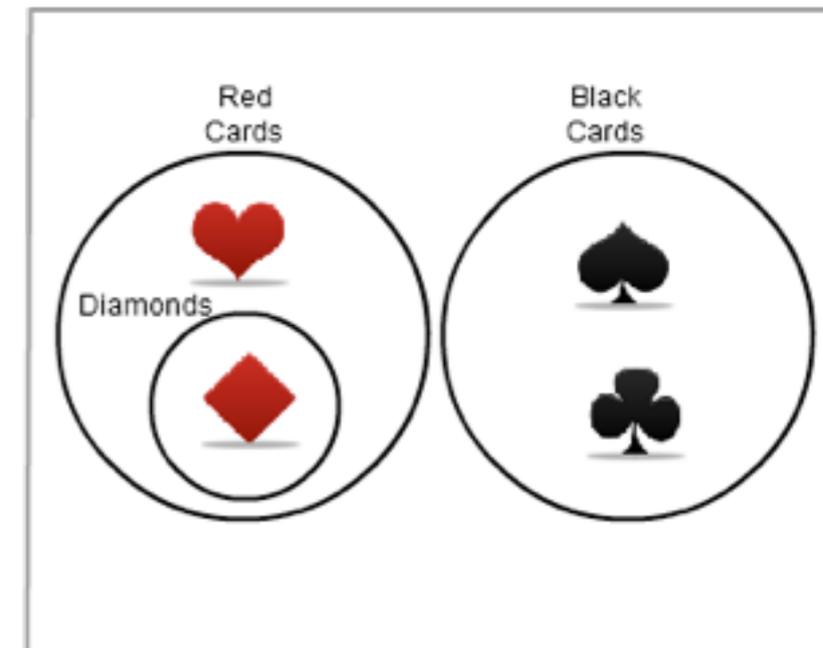


euler diagrams

-show only existing relationships



V
E
N
N

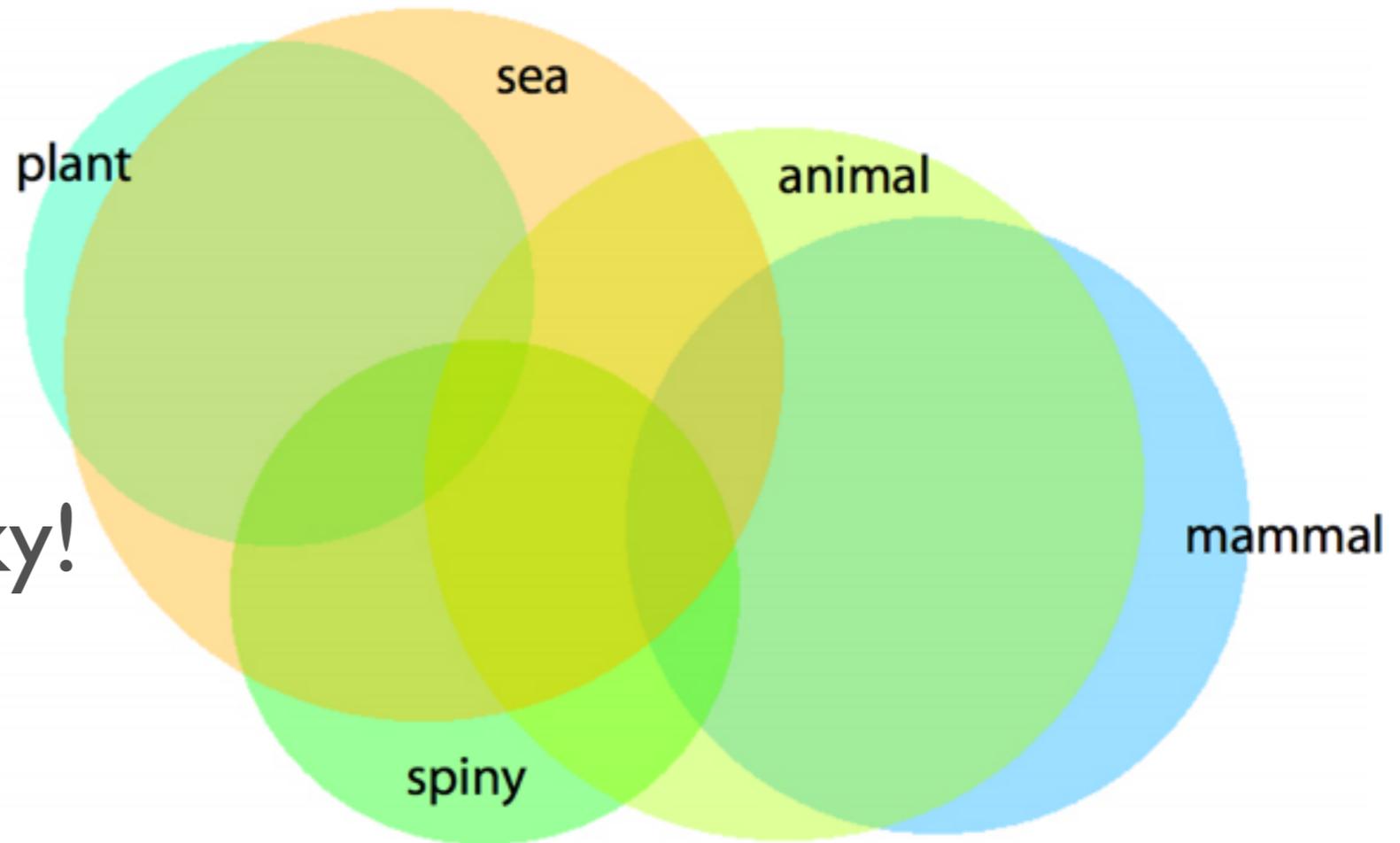


E
U
L
E
R

venn & euler diagrams

-adjust for area

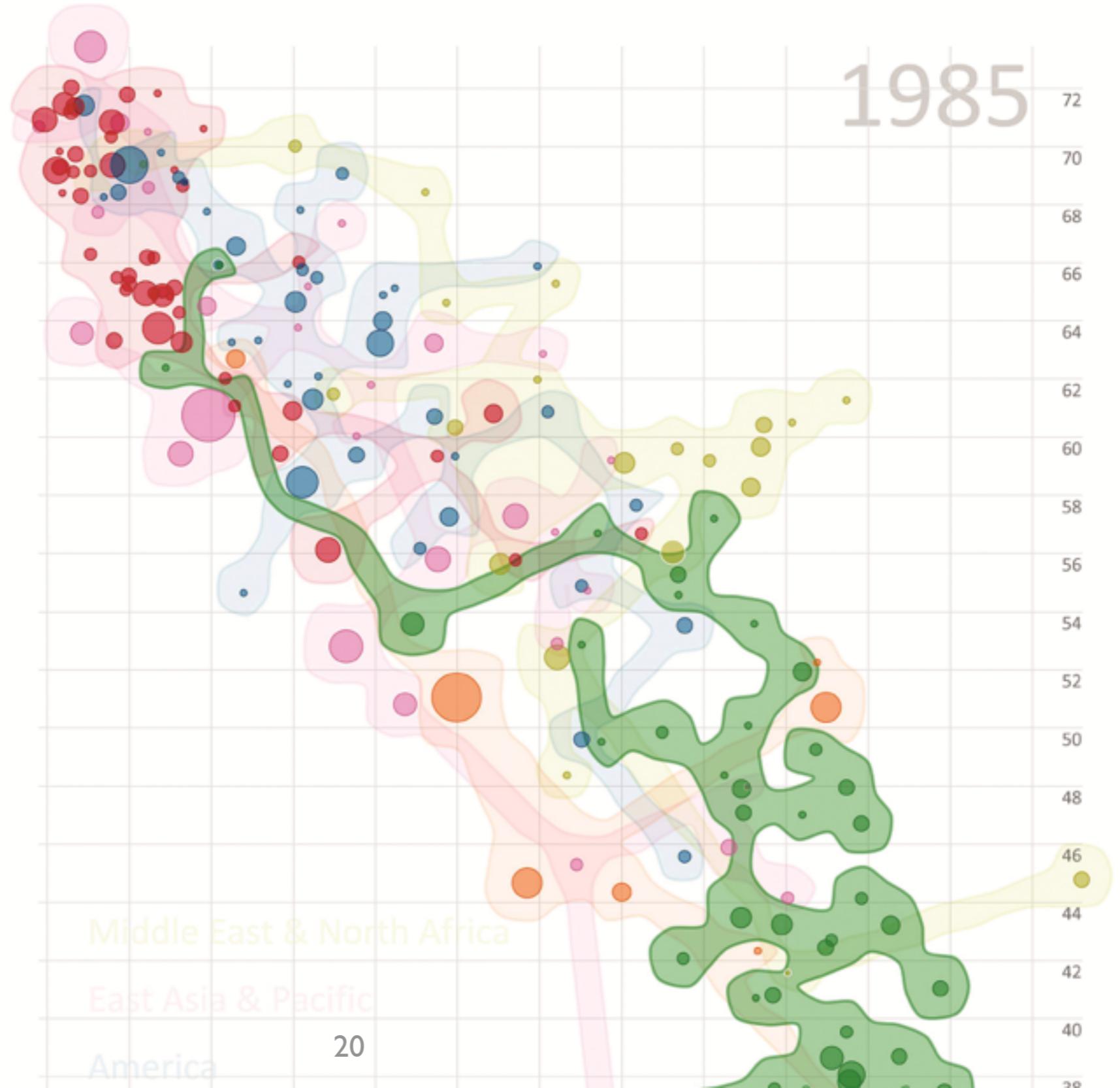
-starts getting tricky!



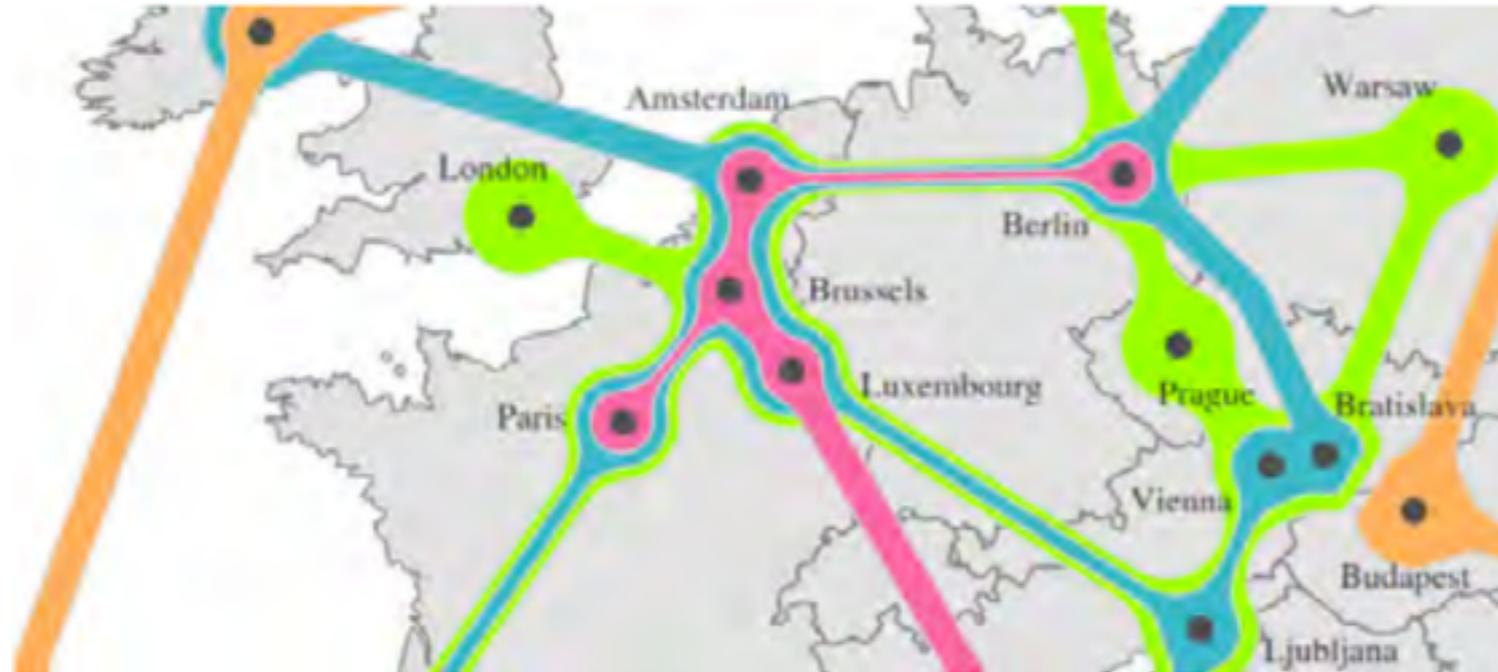
visualizing sets with constraints

bubble sets

-connect
points

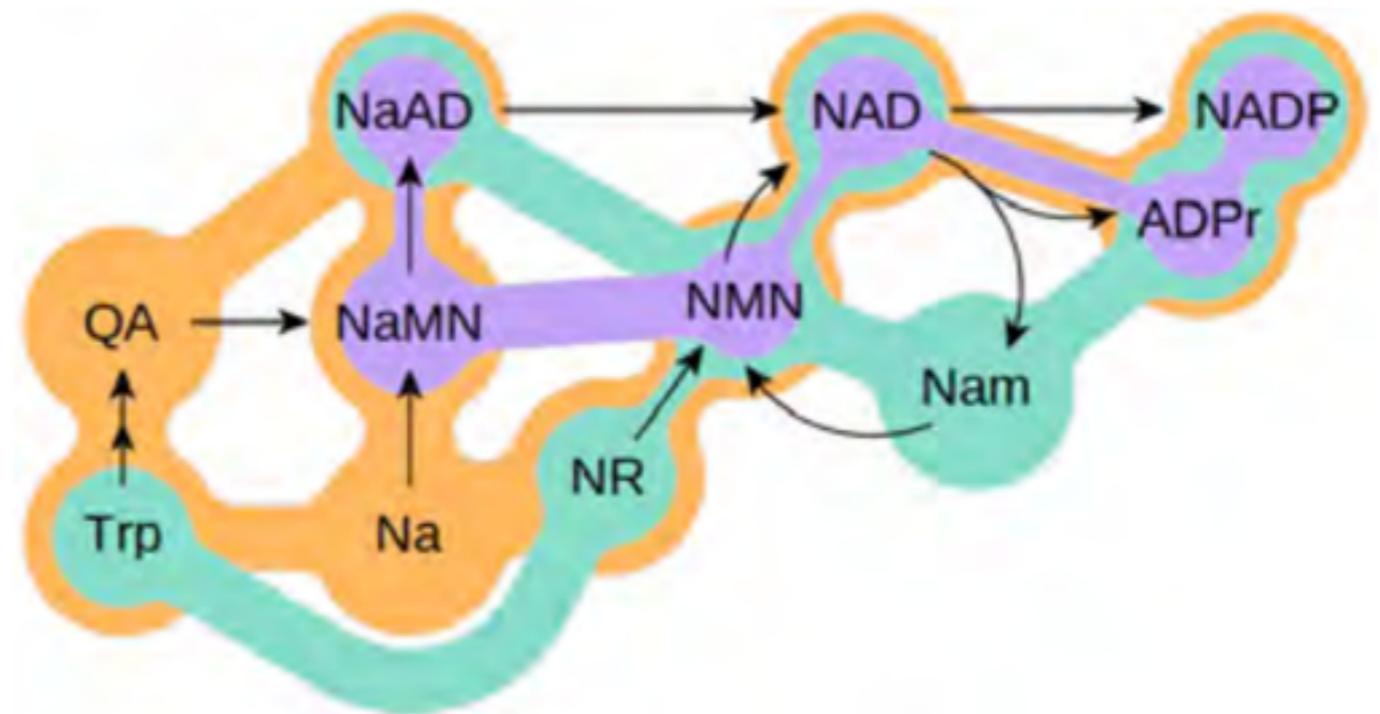


kelp diagrams



-cities on a map

-metabolic network



today . . .

MAPS

- landmarks**
- discrete data**
- continuous data**
- choropleths**
- cartograms**
- projections**

-landmarks

-discrete data

-continuous data

-choropleths

-cartograms

-projections

Toronto

Ottawa

Get Directions

[Search the map](#)

[Find businesses](#)

[Get directions](#)

Maps

[Print](#) [Email](#) [Link to this page](#)

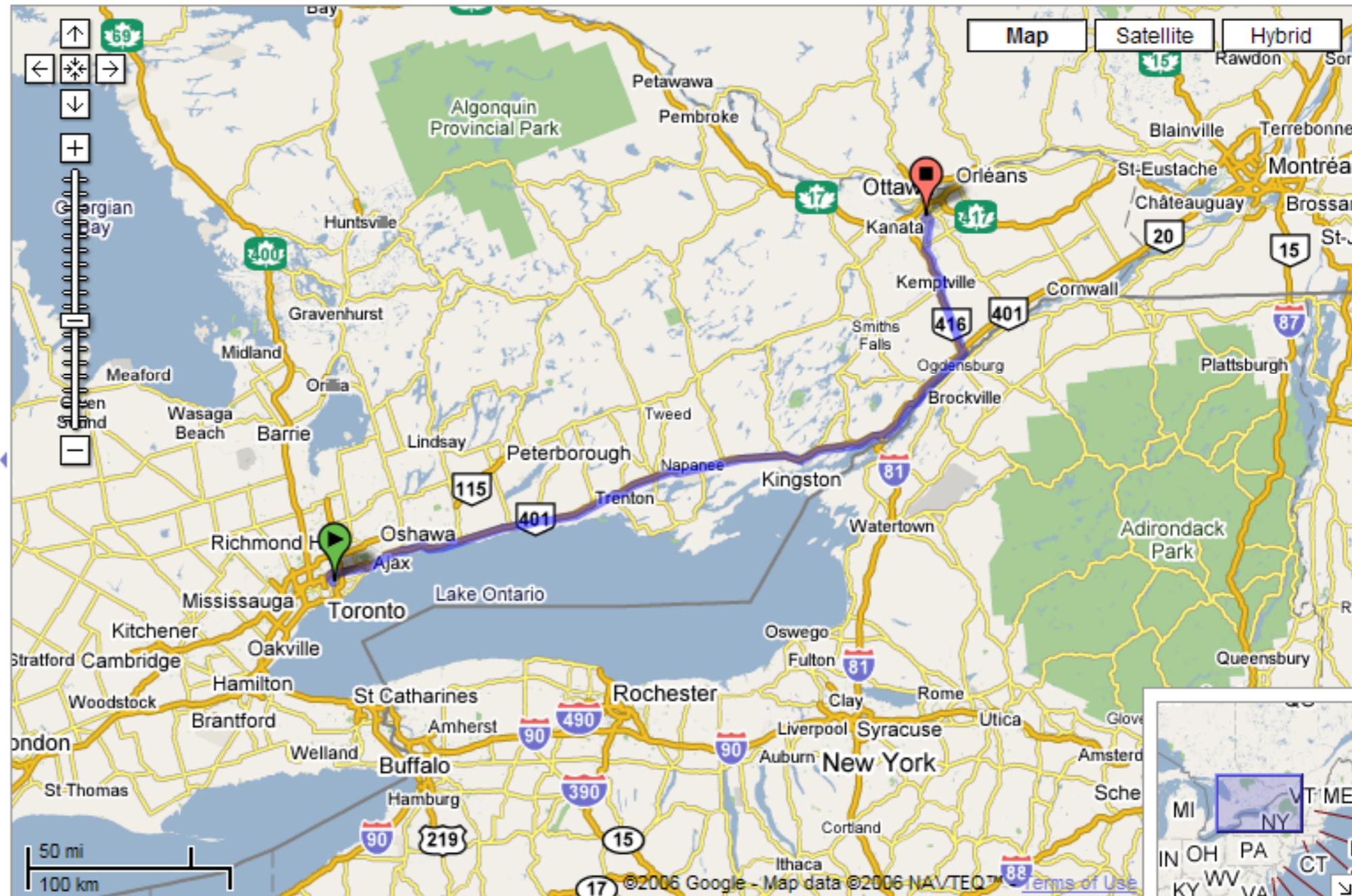
Start address: Toronto, ON
Canada

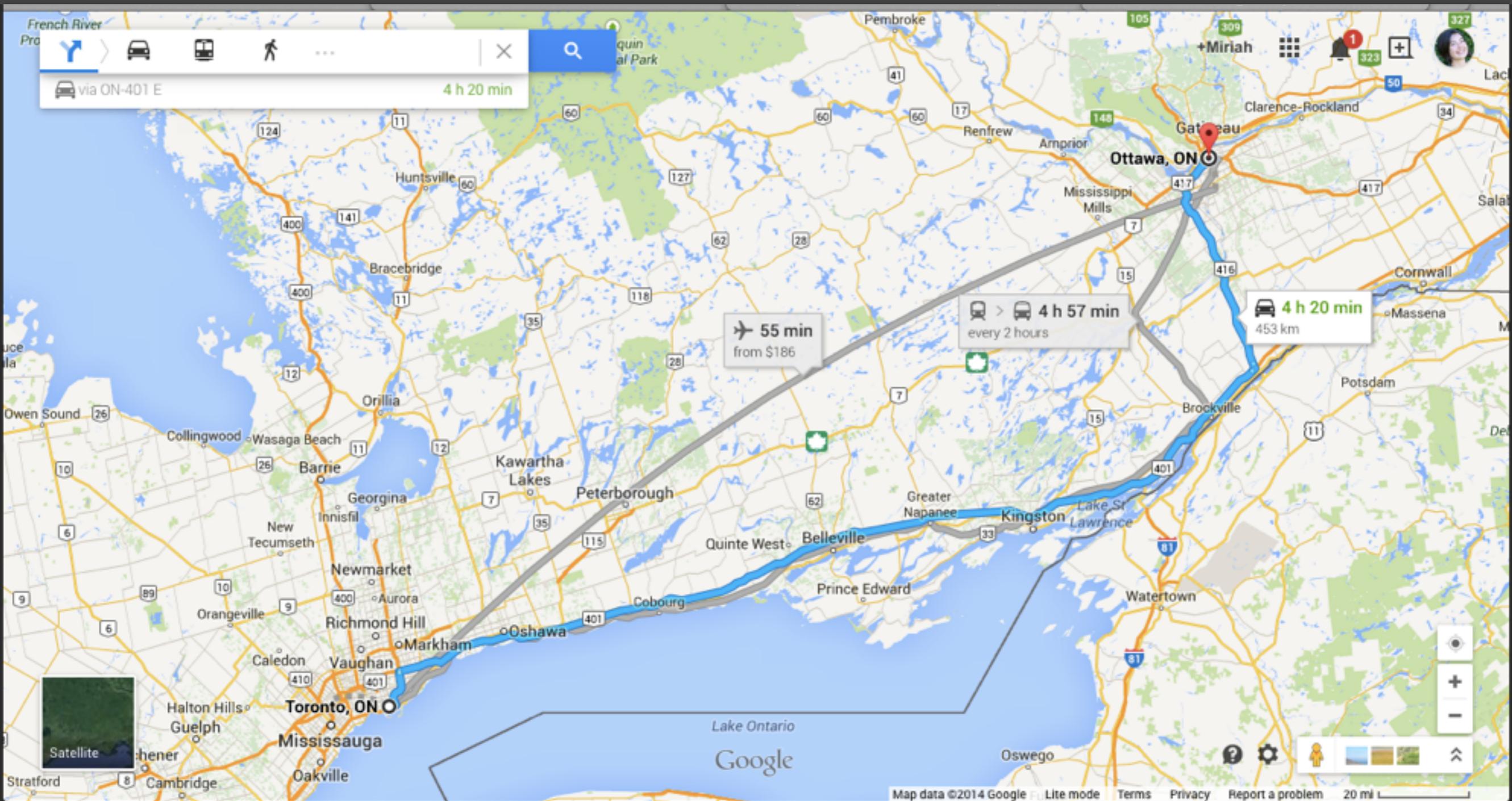
End address: Ottawa, ON
Canada

Distance: 431 km (about 6 hours 2 mins)

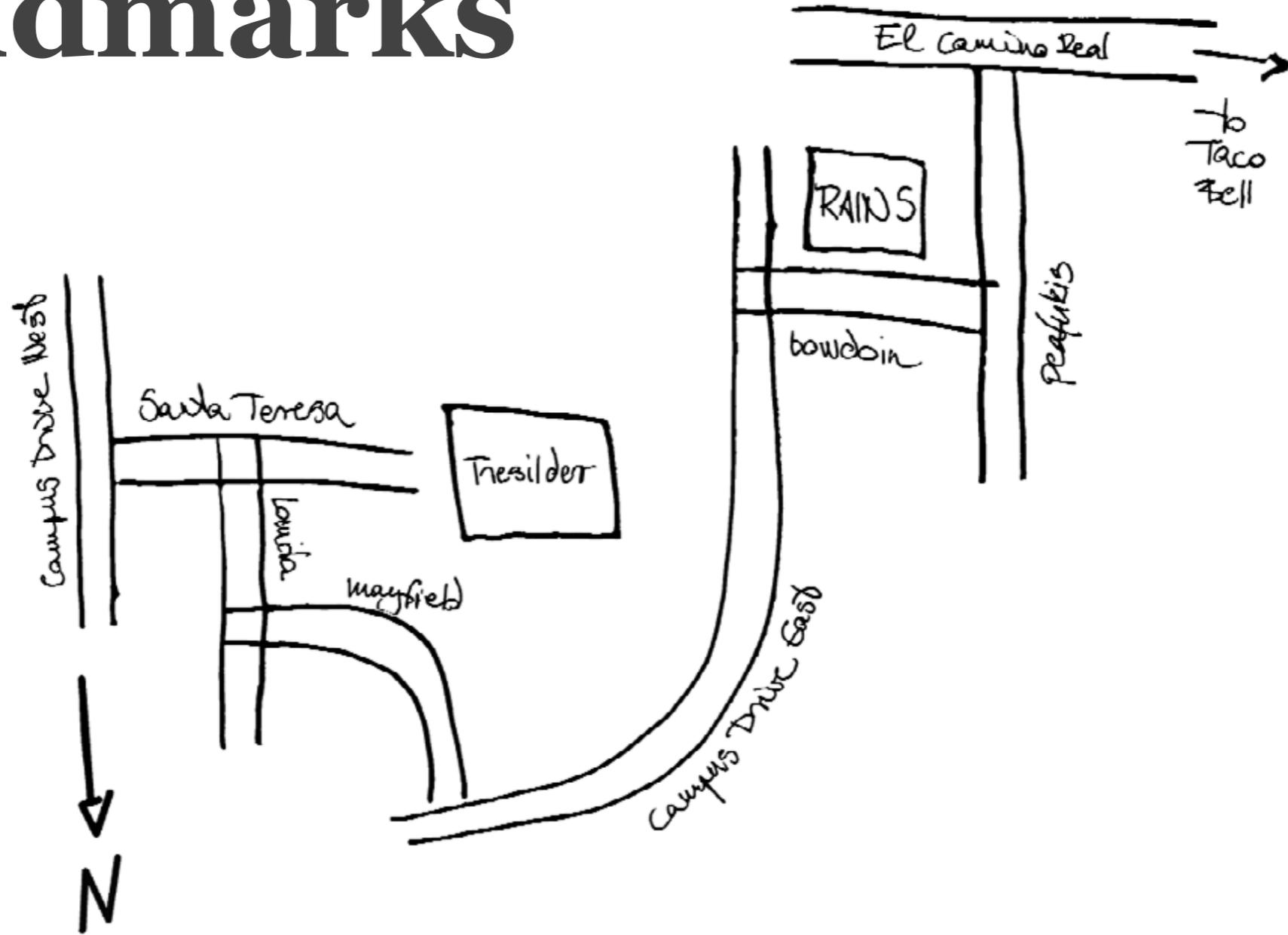
[Get reverse directions](#)

1. Head west from St Leonards Ave - go 0.3 km
2. Turn right at Mt Pleasant Rd - go 0.4 km
3. Turn left at Lawrence Ave E - go 0.5 km
4. Turn right at Yonge St - go 3.1 km
5. Turn right at Lord Seaton Rd - go 83 m
6. Bear left - go 54 m
7. Bear left into the HWY-401 E entry ramp - go 0.3 km
8. Merge into HWY-401 Collectors E - go 29.3 km
9. Continue on HWY-401 E - go 319 km
10. Take the HWY-416 N exit 721A to Kemptville/Ottawa - go 57.7 km
11. Take exit 57 to (HWY-16)/Nepean/Manotick - go 0.6 km
12. Turn right at Bankfield Rd - go 1.7 km
13. Turn left at Prince of Wales Dr - go 13.2 km





landmarks



Rendering Effective Route Maps: Improving Usability Through Generalization

Maneesh Agrawala Chris Stolte
Stanford University*

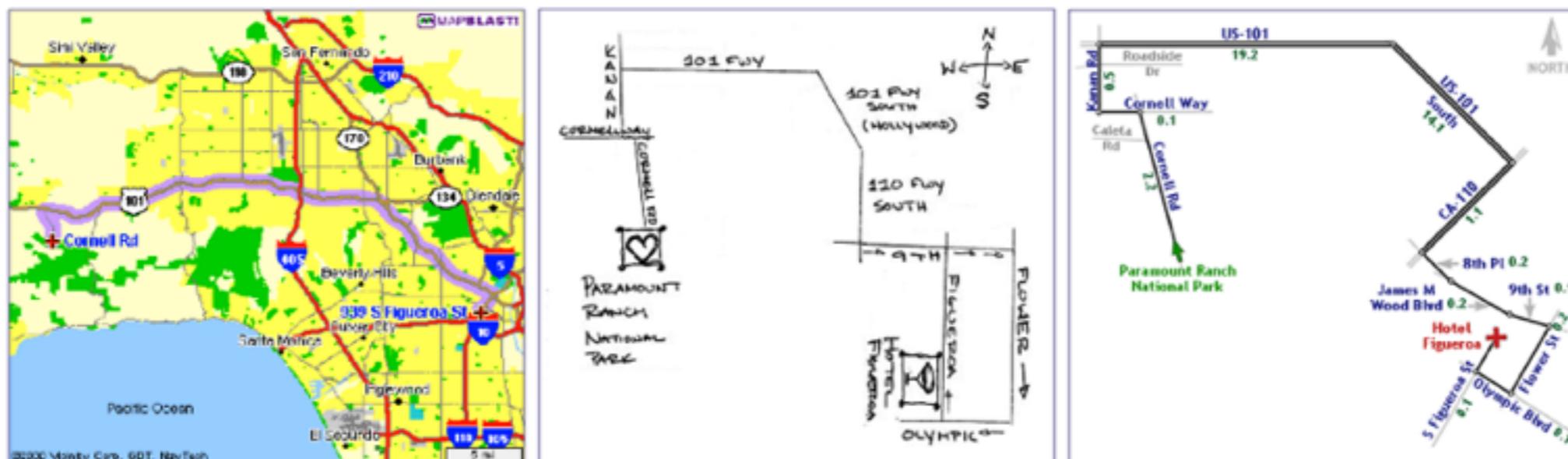


Figure 1: Three route maps for the same route rendered by (left) a standard computer-mapping system, (middle) a person, and (right) LineDrive, our route map rendering system. The standard computer-generated map is difficult to use because its large, constant scale factor causes the short roads to vanish and because it is cluttered with extraneous details such as city names, parks, and roads that are far away from the route. Both the handdrawn map and the LineDrive map exaggerate the lengths of the short roads to ensure their visibility while maintaining a simple, clean design that emphasizes the most essential information for following the route. Note that the handdrawn map was created without seeing either the standard computer-generated map or the LineDrive map. *(Handdrawn map courtesy of Mia Trachinger.)*

Abstract

Route maps, which depict a path from one location to another, have emerged as one of the most popular applications on the Web. Current computer-generated route maps, however, are often very difficult to use. In this paper we present a set of cartographic generalization techniques specifically designed to improve the usability of route maps. Our generalization techniques are based both on cognitive psychology research studying how route maps are used and on an analysis of the generalizations commonly found in hand-drawn route maps. We describe algorithmic implementations of these generalization techniques within LineDrive, a real-time system for automatically designing and rendering route maps. Feedback from over 2200 users indicates that almost all believe LineDrive maps are preferable to using standard computer-generated route maps alone.

Keywords: Information Visualization, Non-Realistic Rendering, WWW Applications, Human Factors

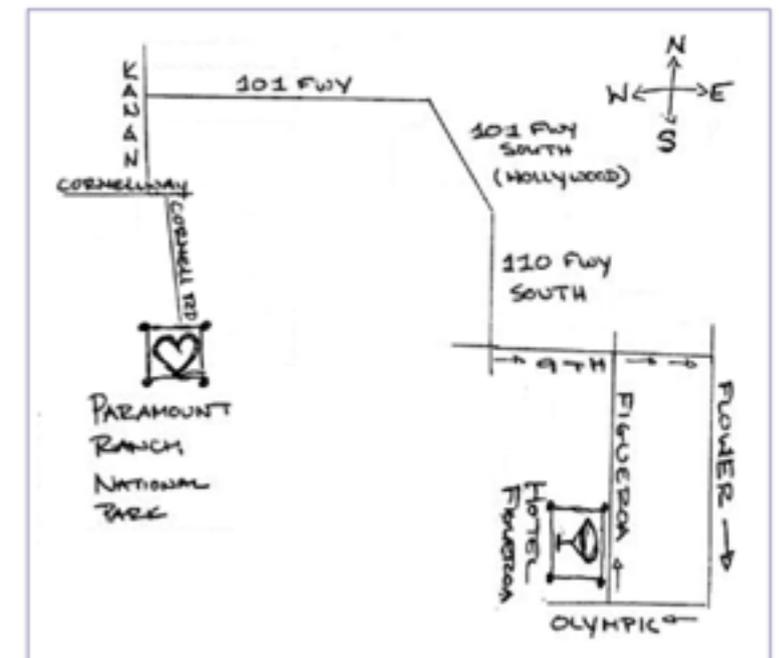
RECOMMENDED READING

clarity of the map and to emphasize the most important information [16, 21]. This type of generalization, performed either consciously or subconsciously, is a critical component both in quickly sketched maps and in professionally designed route maps that appear in print advertisements, invitations, and subway schedules [25, 13].

Recently, route maps in the form of driving directions have become widely available through the Web. In contrast to hand-designed route maps, these computer-generated route maps are often more precise and contain more information. Yet these maps are more difficult to use. The main shortcoming of current systems for automatically generating route maps is that they do not distinguish between essential and extraneous information, and as a result, cannot apply the generalizations used in hand-designed maps to emphasize the information needed to follow the route.

Figure 1 shows several problems arising from the lack of differentiation between necessary and unnecessary information. The primary problem is that current computer-mapping systems maintain a constant scale factor for the entire map. For many routes, the

- straighten wiggly lines
- snap turns to right angles
- expand regions with turns
- contract long straight roads
- label carefully
- maintain overall orientation





Map Size



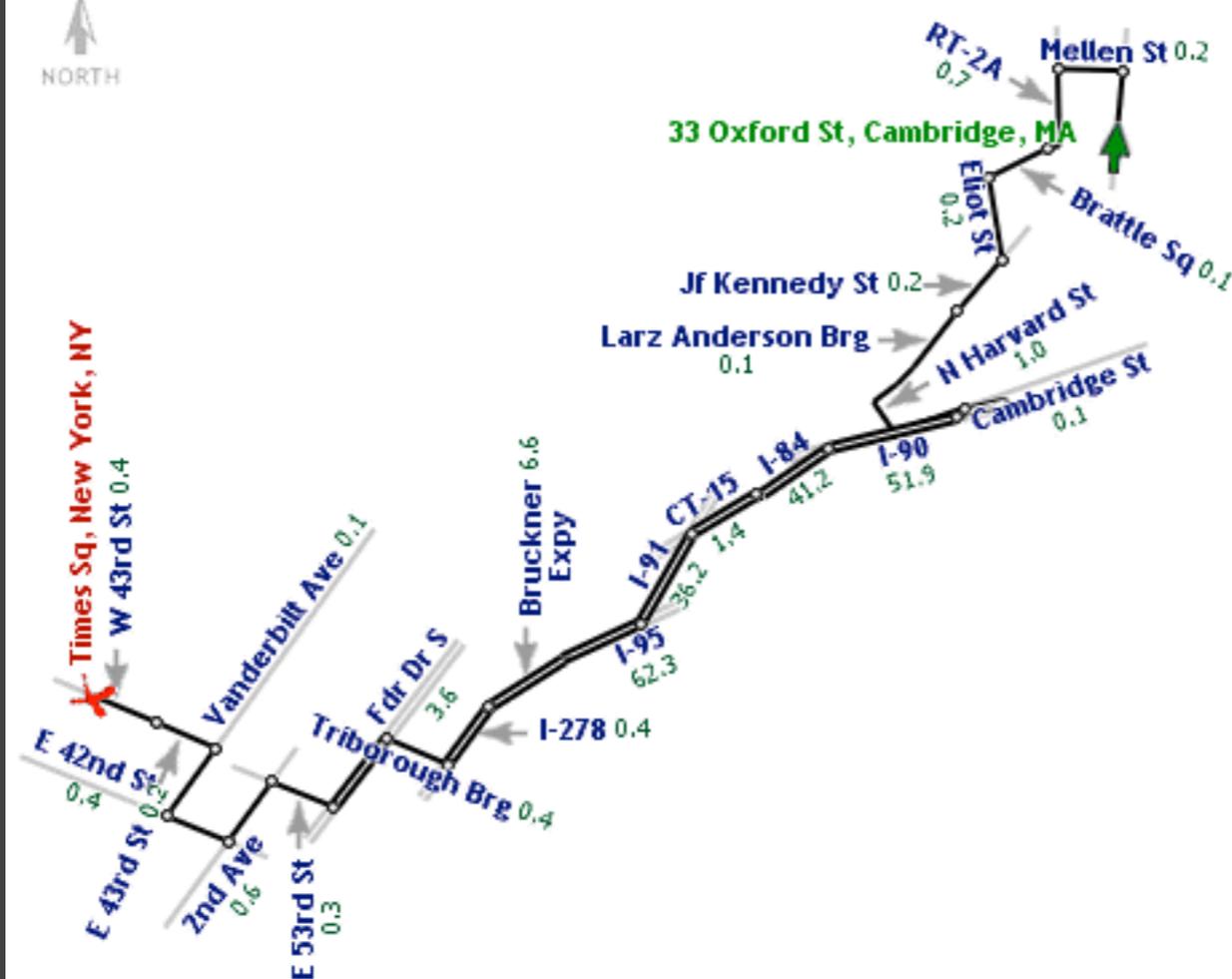
Print



E-mail



NORTH



Start: 33 Oxford St, Cambridge, MA 02138

End: Times Sq, New York, NY 10036

Total Distance: 211.2 Miles

Estimated Total Time: 3 hours, 29 minutes

Route

- Turn-by-Turn Directions
- Reverse Directions
- Change Start
- Change End
- Get New Directions

Local Resources

- [Traffic Maps](#)
- [City Guide](#)
- [Yellow Pages](#)
- [Weather](#)

advertisement

Microsoft®



Maps

Directions My places Map apps



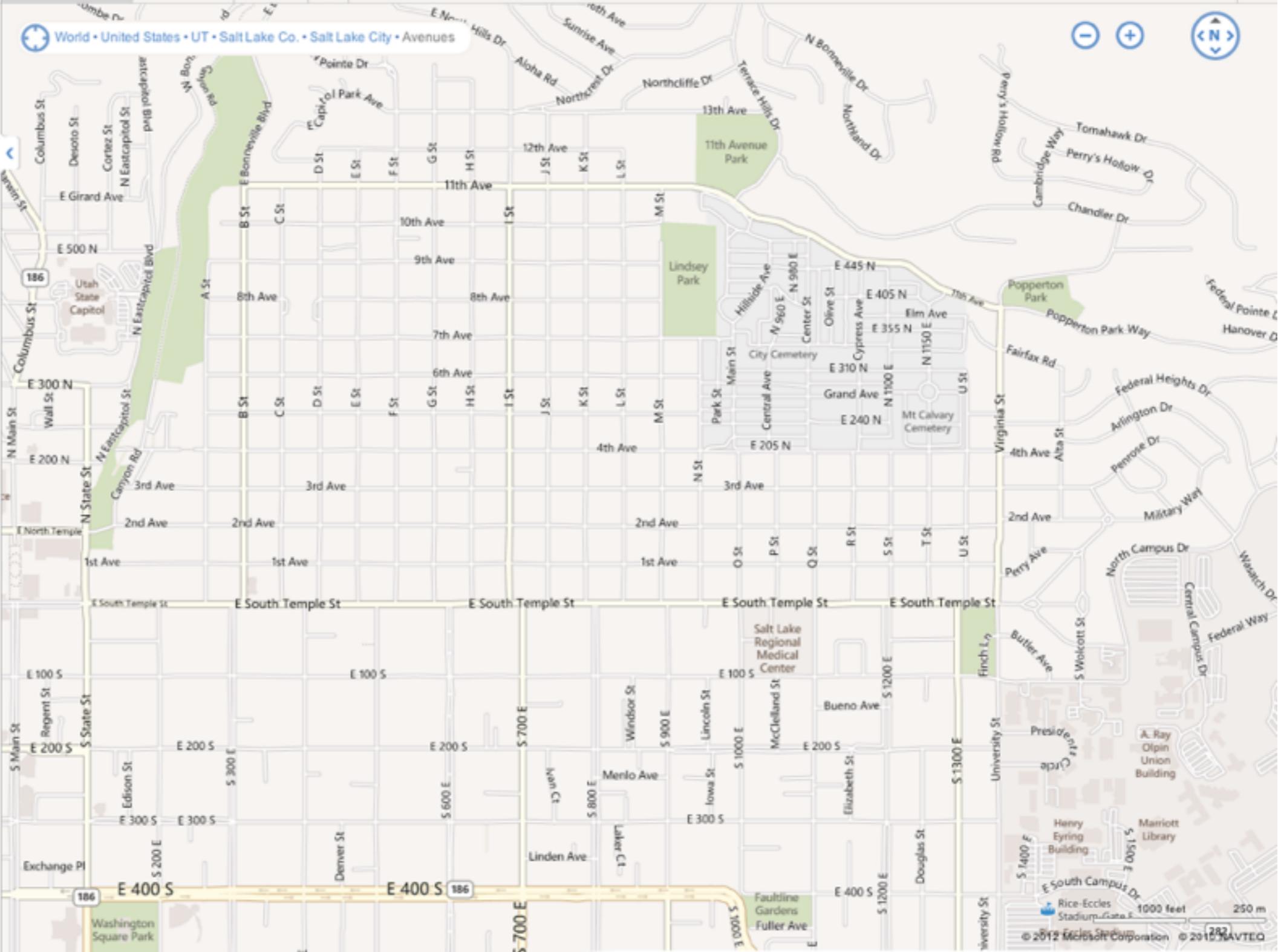
A

B Business, address, or landmark

add destination show options

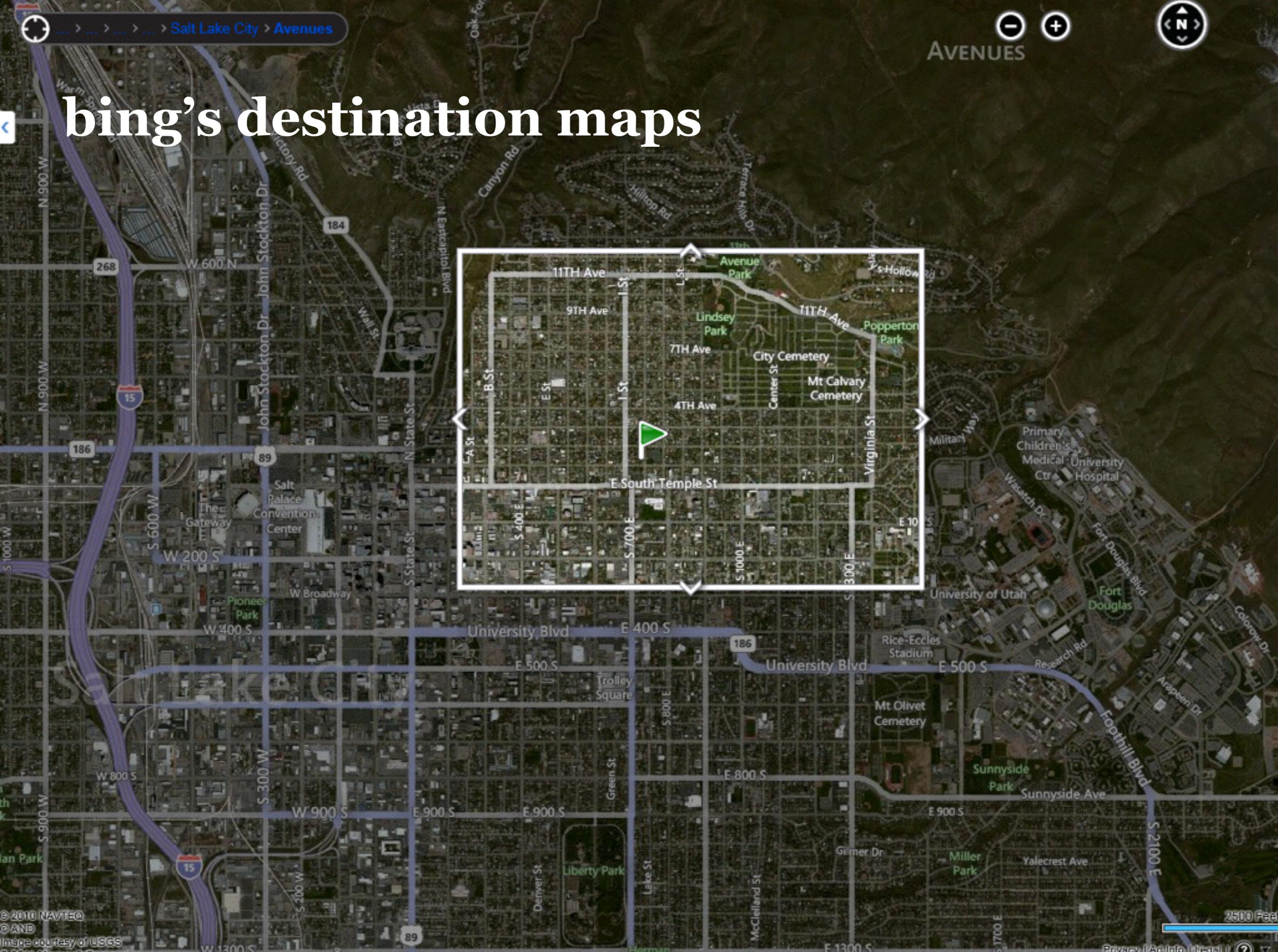
Clear Go

Automatic Aerial Traffic Print Share

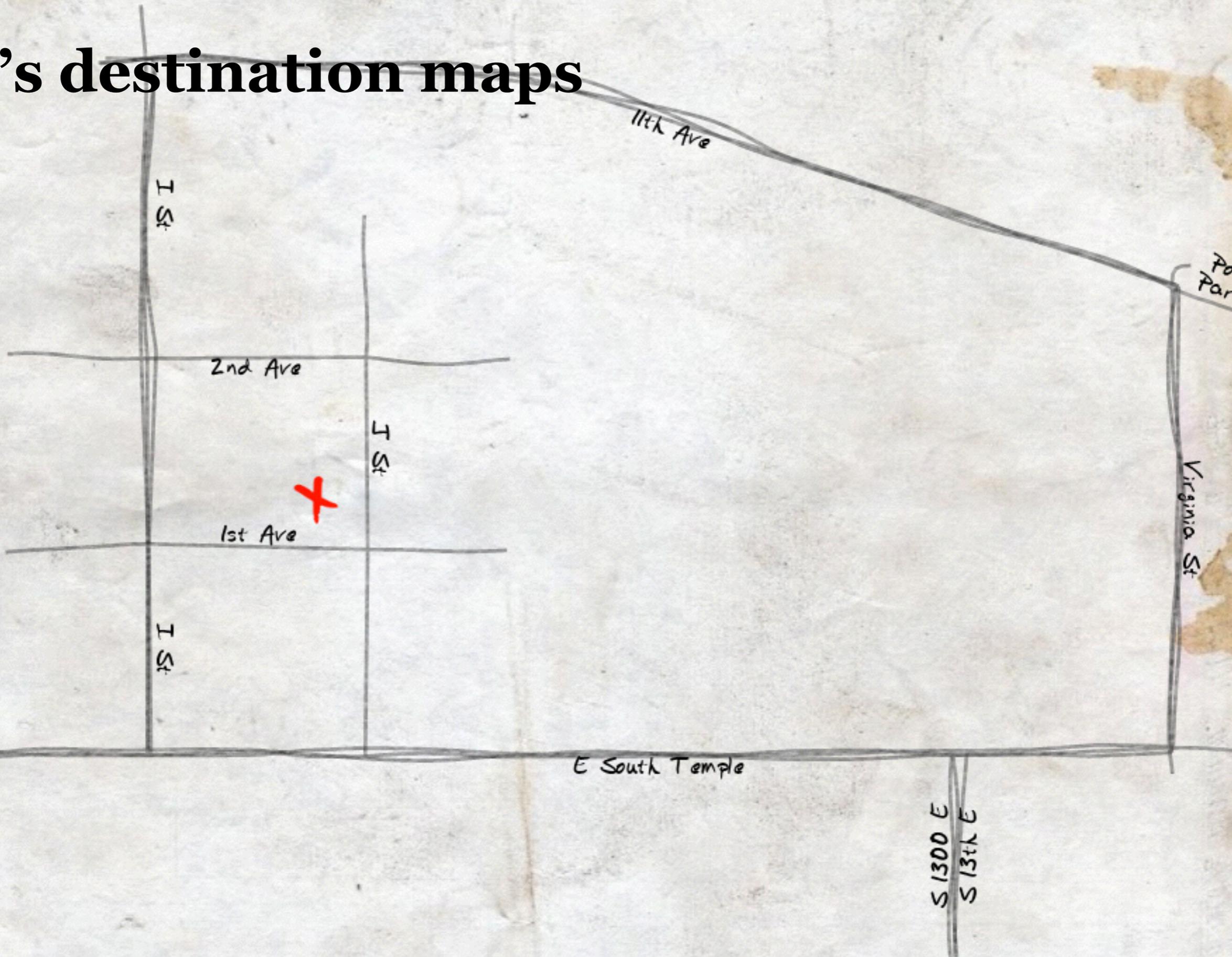


© 2012 Microsoft Corporation © 2012 NAVTEQ

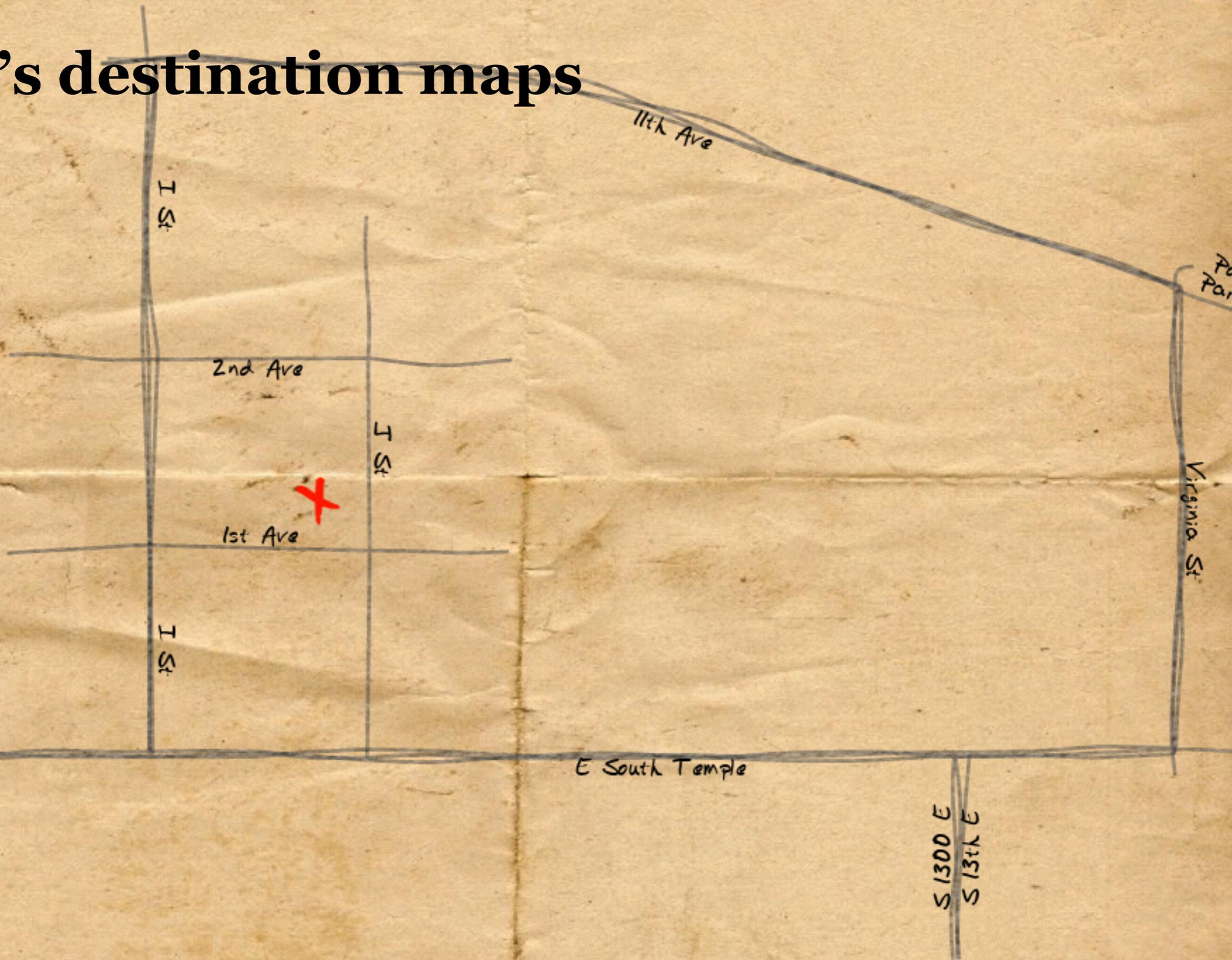
bing's destination maps



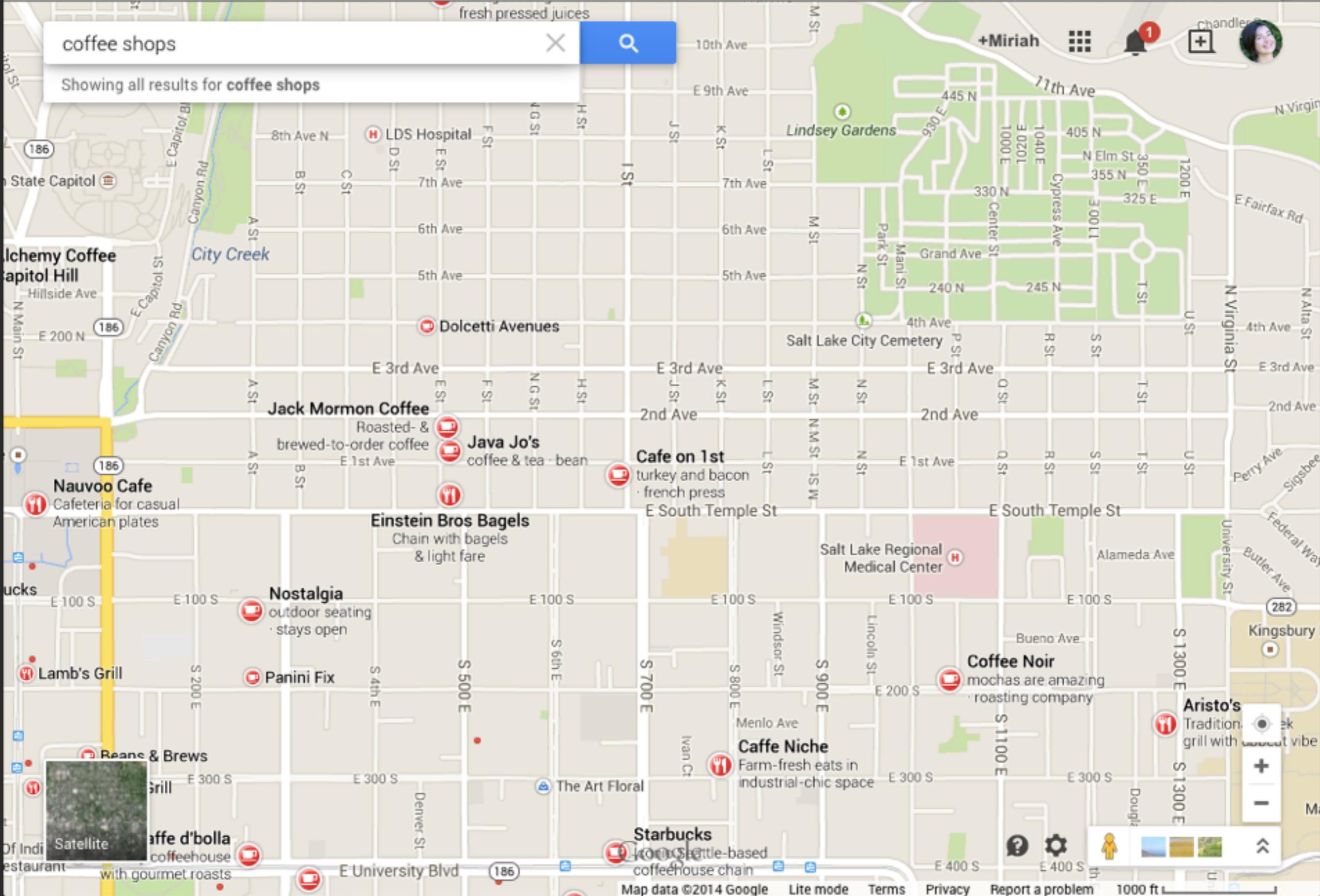
bing's destination maps



bing's destination maps



coffee shops
Showing all results for coffee shops



CLEAR speeds you through airport security. Why wait? \$179 per year. \$50 for a family member.

Advertise on NYTimes.com

UPDATED March 15, 2011

Satellite Photos of Japan, Before and After the Quake and Tsunami

RECOMMEND TWITTER LINKEDIN E-MAIL SHARE

Move the slider to compare satellite images, taken by GeoEye/EyeQ, from before and after the disaster.



Fukushima Daiichi Nuclear Plant

Japan's largest ongoing threat is at this nuclear power plant. There have been explosions at four of its six reactors and all four have released some radioactive material.



-landmarks

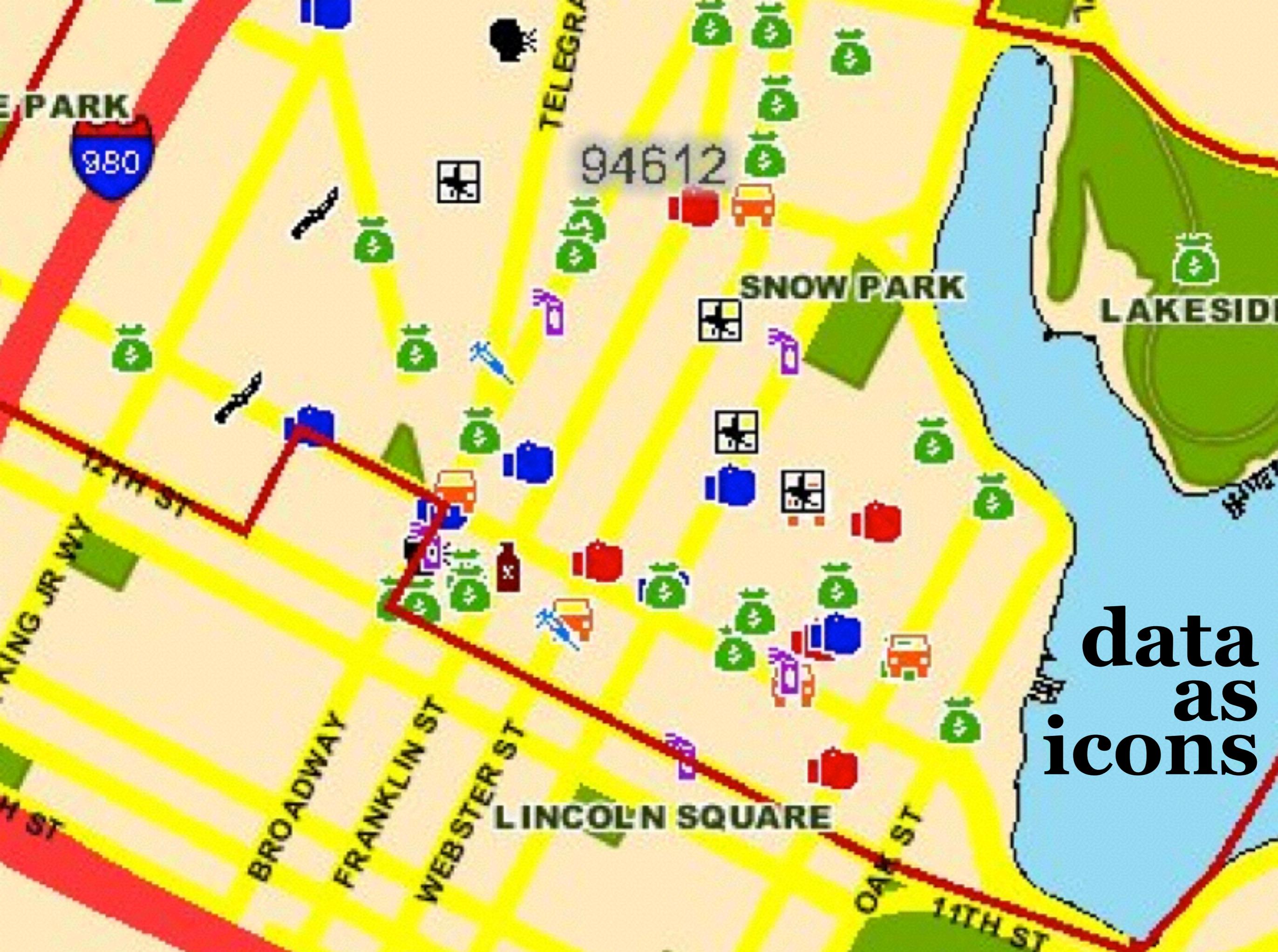
-**discrete data**

-continuous data

-choropleths

-cartograms

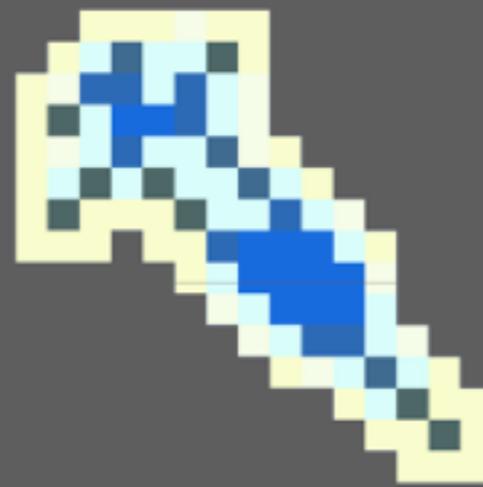
-projections



94612

data
as
icons

what do they mean?



TIME OF DAY
 Show All | Hide All
 Light | Dark [nearest hour]
 Commute | Nightlife
 Day | Night | Swing Shift

NOON
 12
 9 3
 AM 6 6 PM
 3 9
 MIDNIGHT

DATE Past Week
 Feb 2010

FEB 6 2012 FEB 13 2012 FEB 20 2012 FEB 27 2012

T W Th F S S M T W Th F S S M T W Th F S S M

CRIME TYPE Show All | Hide All

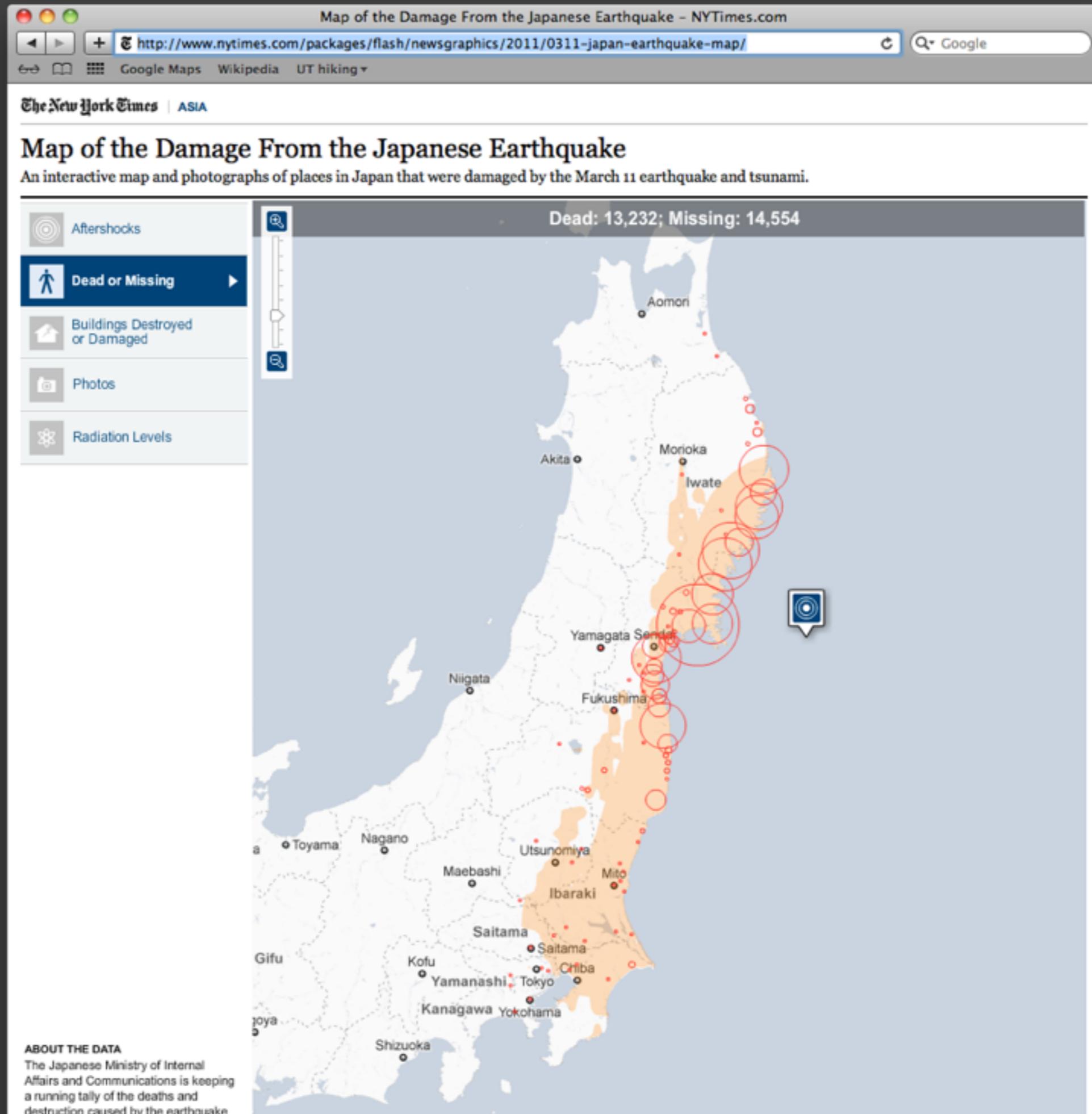
- AA Aggravated Assault
- Mu Murder
- Ro Robbery
- SA Simple Assault
- DP Disturbing the Peace
- Na Narcotics
- Al Alcohol
- Pr Prostitution
- Th Theft
- VT Vehicle Theft
- Va Vandalism
- Bu Burglary
- Ar Arson

Map tiles © CloudMade. Map data © CC-BY-SA OpenStreetMap.org

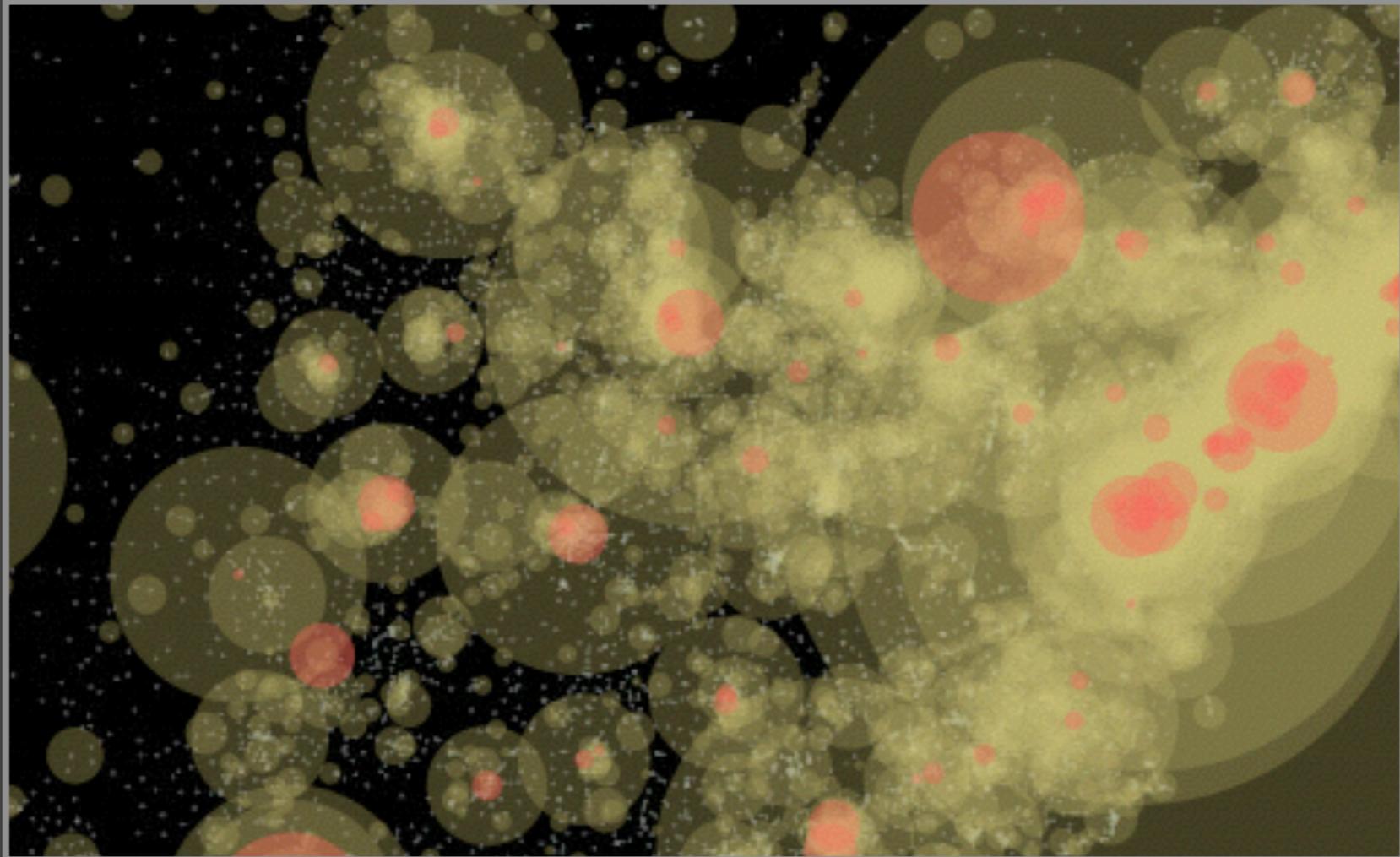
data as
points

data : ordered/
quantitative

encoding : size

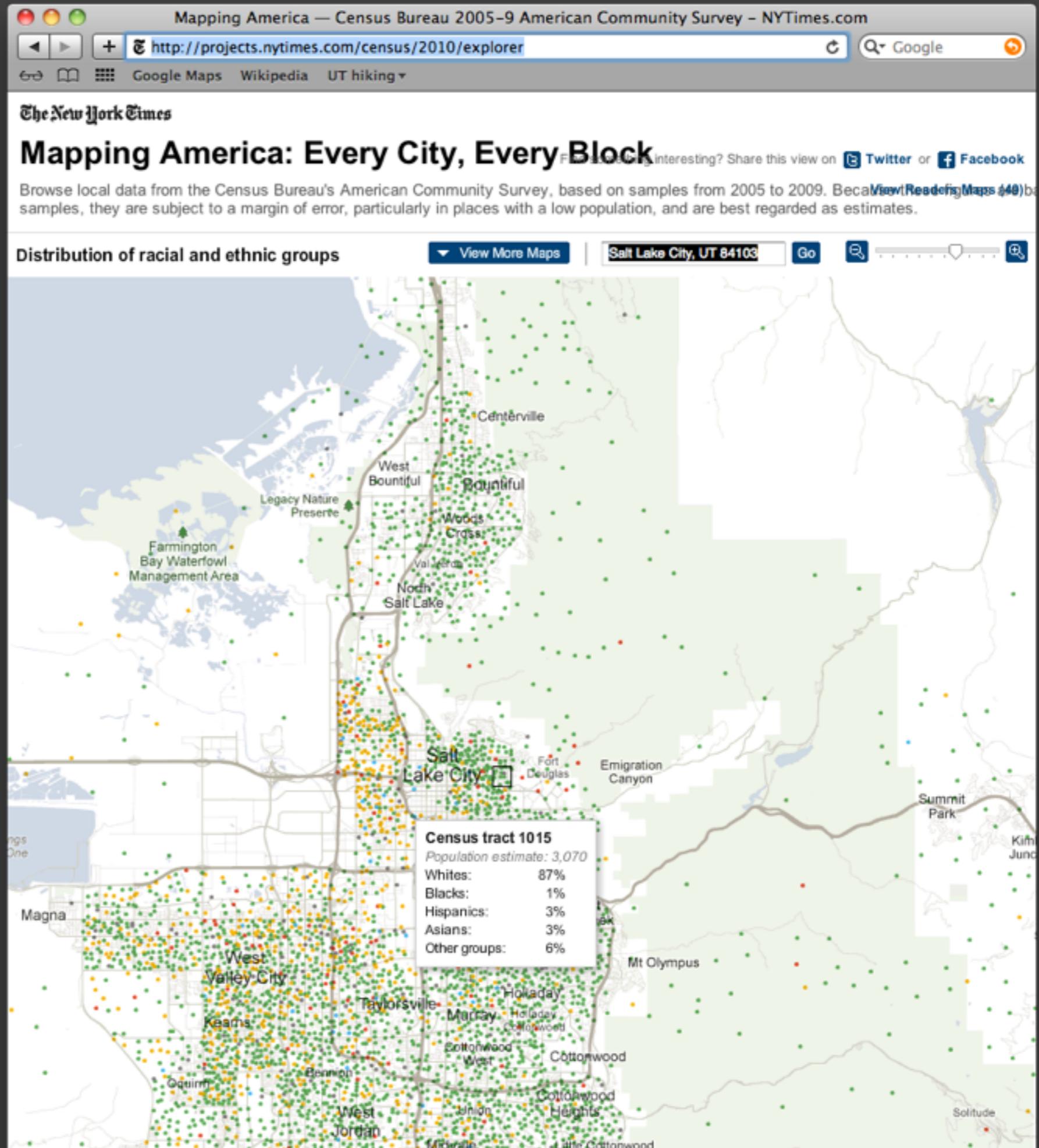


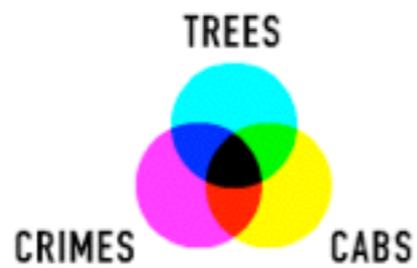
traffic



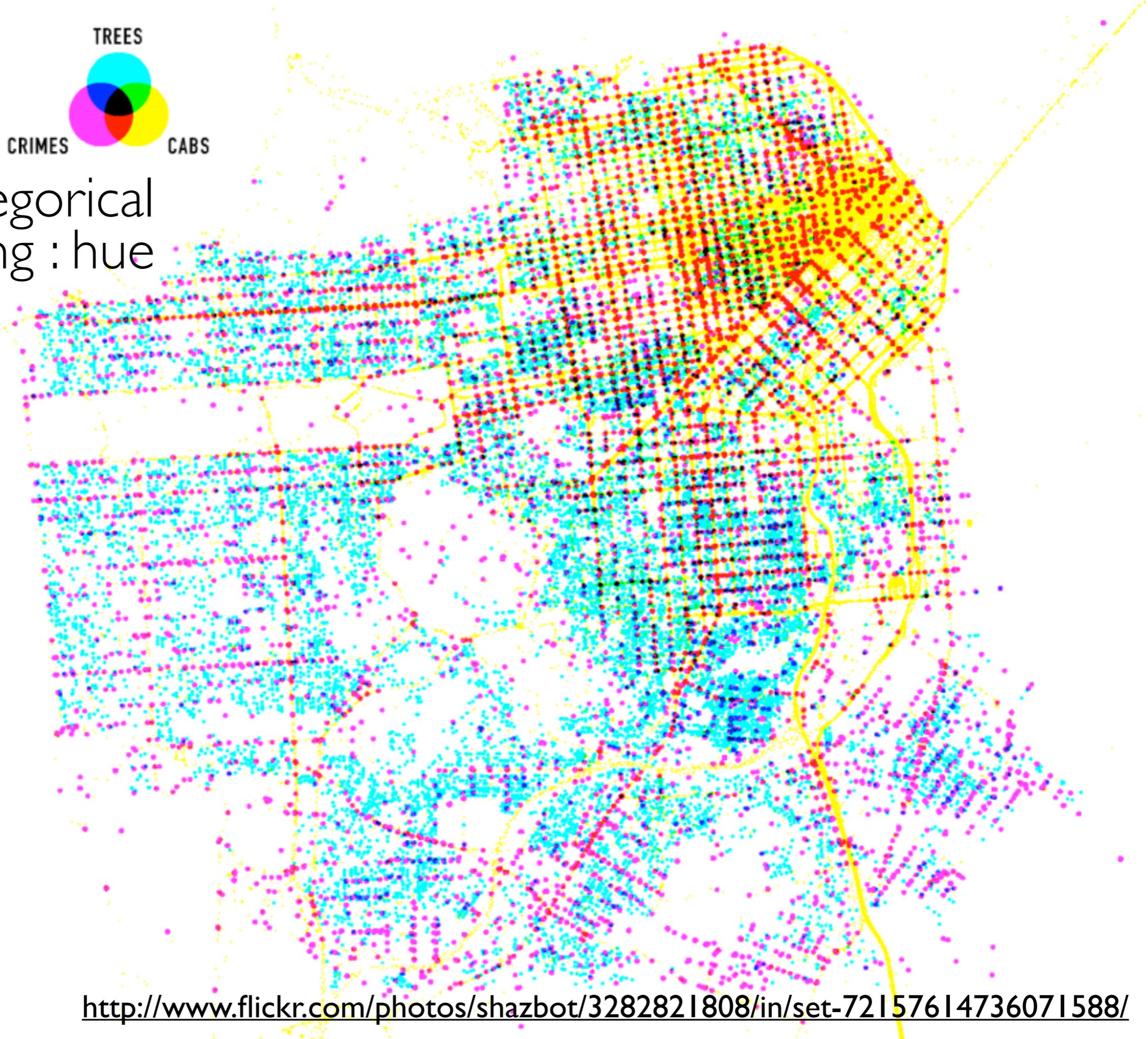
data as
points

data : categorical
encoding : hue





data : categorical
encoding : hue



EBB & FLOW

October 1, 2013

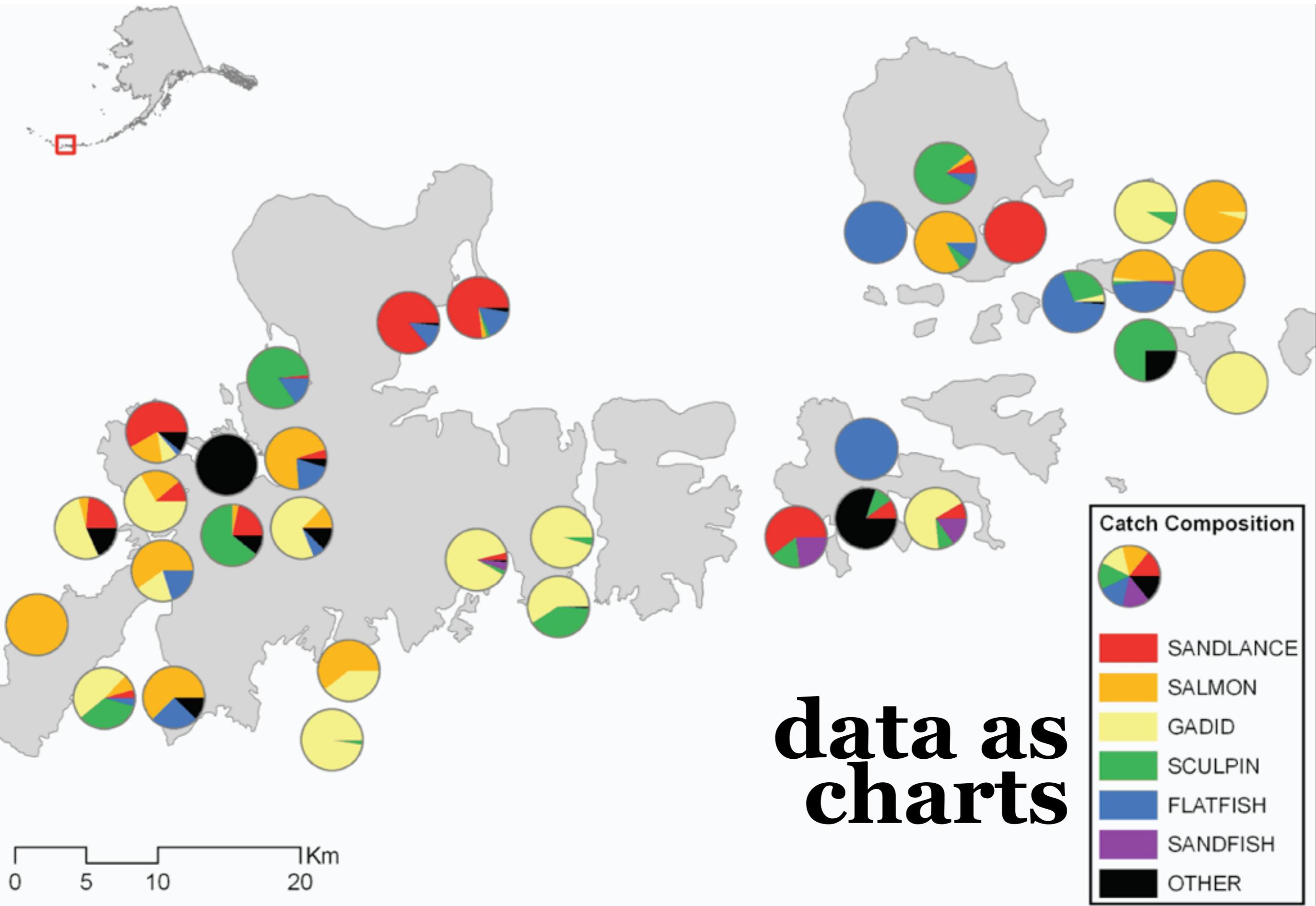
Some really neat data visualization from [foursquare](#) showing the ebb & flow of people in New York City and if you would like to see more they have it for [San Francisco](#), [London](#) and [Tokyo](#) too. For more on how they did it read this [article](#).



Share 3 Likes

data as lines



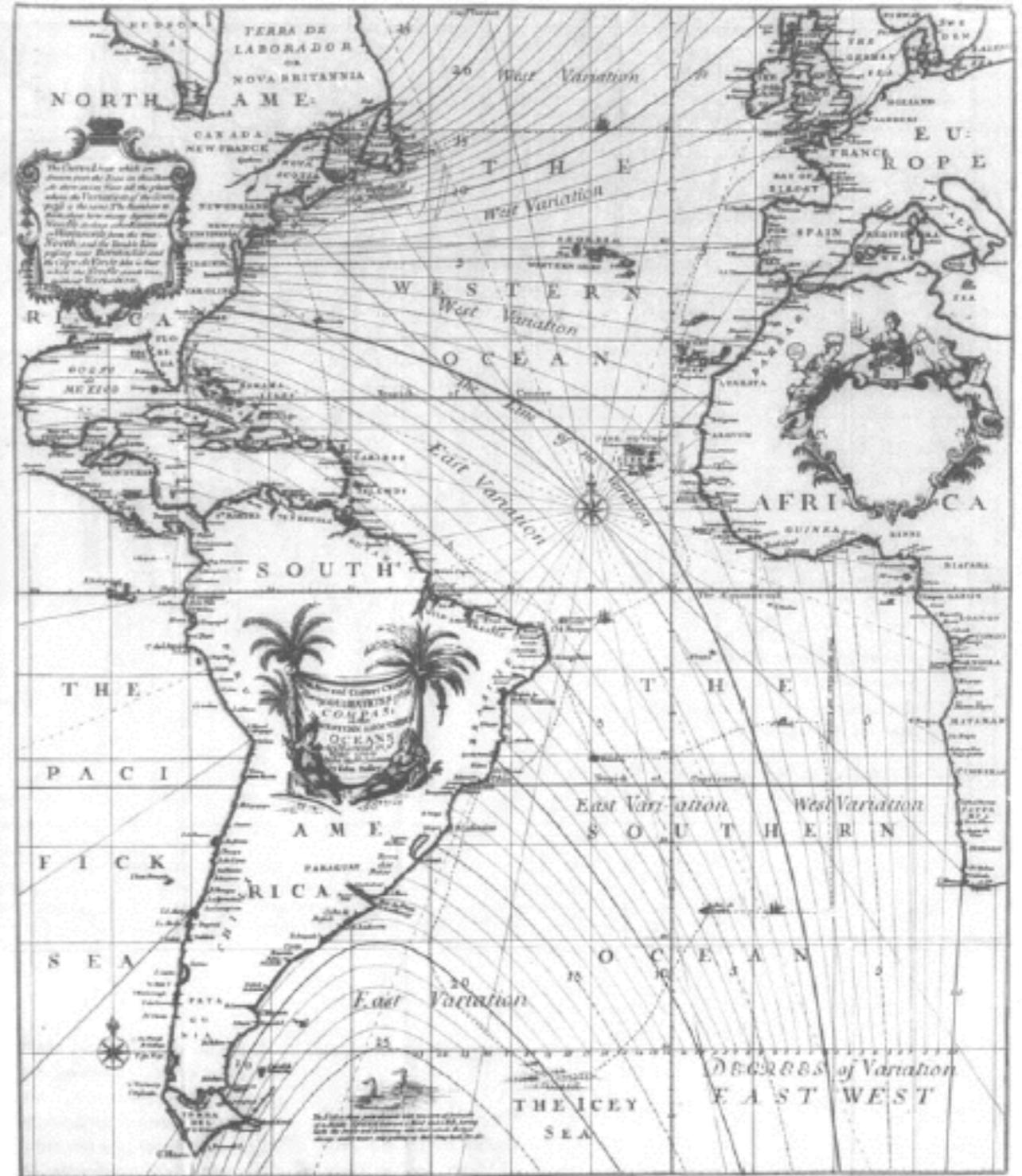


- landmarks
- discrete data
- continuous data**
- choropleths
- cartograms
- projections

isopleth

map which overlays continuous data using a third encoding channel

Lines of Equal Magnetic Declination *first contour map*



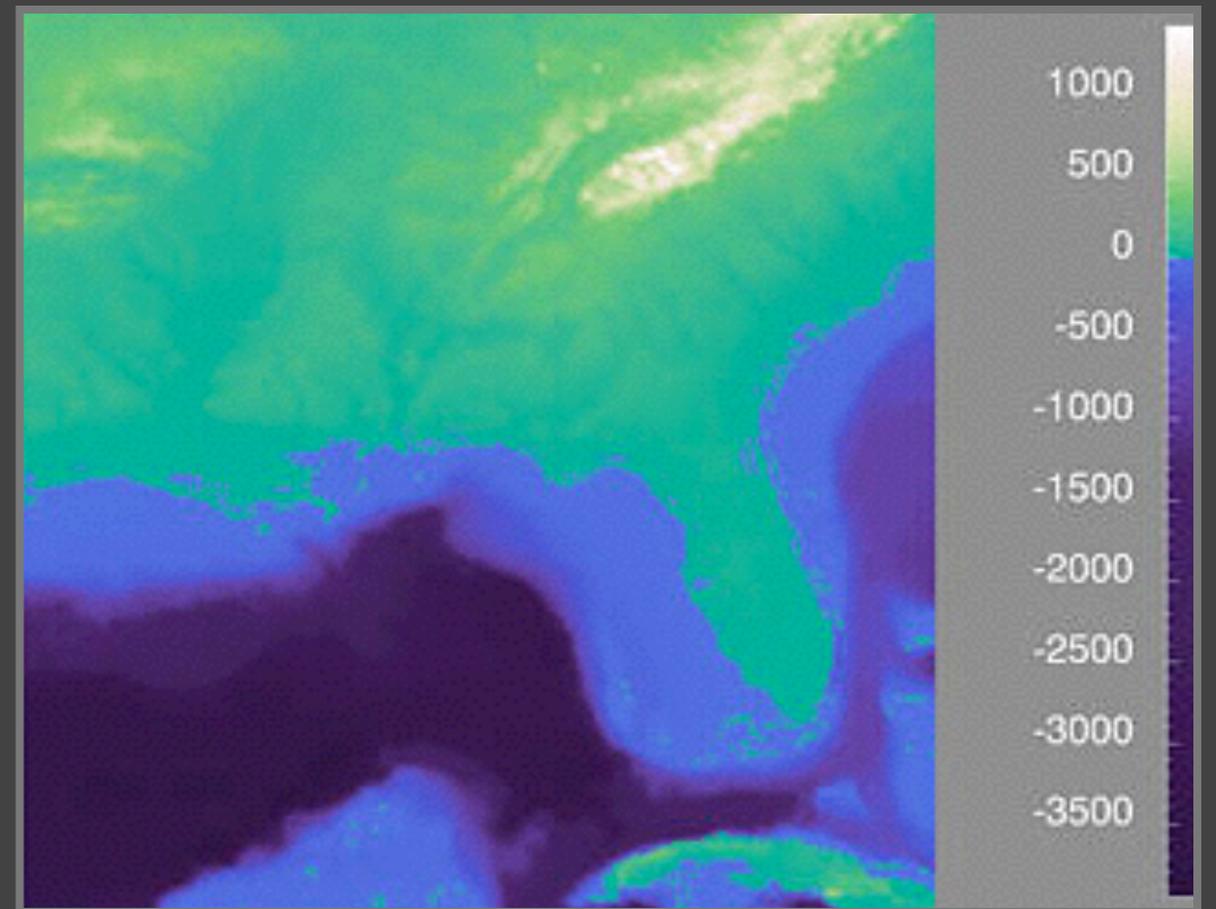
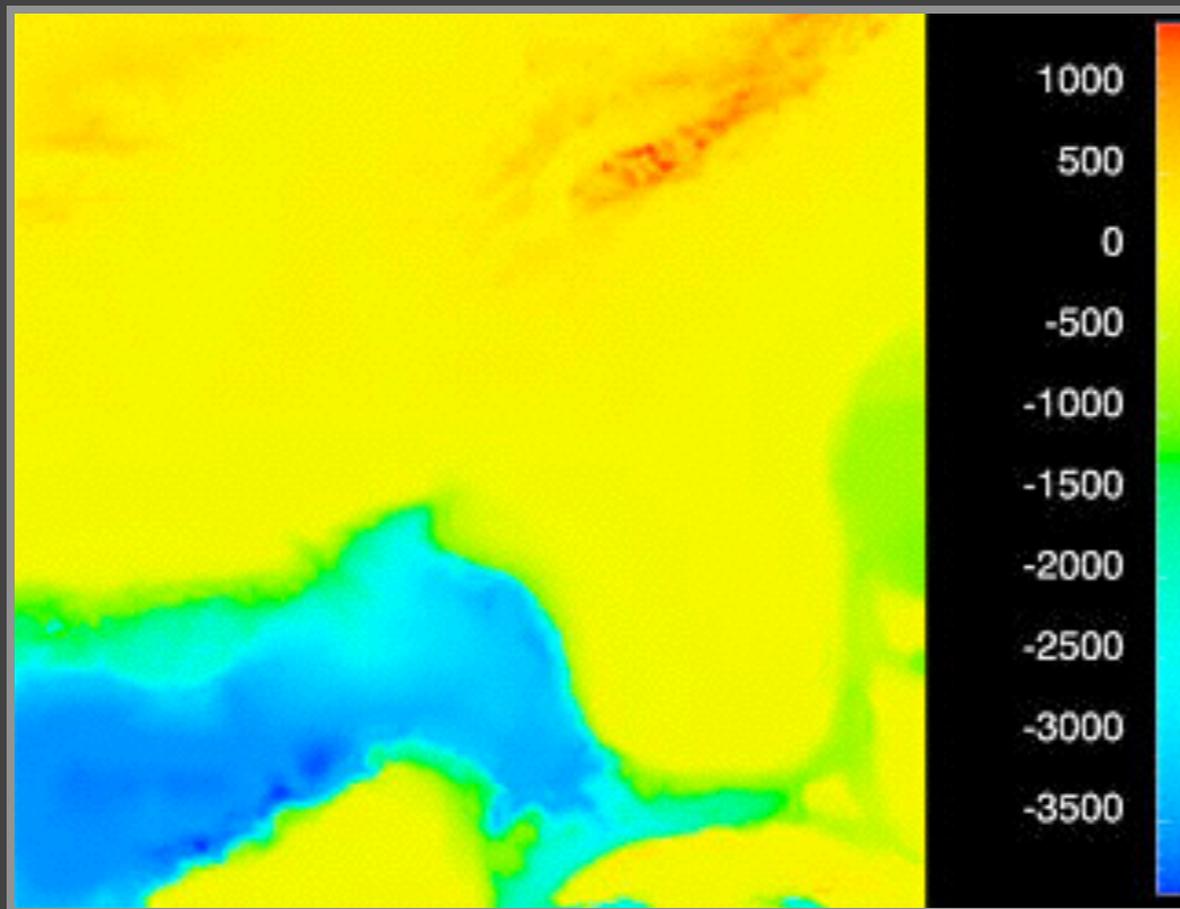
Edmond Halley, 1701

HEATMAPS



**don't hide
the context**

choose the appropriate colormap

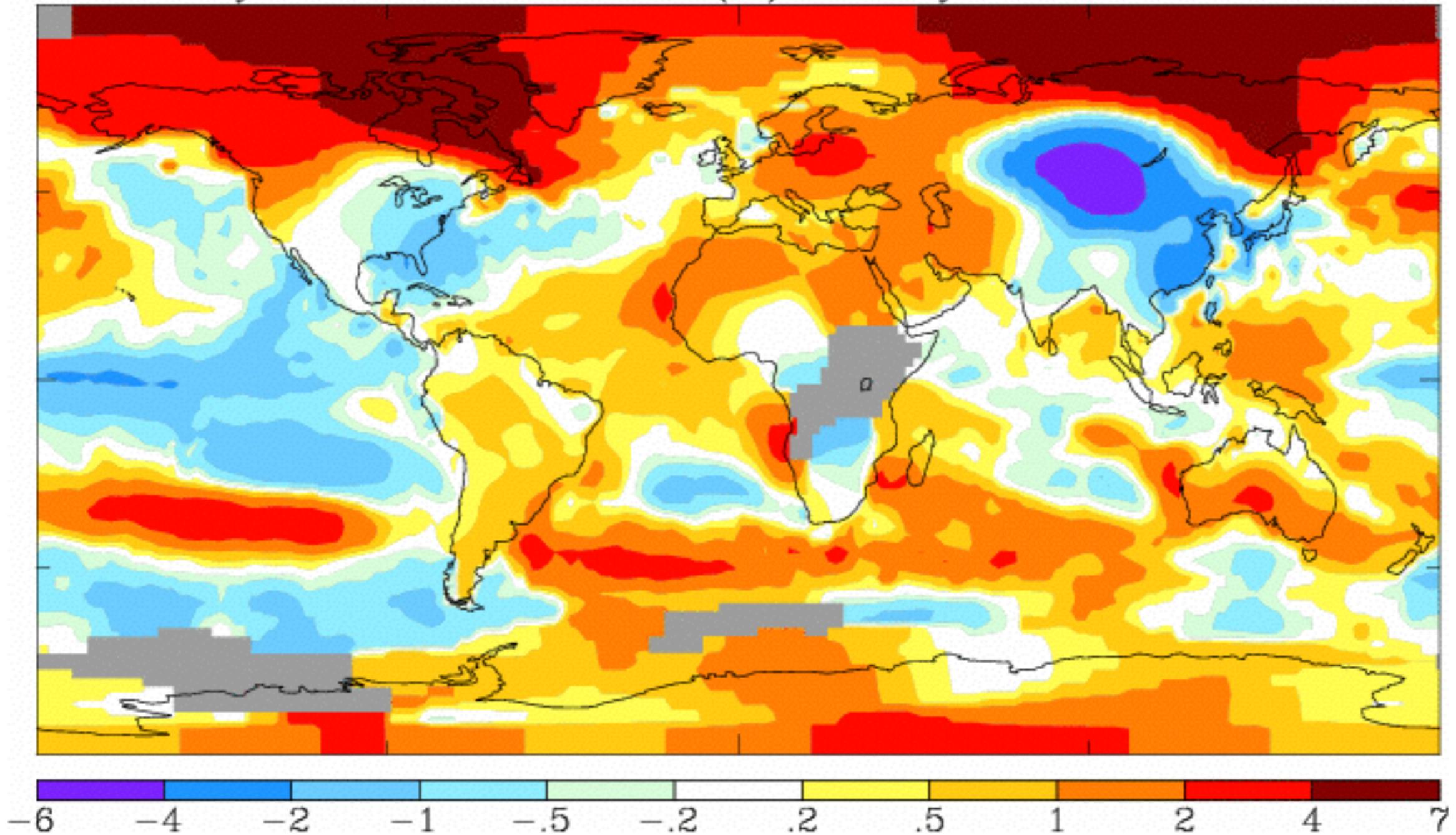


bin the data

January 2011

L-OTI(°C) Anomaly vs 1951-1980

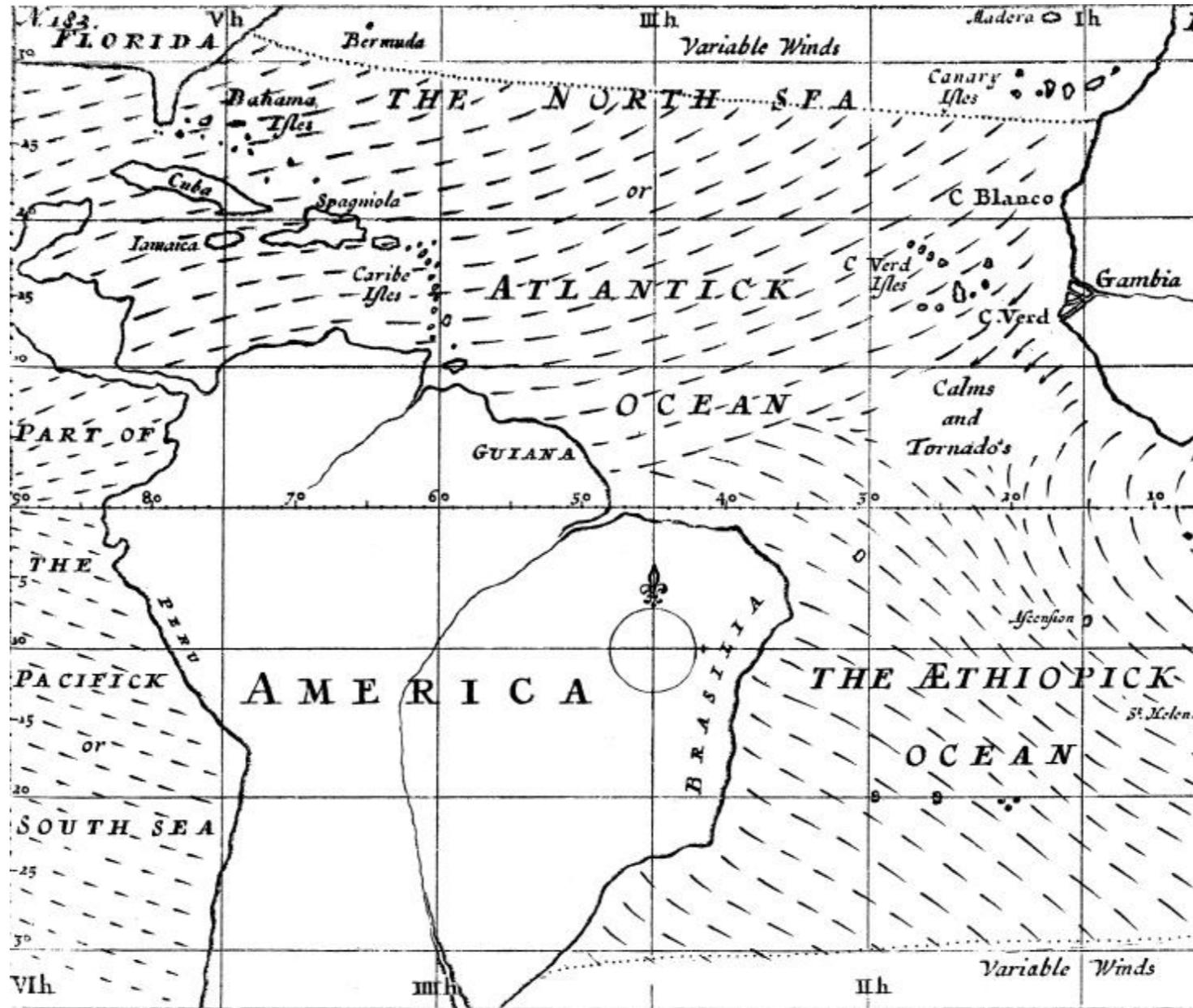
.46



ISOCURVES

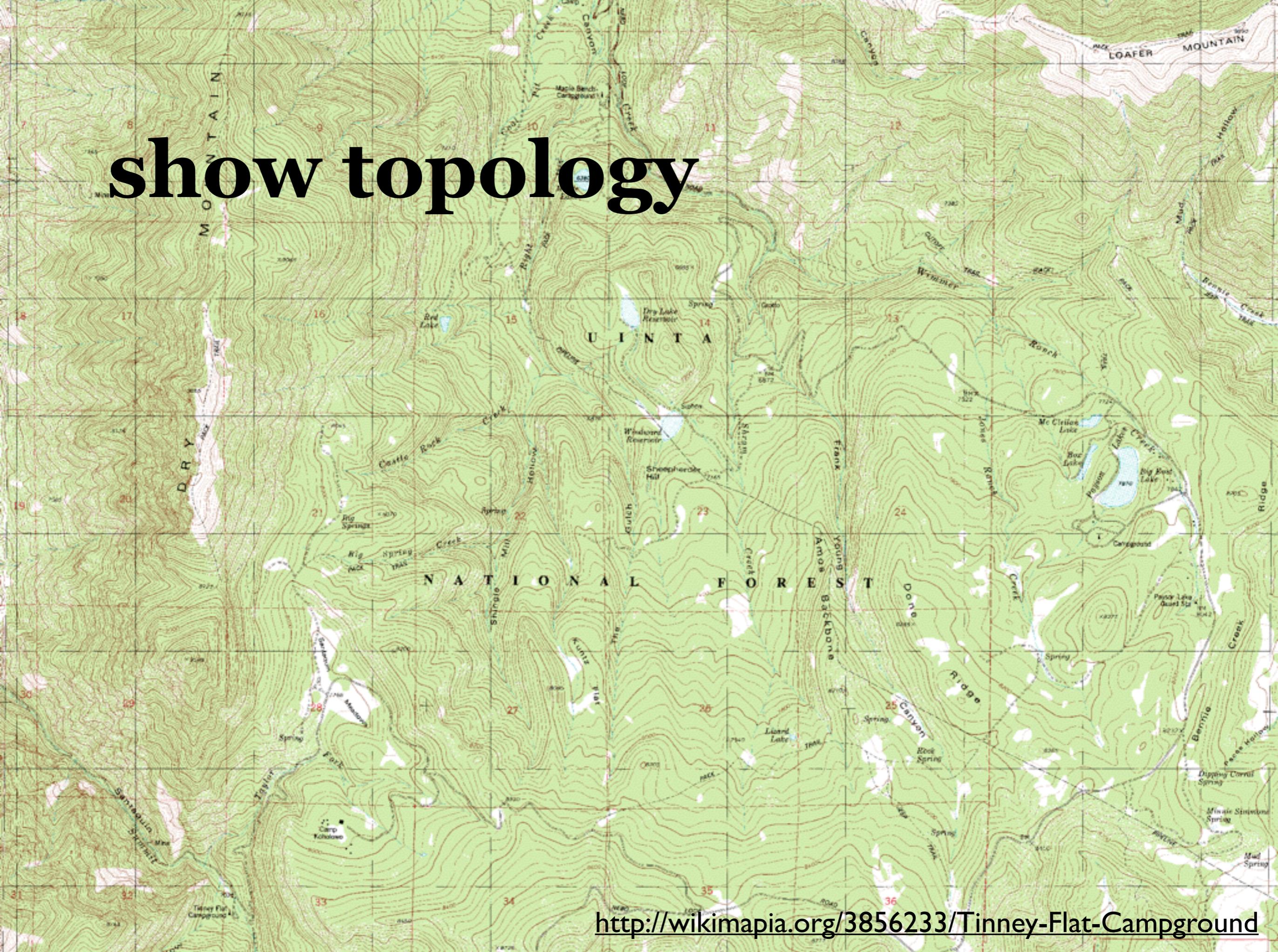
Wind Map

first weather map

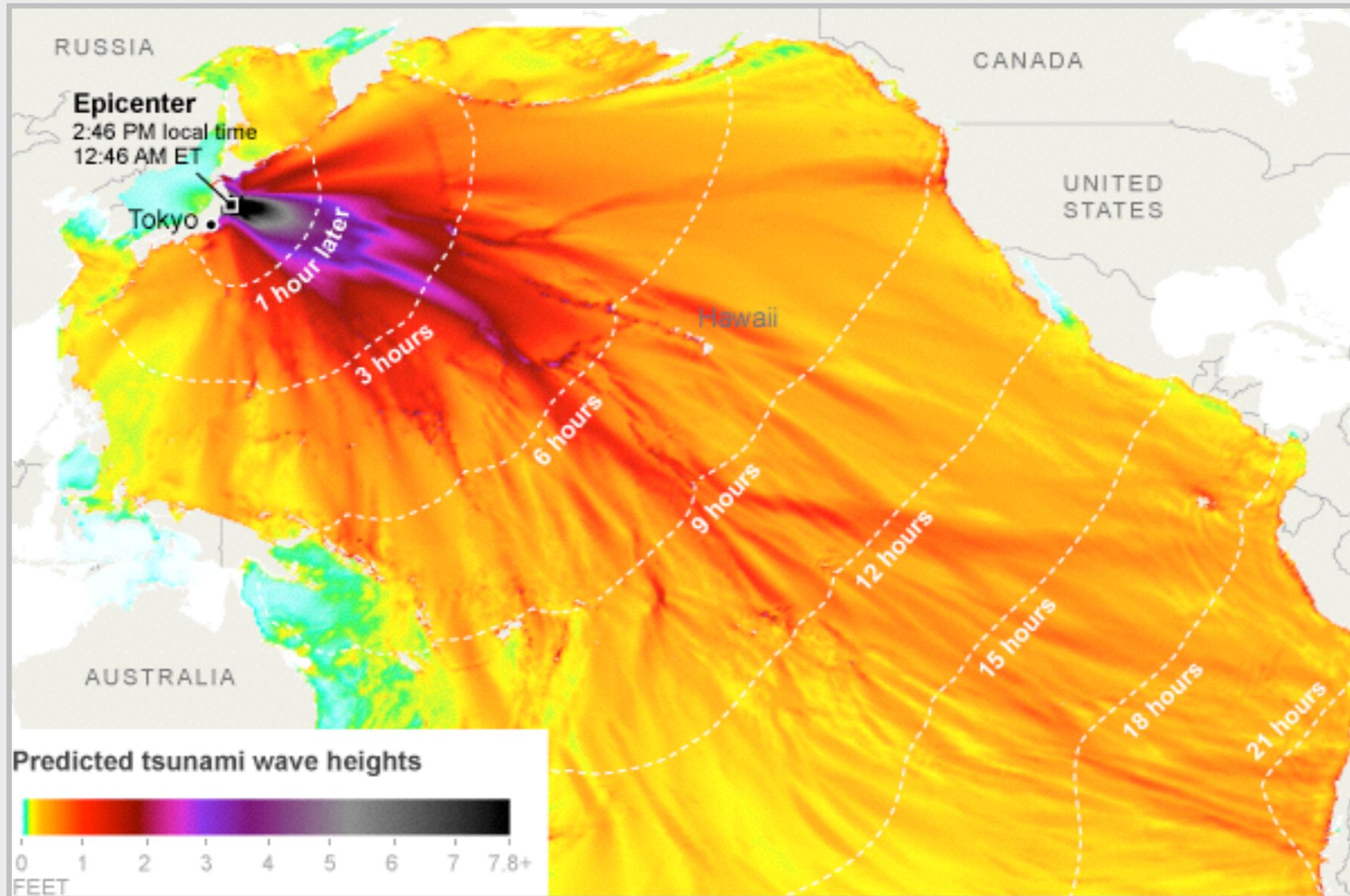


Edmond Halley, 1686

show topology



show time

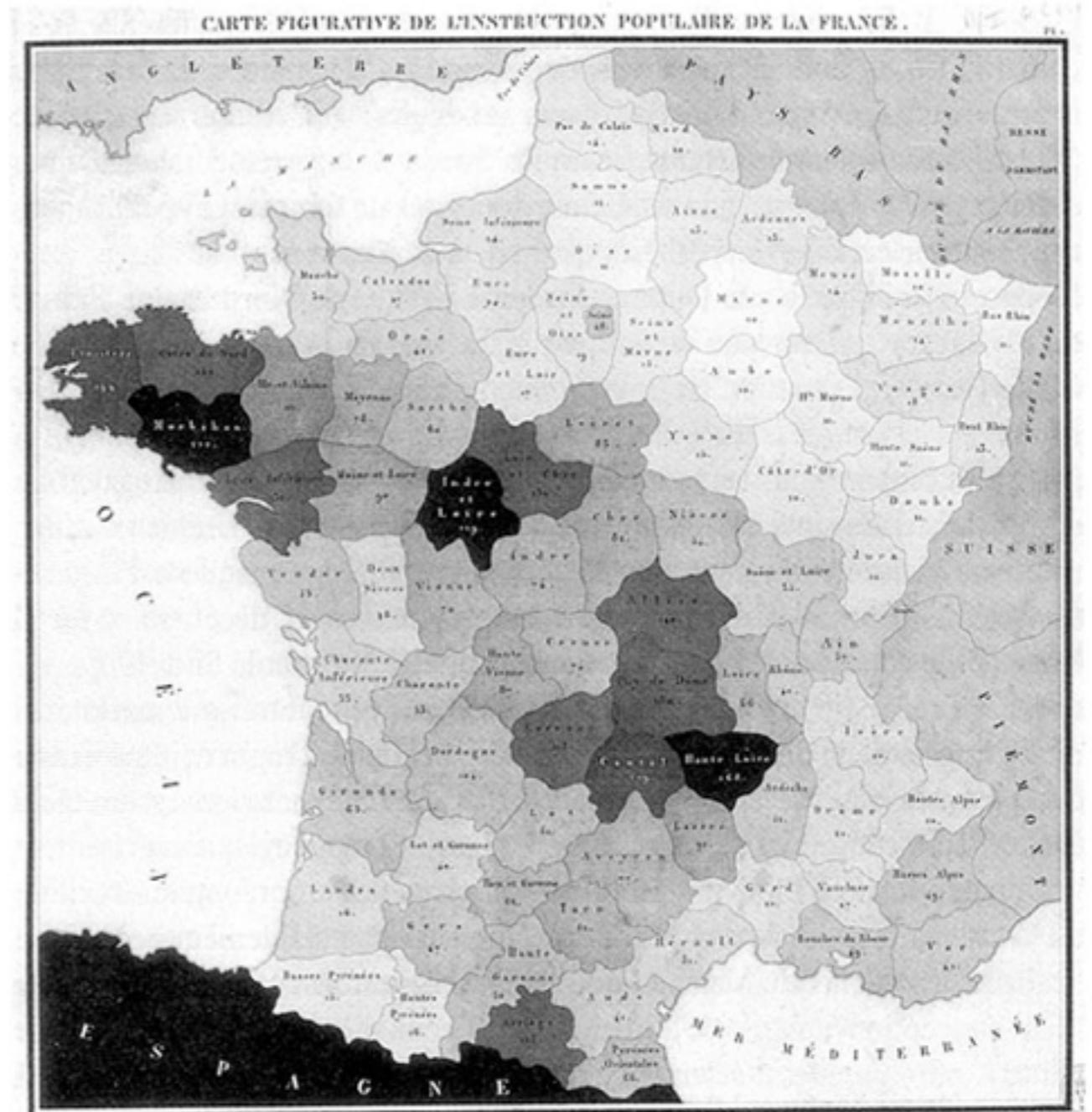


- landmarks
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- projections

choropleth

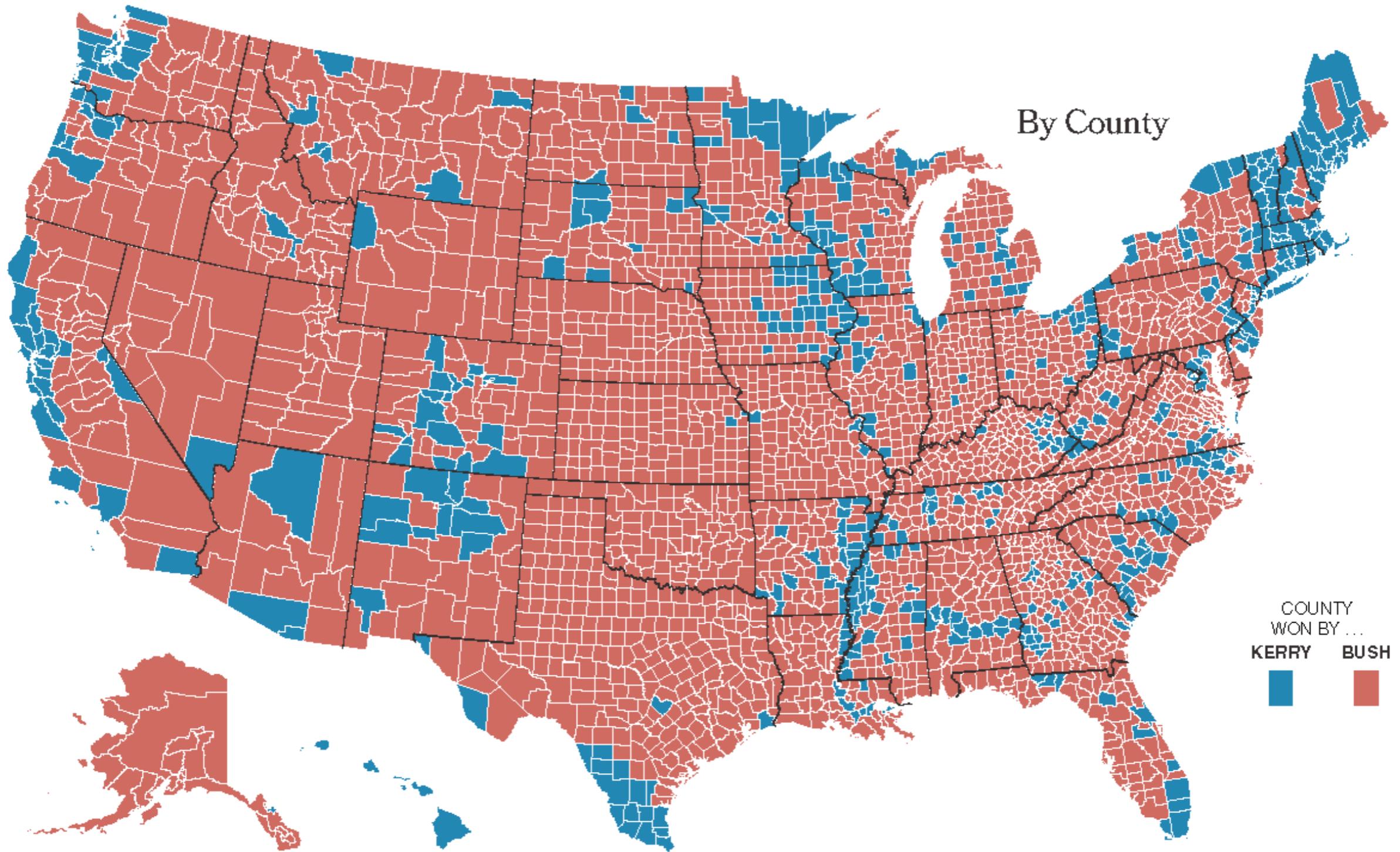
map in which areas are shaded, colored, or patterned relative to a data attribute value

Illiteracy in France *first choropleth map*



Charles Dupin, 1826

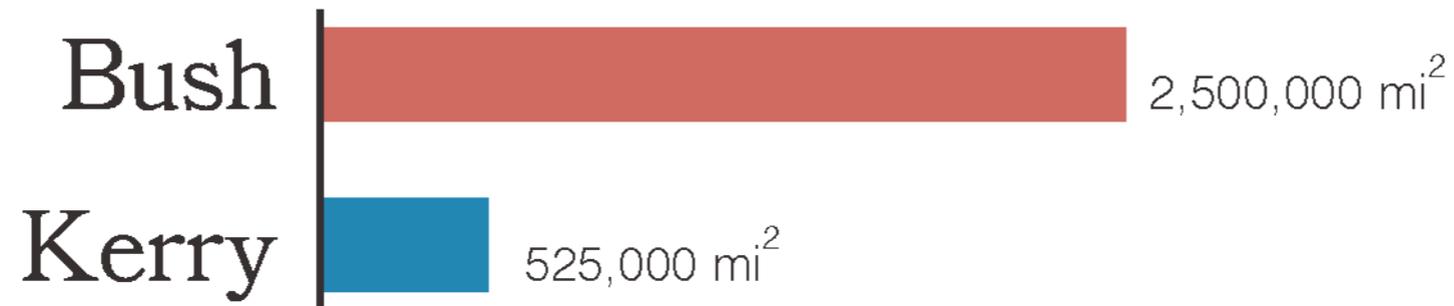
Kerry vs Bush, 2004

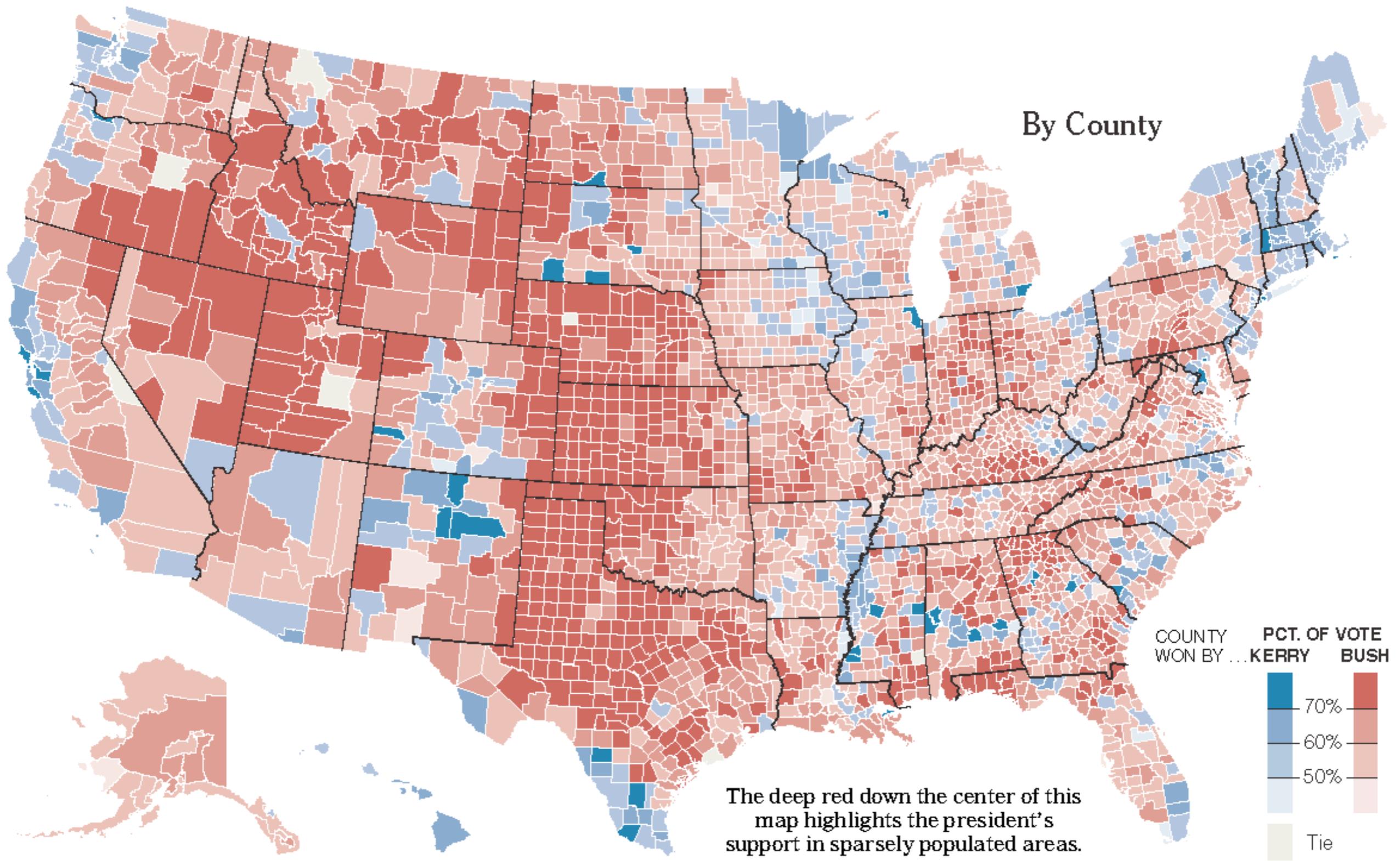


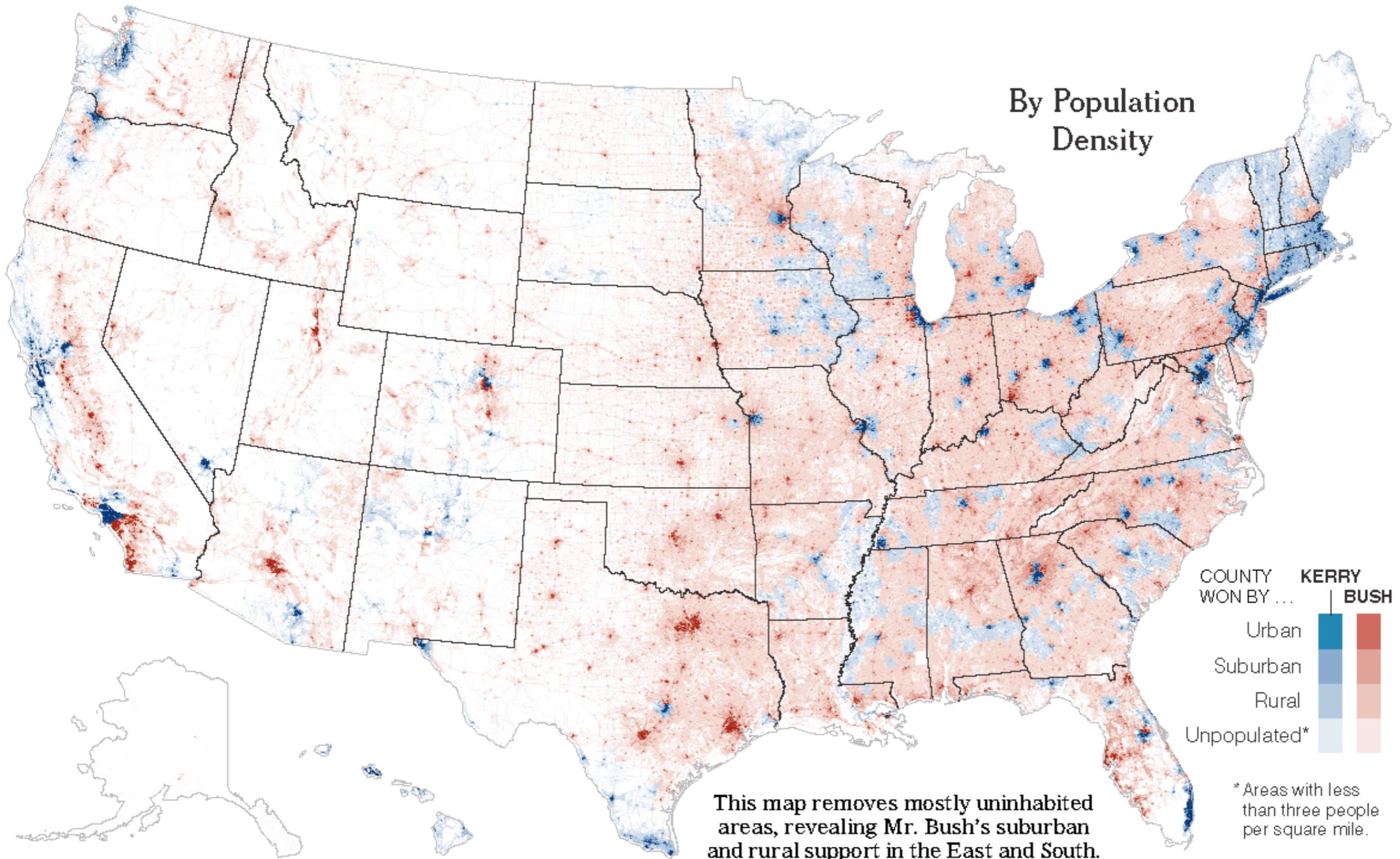
2004 Popular Vote



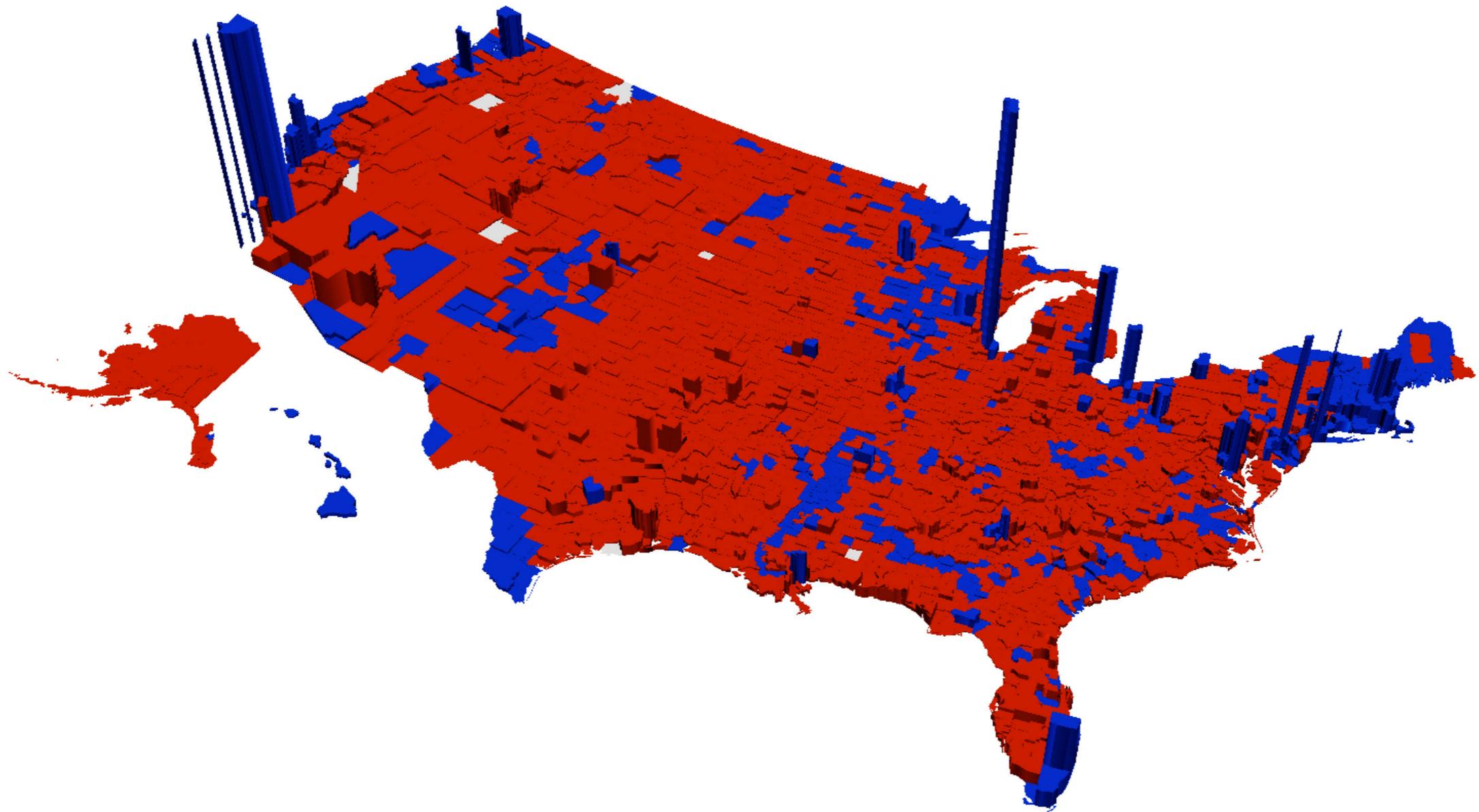
Amount of red and blue shown on map



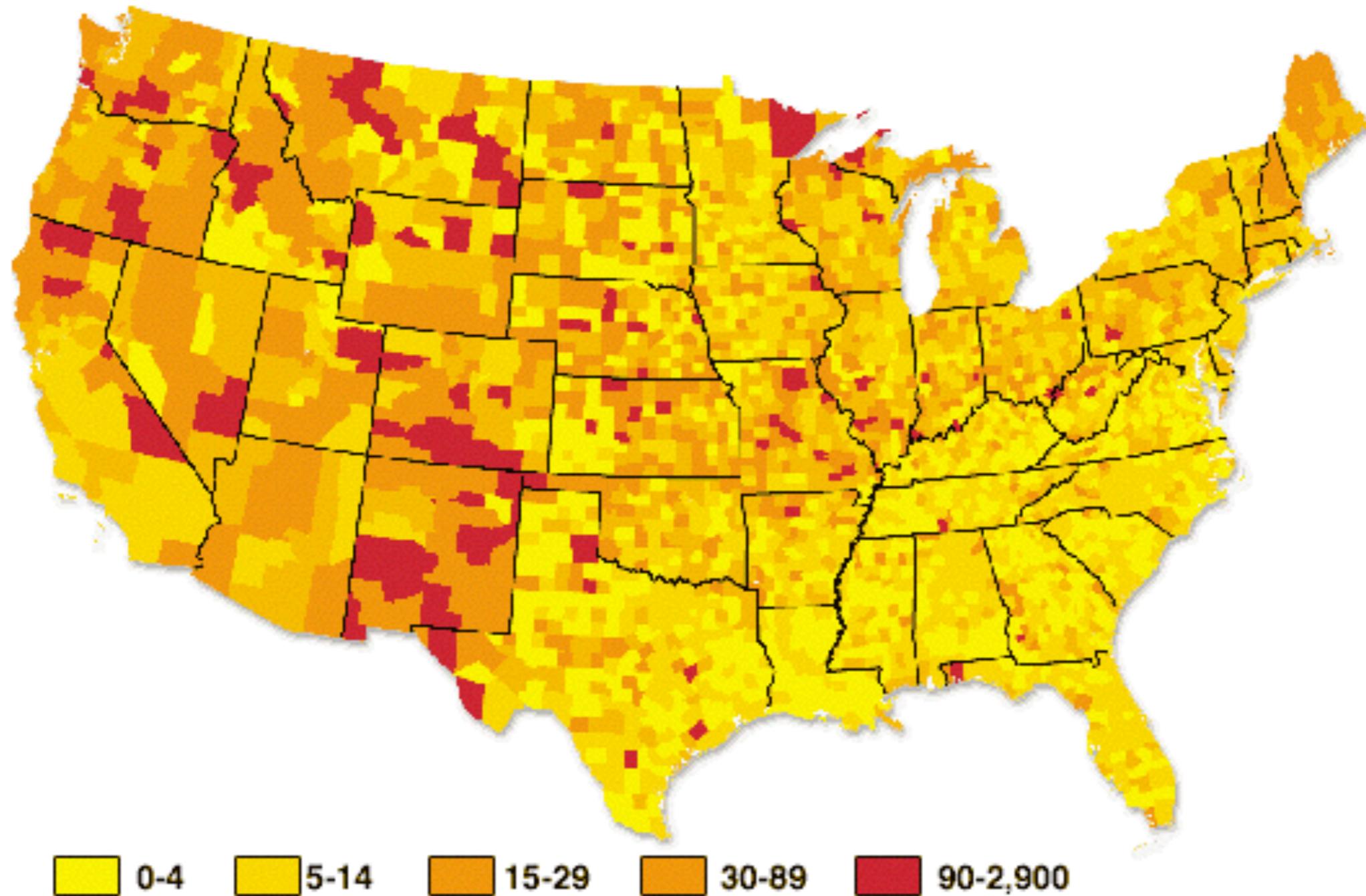




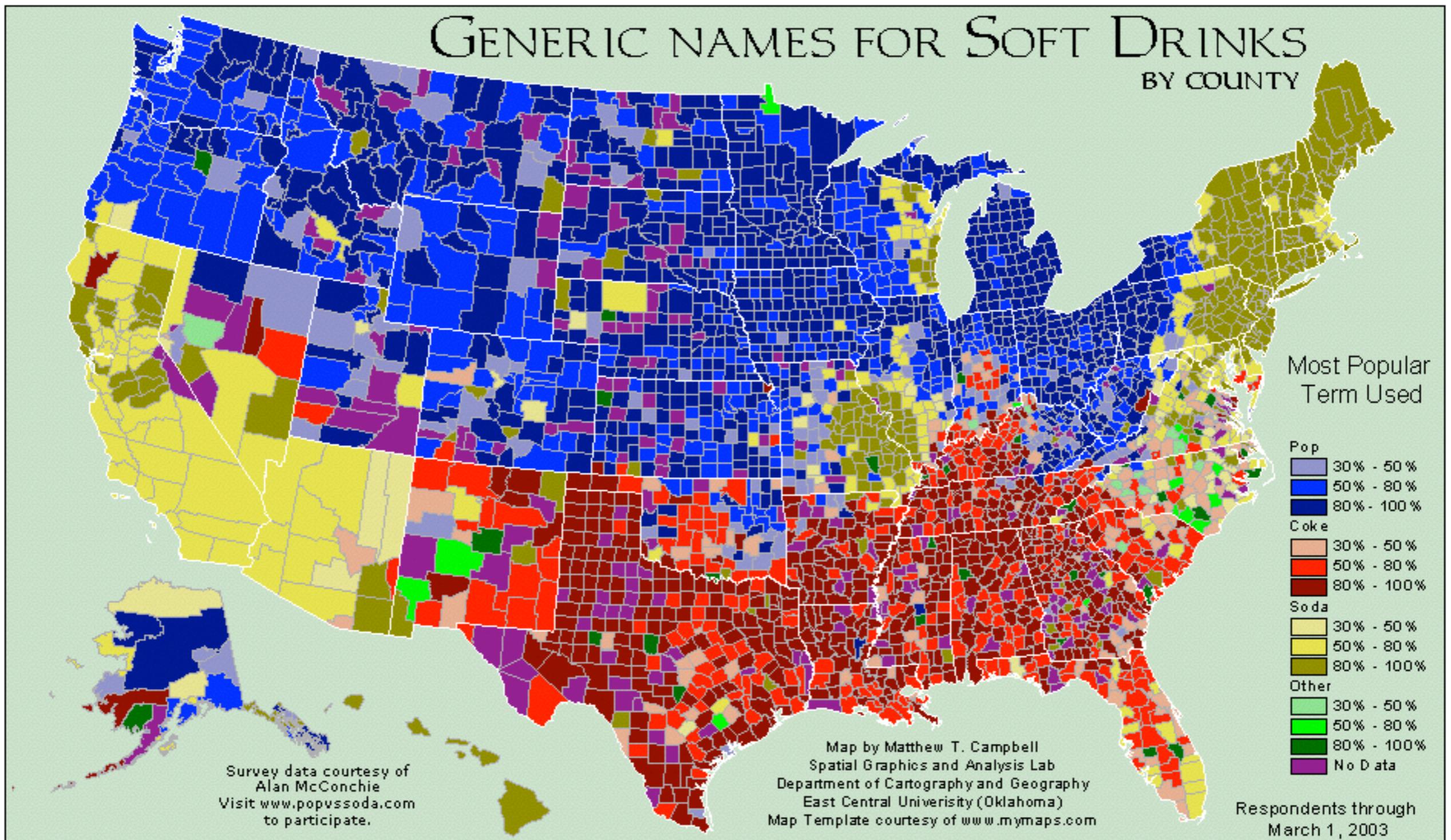
in 3D!



UFO hotspots



GENERIC NAMES FOR SOFT DRINKS BY COUNTY





Advertise on NYTimes.com

March 7, 2009

[E-MAIL](#) | [FEEDBACK](#)

New Yorkers Assess Their City

How New Yorkers feel about everything from fire protection to rat control, taken from a survey of thousands of residents by the city last year.

[Related Article](#)

► **City ratings**

▼ **Crime and safety**

- Citywide crime control
- Citywide emergency services
- Citywide fire protection
- Citywide police relations
- Emergency preparedness
- In a park
- Local crime control
- Neighborhood emergency services
- Neighborhood fire protection
- Neighborhood police relations
- **On the street**
 - On the subway at night
 - On the subway in the daytime
 - Preparedness

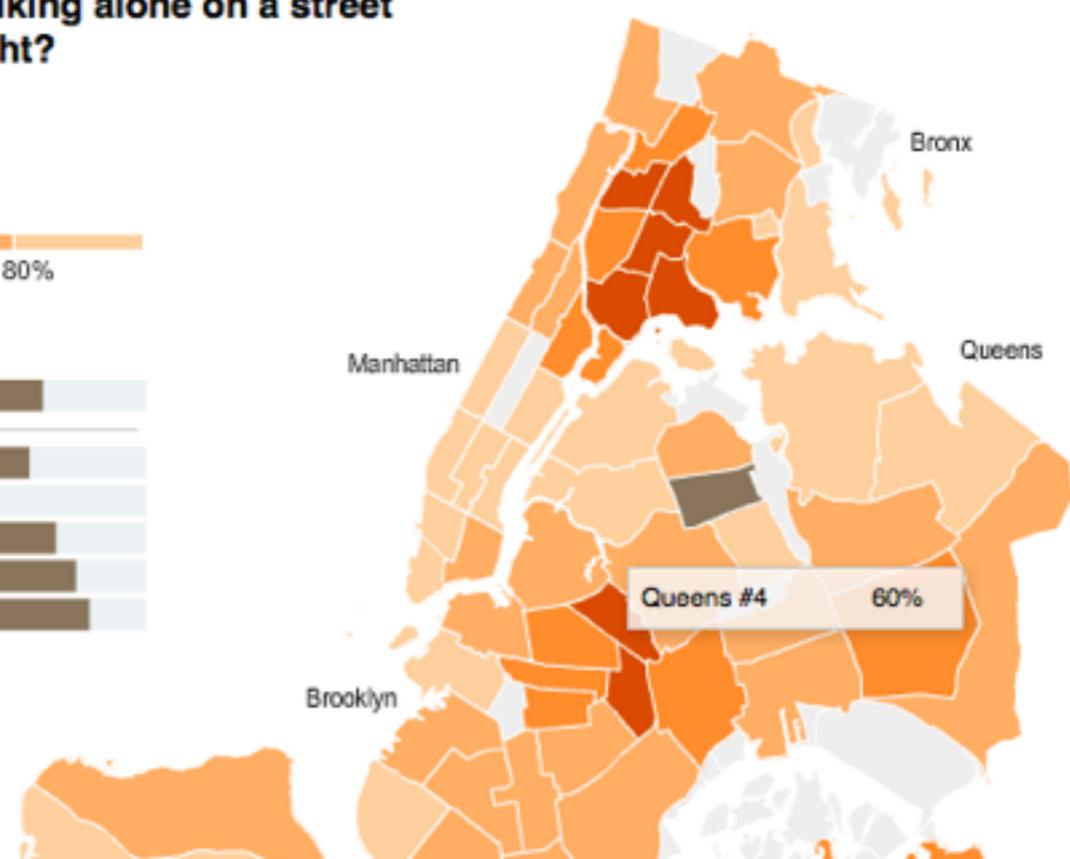
How safe would you feel walking alone on a street in your neighborhood at night?

Percent saying very or somewhat safe



TOTALS

New York City	69%	<div style="width: 69%;"></div>
Brooklyn	65%	<div style="width: 65%;"></div>
Bronx	55%	<div style="width: 55%;"></div>
Queens	73%	<div style="width: 73%;"></div>
Manhattan	79%	<div style="width: 79%;"></div>
Staten Island	83%	<div style="width: 83%;"></div>



► **Education**

- landmarks
- discrete data
- continuous data
- choropleths
- cartograms**
- projections

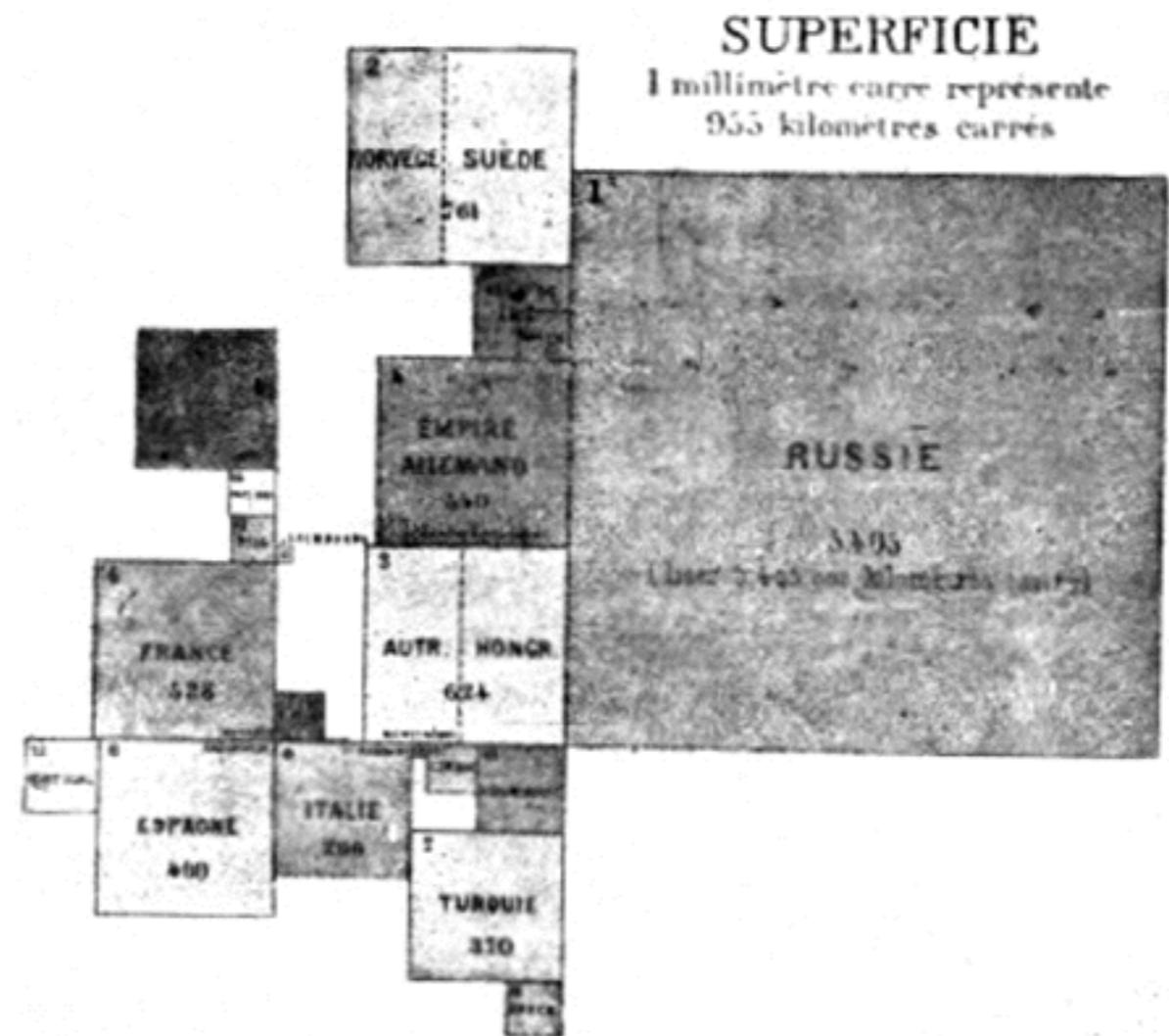
cartogram

map in which areas are scaled and distorted relative to a data attribute value

Land Area

first cartogram

STATISTIQUE FIGURATIVE



Emile Levasseur, 1868

rectangular cartogram

2006 ELECTION GUIDE

SENATE RACES

HOUSE RACES

GOVERNORS' RACES

RACE PROFILES

New York Times ratings

198

Safe Dem.

16

Leaning Dem.

17

Toss up

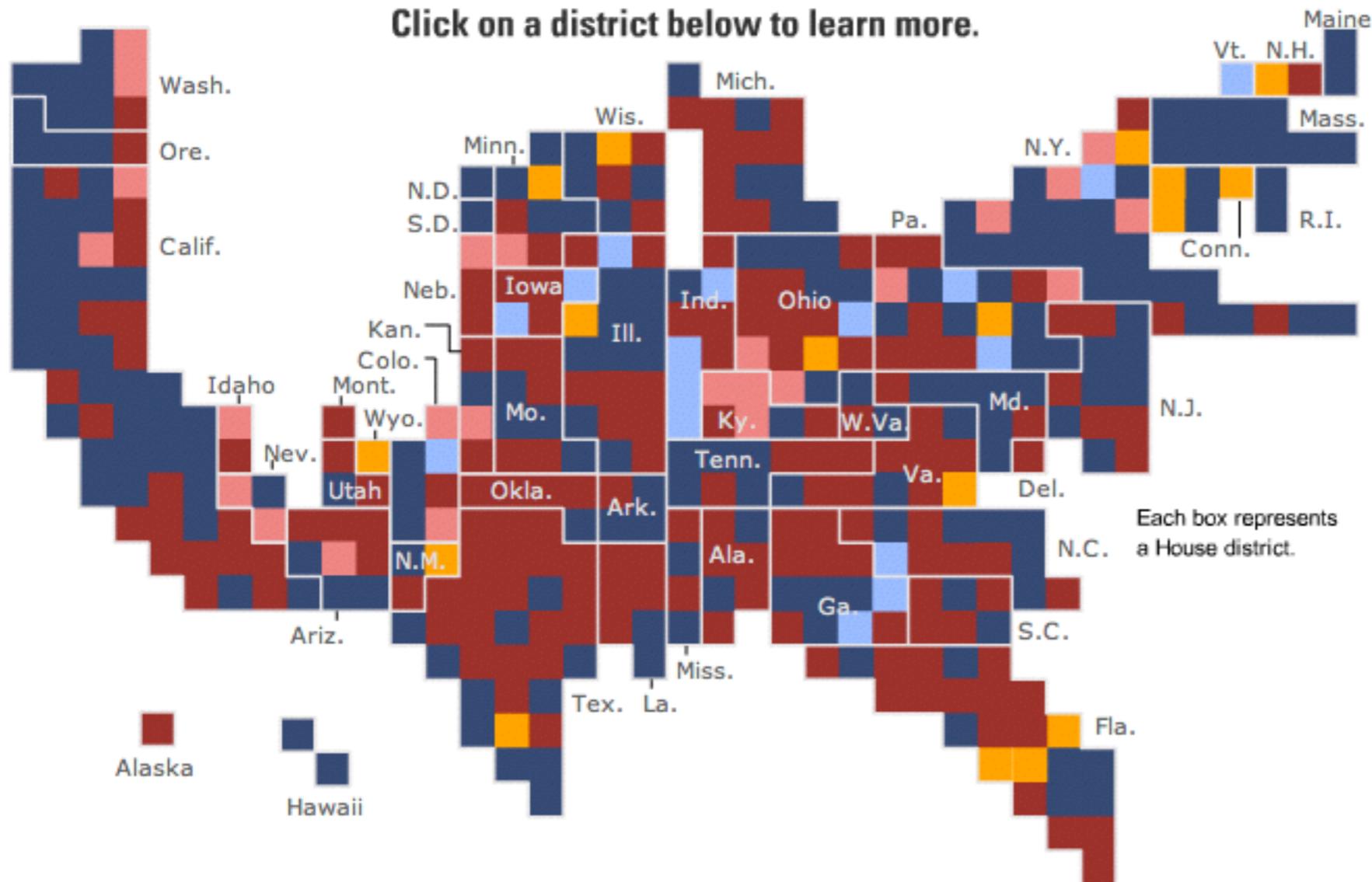
24

Leaning Rep.

180

Safe Rep.

Click on a district below to learn more.



ANALYZE RACES

CREATE OUTCOMES

Shade the map using the pulldown...

New York Times ratings

...then show only certain states

New York Times ratings ?

Democrat: Safe Leaning Toss Up
Republican: Safe Leaning

Current Rep. Dem. Rep.

Margin in 2004 House race

Democrat: >50% 25-50% <25%
Republican: >50% 25-50% <25%

Votes for president Kerry Gore
 Bush Bush

Appearances by big fundraisers ?

George W. Bush Bill Clinton

Races to watch ?

Open races

Switch districts ?

Urbanization

Urban Suburban Rural Mixed

Race/Ethnicity

White Black Hispanic

Median income

<\$30K \$30-50K >\$50K

RESET

Search Business

Go

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More in Business »

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- Markets
- Economy
- DealBook
- Media & Advertising
- Small Business
- Your Money

September 4, 2008

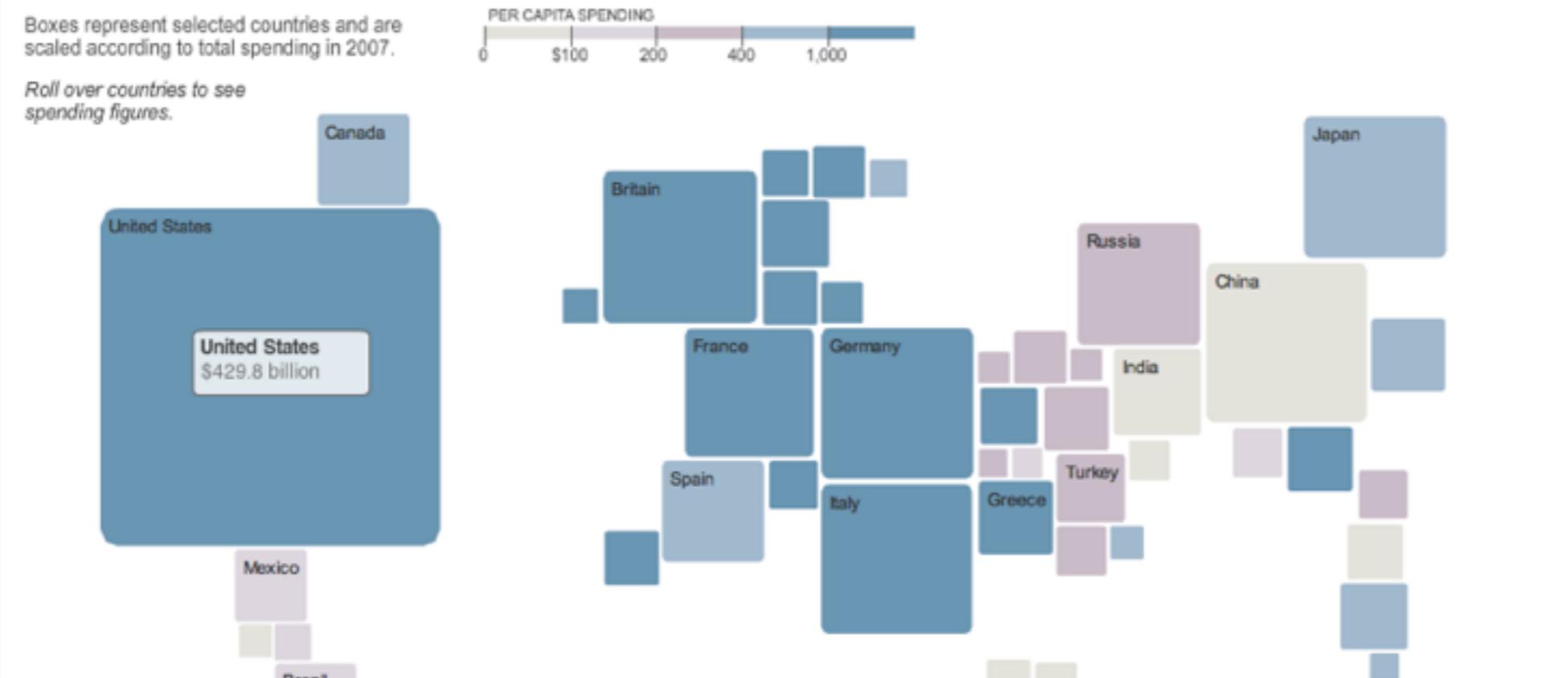
E-MAIL FEEDBACK

What Your Global Neighbors Are Buying

How people spend their discretionary income – the cash that goes to clothing, electronics, recreation, household goods, alcohol – depends a lot on where they live. People in Greece spend almost 13 times more money on clothing as they do on electronics. People living in Japan spend more on recreation than they do on clothing, electronics and household goods combined. Americans spend a lot of money on everything.

[Related Article](#)

- CLOTHING & FOOTWEAR
- ELECTRONICS
- ALCOHOL & TOBACCO
- HOUSEHOLD GOODS
- RECREATION



Population

first nonrectangular cartogram

Apportionment Map of the United States

BY WILLIAM B. BAILEY, Ph.D.

ASSISTANT PROFESSOR OF POLITICAL ECONOMY IN YALE UNIVERSITY.



William Bailey, 1910

Population and Federal Taxes

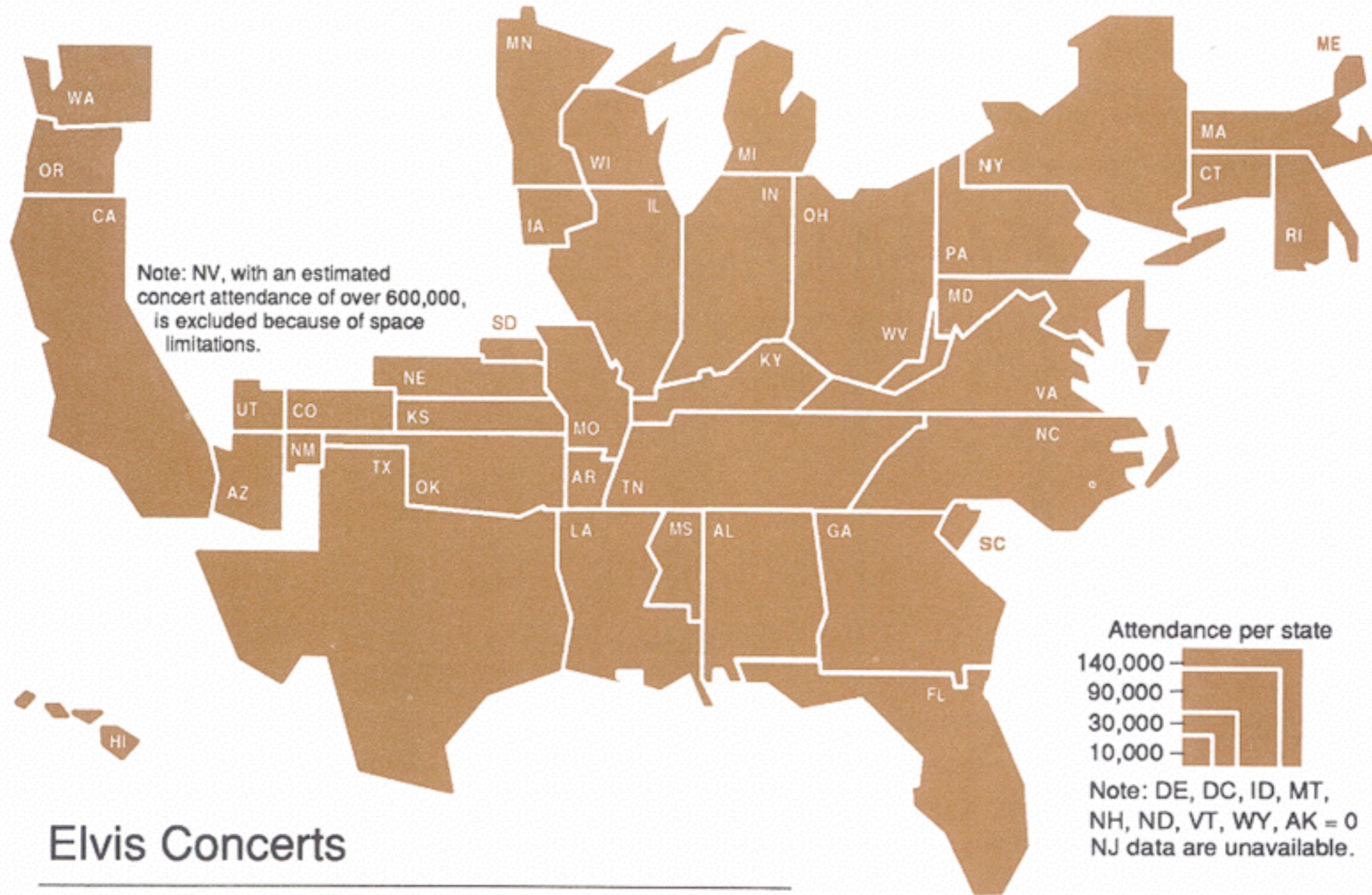
first modern cartogram

GRUNDY'S MAP OF THE UNITED STATES



Joseph Grundy, 1929

area scaled by data



Elvis Concerts

Attendance per State, 1970 - 1977

Source: Stanley, David E., with Frank Coffey. The Elvis Encyclopedia. Santa Monica, CA.: General Publishing Group, Inc , 1994.

© 1995 Andrew Dent and Linda Turnbull

Kerry vs. Bush, 2004



Michael Gastner,
Cosma Shalizi, and
Mark Newman

University of Michigan

distortion via diffusion

Diffusion-based method for producing density-equalizing maps

Michael T. Gastner and M. E. J. Newman*

Center for the Study of Complex Systems and Department of Physics, University of Michigan, Ann Arbor, MI 48109

Edited by Michael F. Goodchild, University of California, Santa Barbara, CA, and approved April 2, 2004 (received for review January 13, 2004)

Map makers have for many years searched for a way to construct cartograms, maps in which the sizes of geographic regions such as countries or provinces appear in proportion to their population or some other analogous property. Such maps are invaluable for the representation of census results, election returns, disease incidence, and many other kinds of human data. Unfortunately, to scale regions and still have them fit together, one is normally forced to distort the regions' shapes, potentially resulting in maps that are difficult to read. Many methods for making cartograms have been proposed, some of them are extremely complex, but all suffer either from this lack of readability or from other pathologies, like overlapping regions or strong dependence on the choice of coordinate axes. Here, we present a technique based on ideas borrowed from elementary physics that suffers none of these drawbacks. Our method is conceptually simple and produces useful, elegant, and easily readable maps. We illustrate the method with applications to the results of the 2000 U.S. presidential

election, lung cancer cases in the State of New York, and the geographical distribution of some other variables.

Suppose we wish to represent on a map some data concerning, to take the most common example, the human population. For instance, we might wish to show votes in an election, incidence of a disease, number of cars, televisions, or phones in use, numbers of people falling in one group or another of the population, by age or income, or any of very many other variables of statistical, medical, or demographic interest. The typical course under such circumstances would be to choose one of the standard projections for the area of interest and plot the data on it with some color code or similar representation. Such maps, however, can be misleading. A plot of disease incidence, for example, will inevitably show high incidence in cities and low incidence in rural areas, solely because more people live in cities.

this kind are known as value-by-area maps, density-equalizing maps, or cartograms.

The construction of cartograms is a challenging undertaking. A variety of methods have been put forward, but none is entirely satisfactory. In particular, many of these methods produce highly distorted maps that are difficult to read or projections that are badly behaved under some circumstances, with overlapping regions or strong dependence on coordinate axes. In many cases the methods proposed are also computationally demanding, sometimes taking hours to produce a single map. In this article we propose a method that is, we believe, intuitive, but also produces elegant, well behaved, and useful cartograms, whose calculation makes relatively low demands on our computational resources.

Previous Methods for Constructing Cartograms

Mathematically, the construction of a (flat 2D) cartogram involves finding a transformation $\mathbf{r} \rightarrow \mathbf{T}(\mathbf{r})$ of a plane to another plane such that the Jacobian $J = \partial(x, y) / \partial(x', y')$ of the transformation is proportional to the (population) density $\rho(\mathbf{r})$, thus:

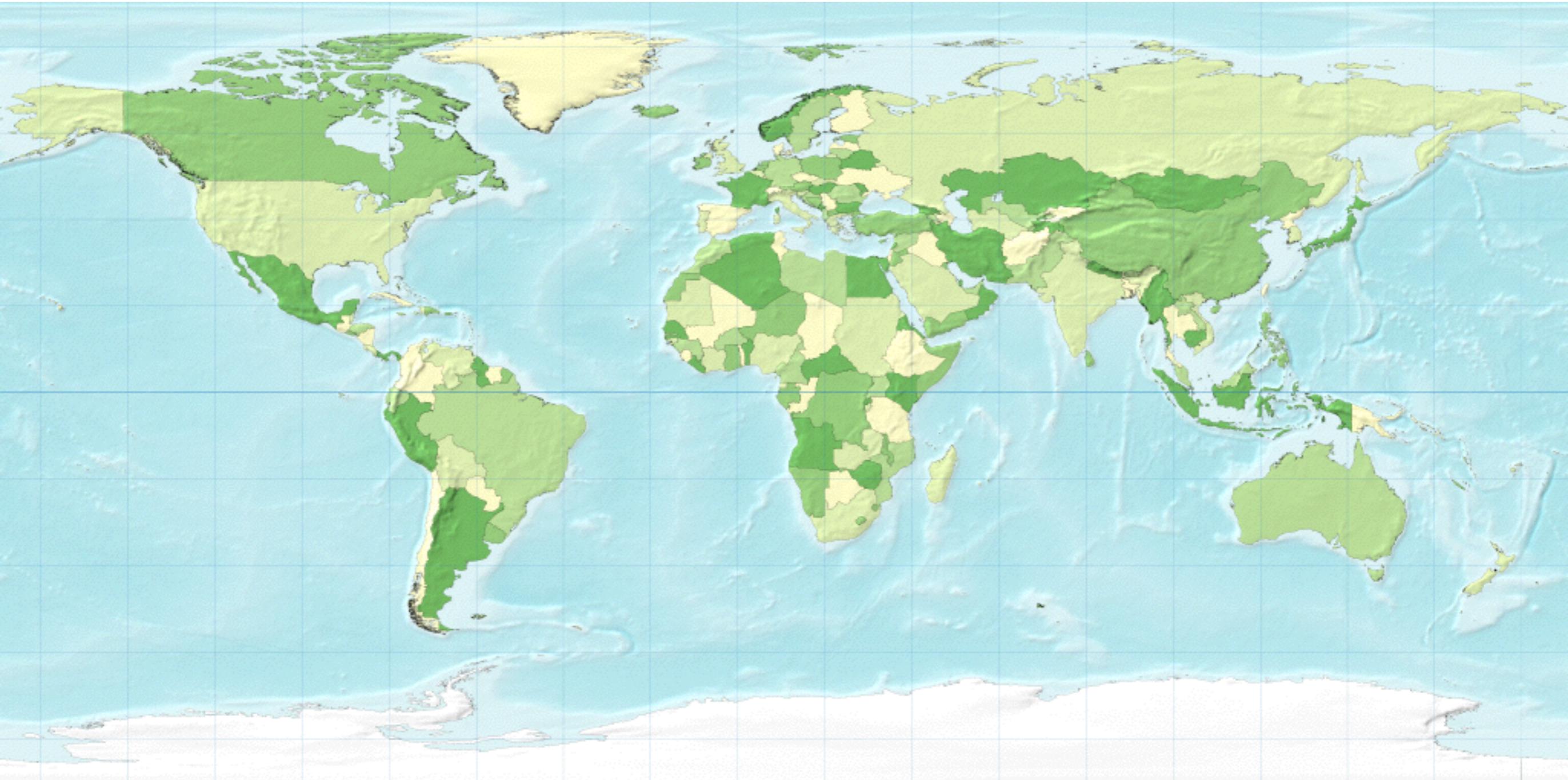
$$\frac{\partial(T_x, T_y)}{\partial(x, y)} \equiv \frac{\partial T_x}{\partial x} \frac{\partial T_y}{\partial y} - \frac{\partial T_x}{\partial y} \frac{\partial T_y}{\partial x} = \frac{\rho(\mathbf{r})}{\bar{\rho}}, \quad [1]$$

where $\bar{\rho}$ is the mean population density averaged over the area to be mapped. (This choice of normalization for the Jacobian ensures that the total area before and after the transformation is the same.)

Eq. 1 does not determine the cartogram projection uniquely. To do that, we need one more constraint; two constraints are needed to fix the projection for a 2D cartogram. Different choices of the second constraint give different projections, and no single choice appears to be the obvious candidate, which is

RECOMMENDED READING

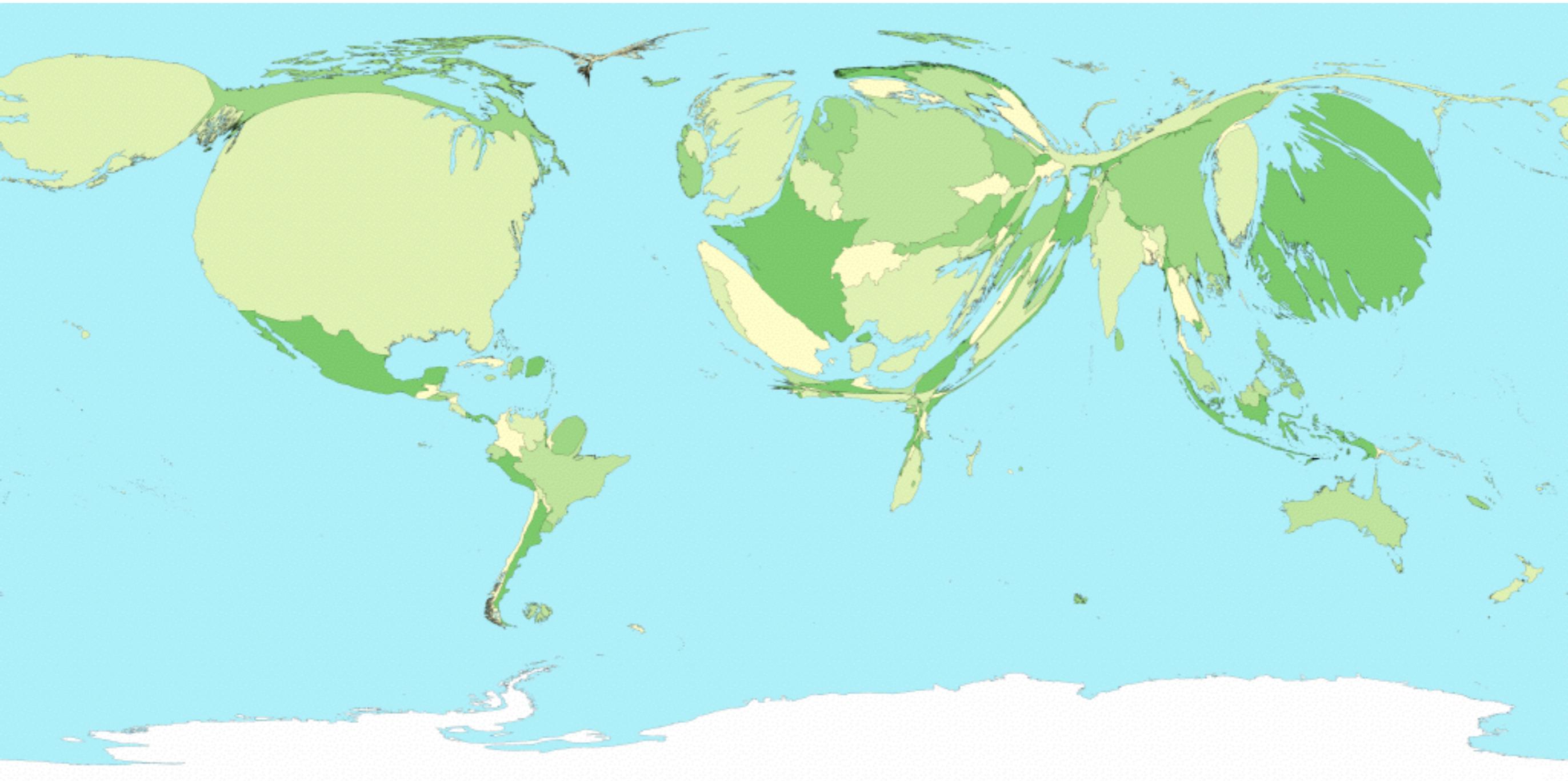
the world



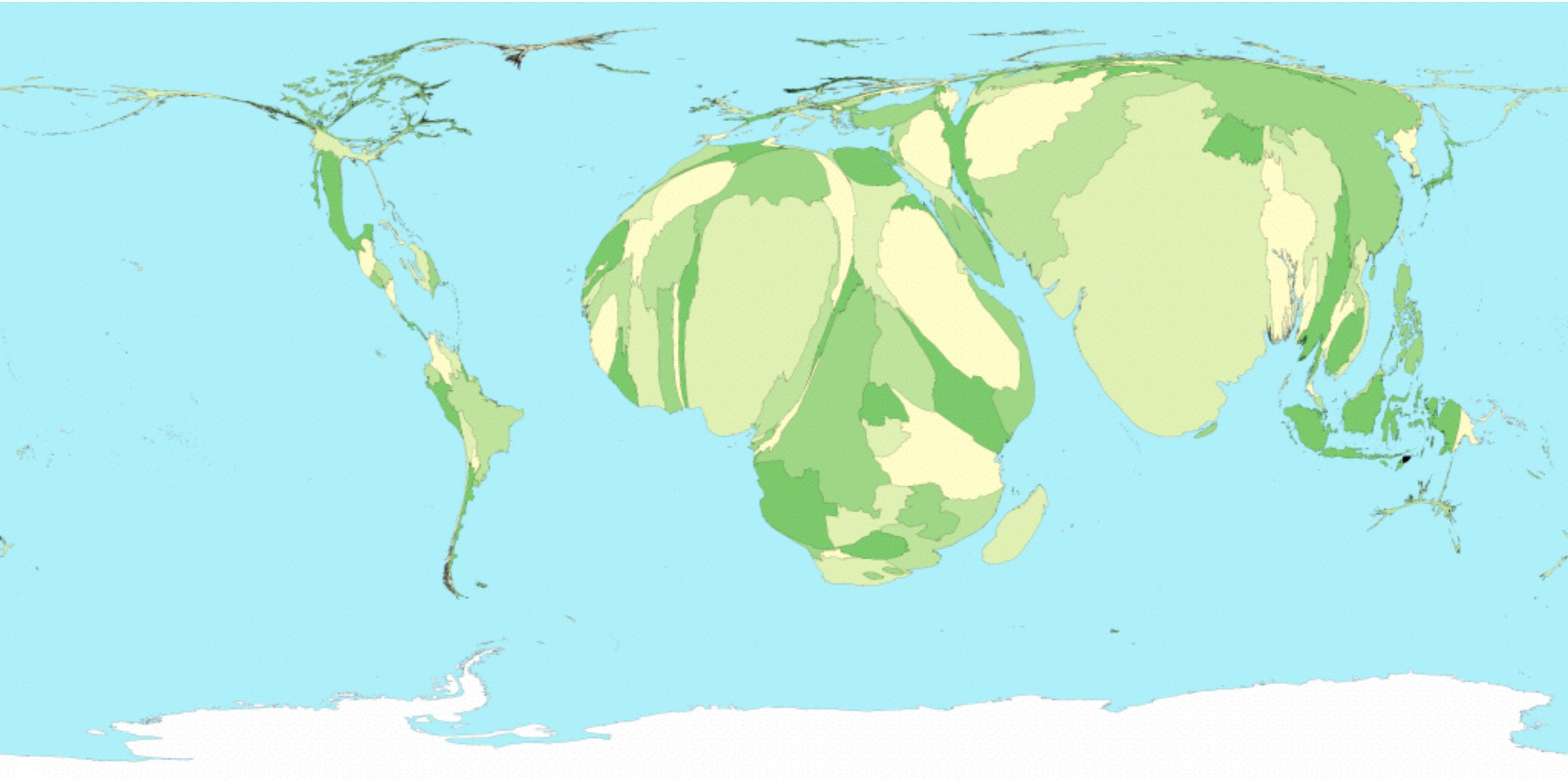
population



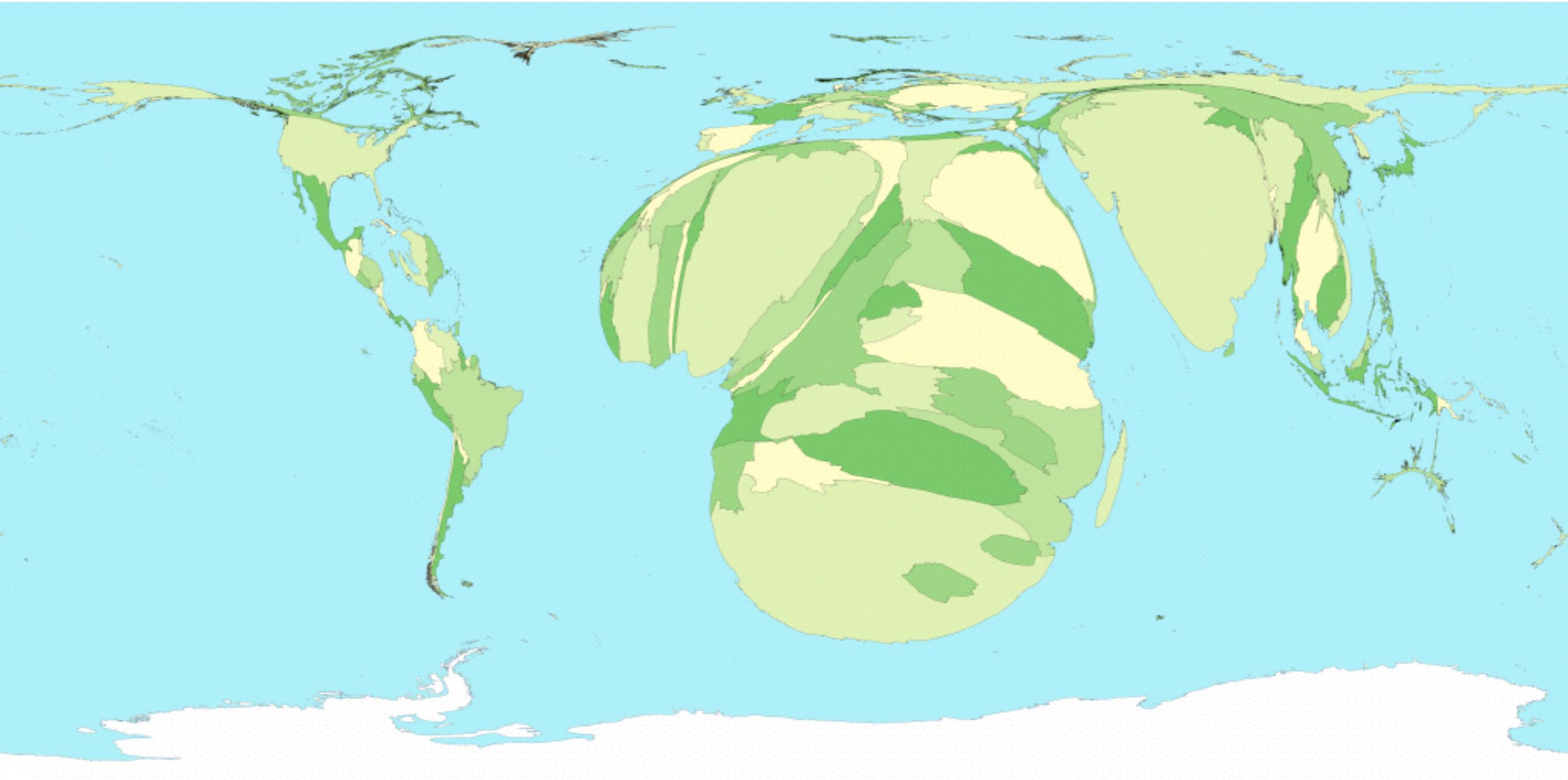
GDP



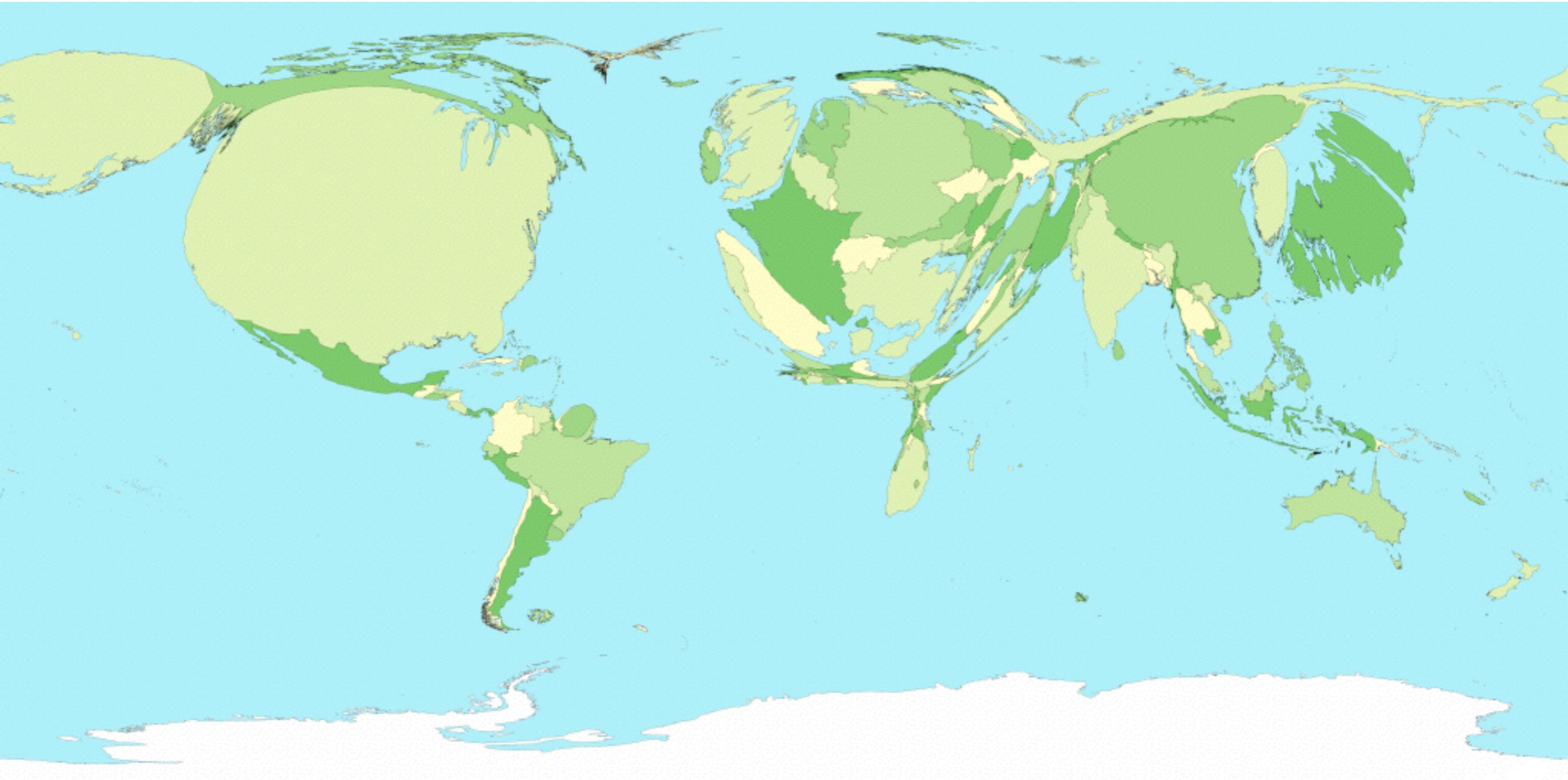
child mortality



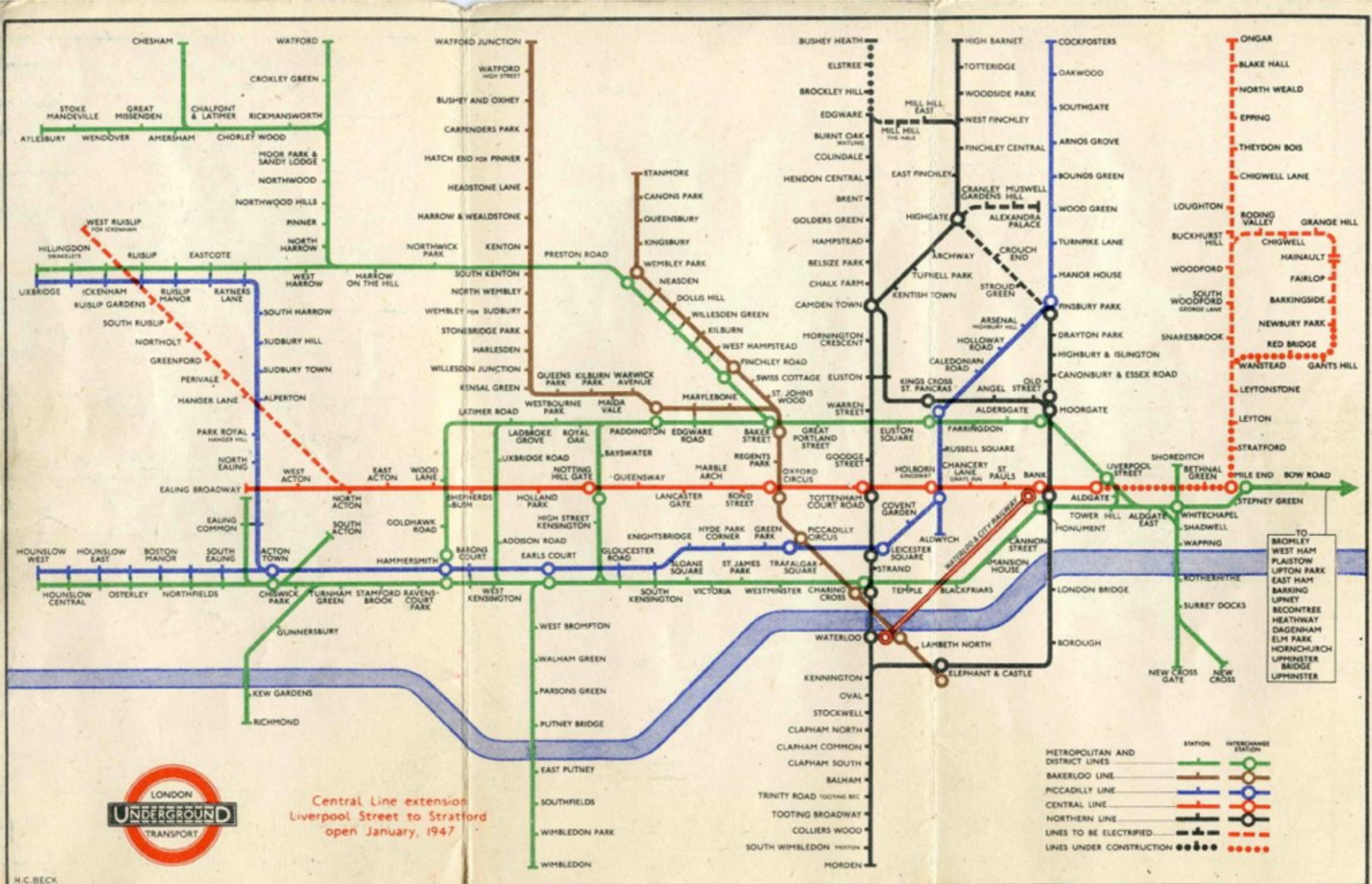
people living with HIV/AIDS



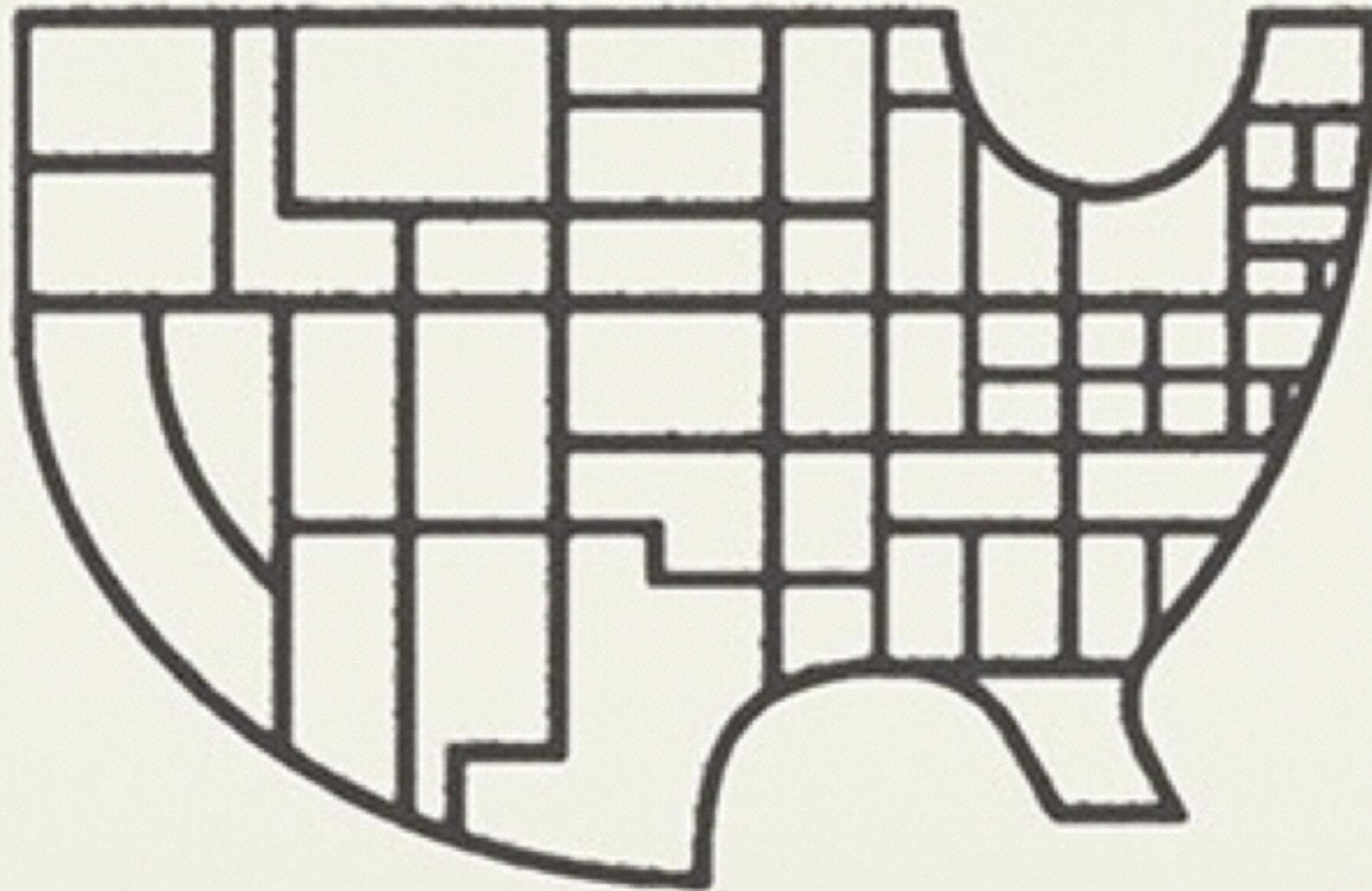
total spending on healthcare



Harry Beck's Tube Map



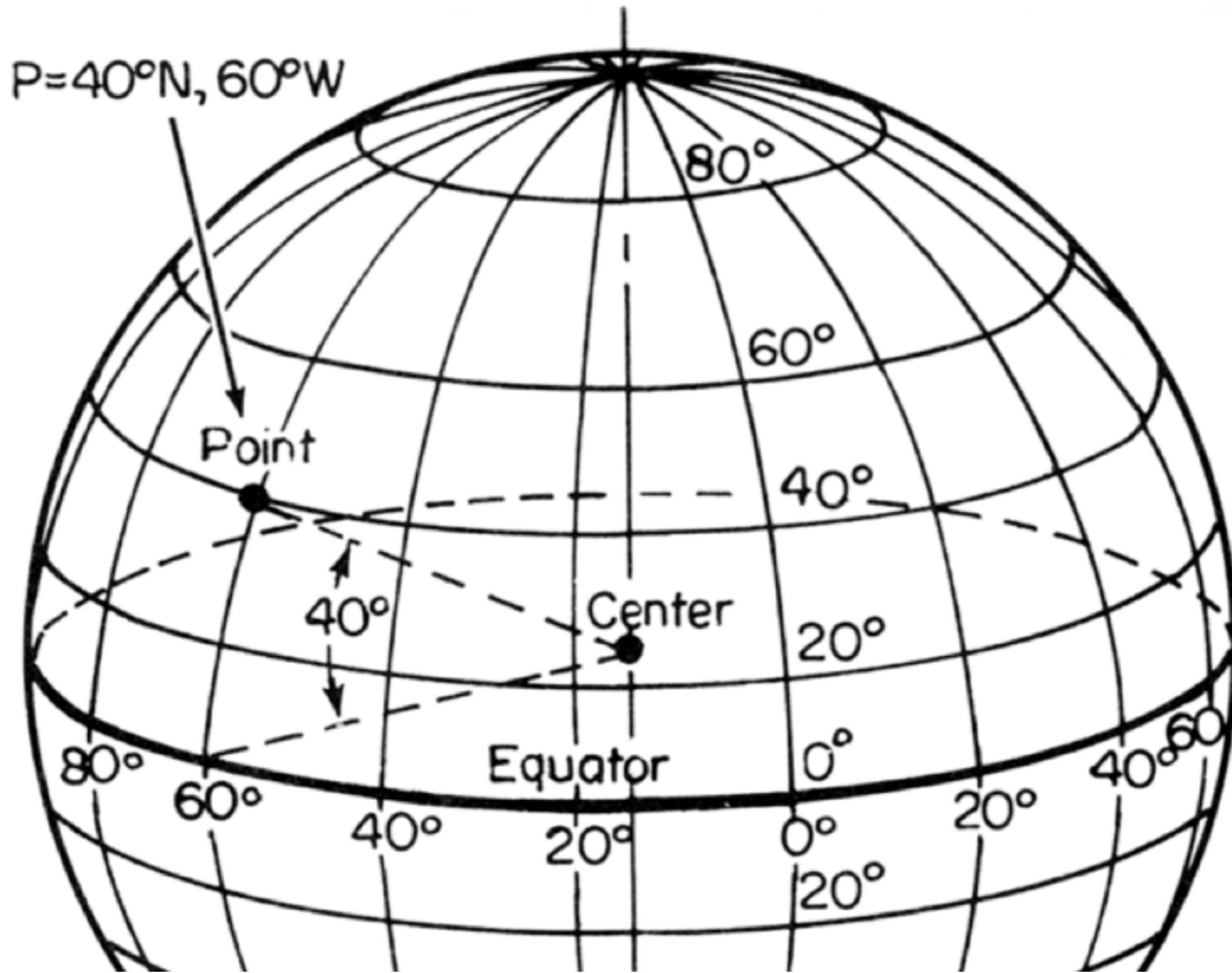
do they work?



- landmarks
- discrete data
- continuous data
- choropleths
- cartograms
- projections**

FLATTENING THE GLOBE

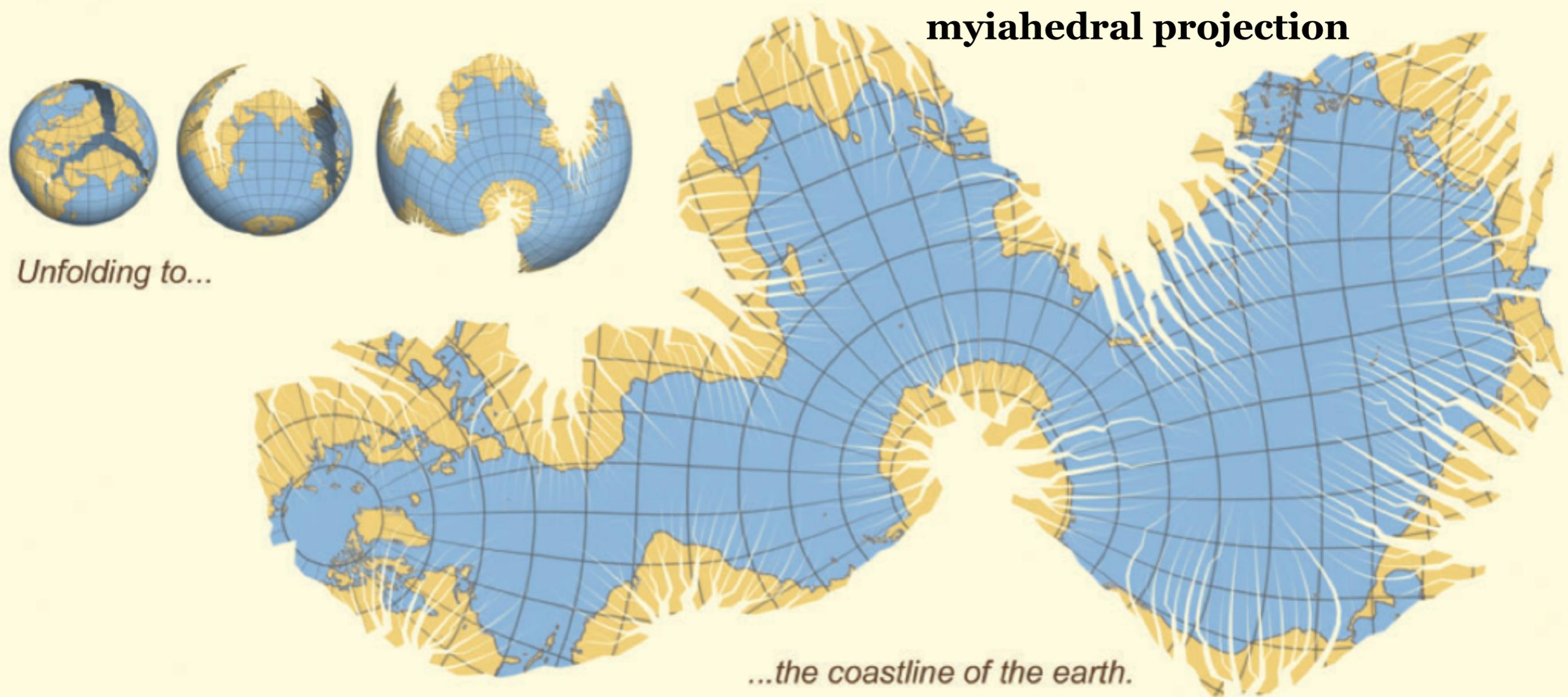
latitude, longitude



**a sphere tears
when you flatten it**



many ways to tear...



WHAT YOUR FAVORITE
MAP PROJECTION
SAYS ABOUT YOU

MERCATOR



YOU'RE NOT REALLY INTO MAPS.

ROBINSON



YOU HAVE A COMFORTABLE PAIR OF RUNNING SHOES THAT YOU WEAR EVERYWHERE. YOU LIKE COFFEE AND ENJOY THE BEATLES. YOU THINK THE ROBINSON IS THE BEST-LOOKING PROJECTION, HANDS DOWN.

VAN DER GRINTEN



YOU'RE NOT A COMPLICATED PERSON. YOU LOVE THE MERCATOR PROJECTION; YOU JUST WISH IT WEREN'T SQUARE. THE EARTH'S NOT A SQUARE, IT'S A CIRCLE. YOU LIKE CIRCLES. TODAY IS GONNA BE A GOOD DAY!

DYMAXION



YOU LIKE ISAAC ASIMOV, XML, AND SHOES WITH TOES. YOU THINK THE SEGWAY GOT A BAD RAP. YOU OWN 3D GOGGLES, WHICH YOU USE TO VIEW ROTATING MODELS OF BETTER 3D GOGGLES. YOU TYPE IN DVORAK.

WINKEL-TRIPPEL



NATIONAL GEOGRAPHIC ADOPTED THE WINKEL-TRIPPEL IN 1998, BUT YOU'VE BEEN A WT FAN SINCE LONG BEFORE "NAT GEO" SHOWED UP. YOU'RE WORRIED IT'S GETTING PLAYED OUT, AND ARE THINKING OF SWITCHING TO THE KAVRAYSKIY. YOU ONCE LEFT A PARTY IN DISGUST WHEN A GUEST SHOWED UP WEARING SHOES WITH TOES. YOUR FAVORITE MUSICAL GENRE IS "POST-".

GOODE HOMOLOGINE



THEY SAY MAPPING THE EARTH ON A 2D SURFACE IS LIKE FLATTENING AN ORANGE PEEL, WHICH SEEMS EASY ENOUGH TO YOU. YOU LIKE EASY SOLUTIONS. YOU THINK WE WOULDN'T HAVE SO MANY PROBLEMS IF WE'D JUST ELECT *NORMAL* PEOPLE TO CONGRESS INSTEAD OF POLITICIANS. YOU THINK AIRLINES SHOULD JUST BUY FOOD FROM THE RESTAURANTS NEAR THE GATES AND SERVE *THAT* ON BOARD. YOU CHANGE YOUR CAR'S OIL, BUT SECRETLY WONDER IF YOU REALLY *NEED* TO.

HOBO-DYER



YOU WANT TO AVOID CULTURAL IMPERIALISM, BUT YOU'VE HEARD BAD THINGS ABOUT GALL-PETERS. YOU'RE CONFLICT-AVERSE AND BUY ORGANIC. YOU USE A RECENTLY-INVENTED SET OF GENDER-NEUTRAL PRONOUNS AND THINK THAT WHAT THE WORLD NEEDS IS A REVOLUTION IN CONSCIOUSNESS.

PLATE CARRÉE (EQUIRECTANGULAR)



YOU THINK THIS ONE IS FINE. YOU LIKE HOW X AND Y MAP TO LATITUDE AND LONGITUDE. THE OTHER PROJECTIONS OVERCOMPLICATE THINGS. YOU WANT ME TO STOP ASKING ABOUT MAPS SO YOU CAN ENJOY DINNER.

A GLOBE!



WATERMAN BUTTERFLY



YOU'RE CONFLICT-AVERSE AND BUY ORGANIC. YOU USE A RECENTLY-INVENTED SET OF GENDER-NEUTRAL PRONOUNS AND THINK THAT WHAT THE WORLD NEEDS IS A REVOLUTION IN CONSCIOUSNESS.

A GLOBE!



YES, YOU'RE VERY CLEVER.

PEIRCE QUINCUNCIAL



YOU THINK THAT WHEN WE LOOK AT A MAP, WHAT WE REALLY SEE IS OURSELVES. AFTER YOU FIRST SAW *INCEPTION*, YOU SAT SILENT IN THE THEATER FOR SIX HOURS. IT FREAKS YOU OUT TO REALIZE THAT EVERYONE AROUND YOU HAS A SKELETON INSIDE THEM. YOU *HAVE* REALLY LOOKED AT YOUR HANDS.

MAP TO LATITUDE AND LONGITUDE. THE OTHER PROJECTIONS OVERCOMPLICATE THINGS. YOU WANT ME TO STOP ASKING ABOUT MAPS SO YOU CAN ENJOY DINNER.

WATERMAN BUTTERFLY



REALLY? YOU KNOW THE WATERMAN? HAVE YOU SEEN THE 1909 CAHILL MAP IT'S BASED— ... YOU HAVE A FRAMED REPRODUCTION AT HOME?! WHOA. ...LISTEN, FORGET THESE QUESTIONS. ARE YOU DOING ANYTHING TONIGHT?

GALL-PETERS



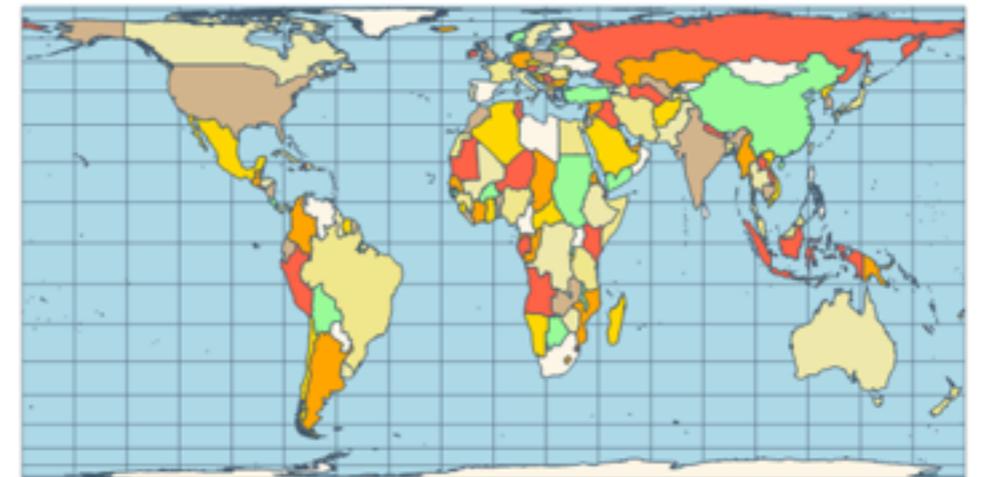
I HATE YOU.

CLASSES OF PROJECTION

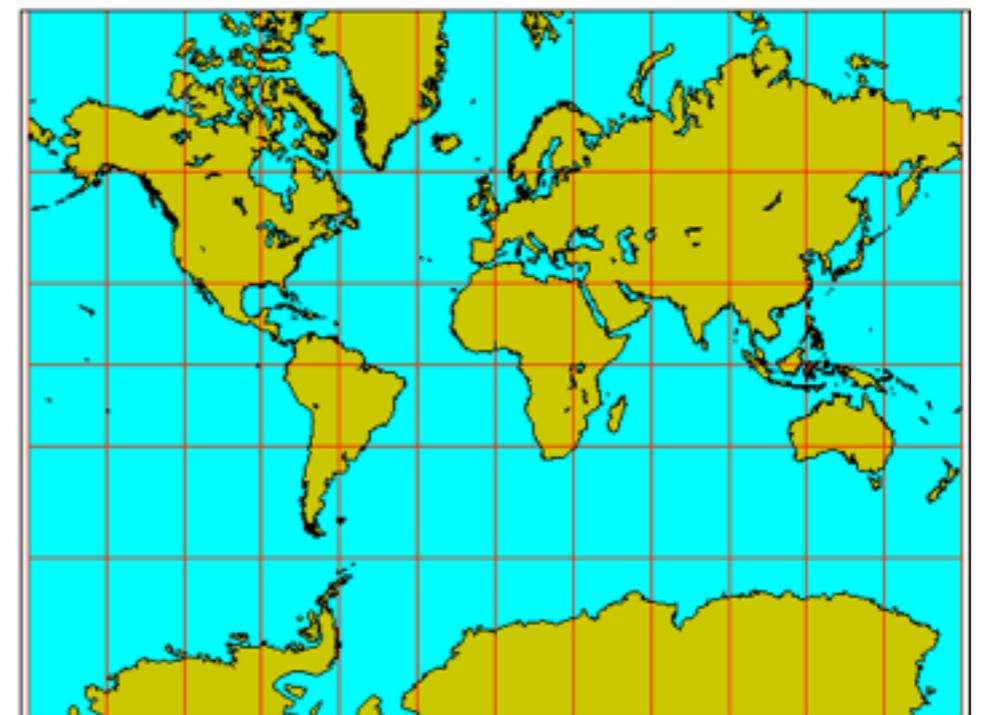
azimuthal
preserves direction



equal-area
preserves area



conformal
preserves local shapes



L16: Grids

REQUIRED READING

Facets

Increasingly, 3D data acquisition techniques, such as laser scanners, are used to bring the physical world into a digital form. Powerful computers that are equipped with large amounts of memory can be credited for this increase in use. Laser scanners produce *point clouds*—unorganized 3D points. Analysis and rendering of these point clouds are most efficiently done when the points are connected to form *facets*—planar entities.

13.1 Triangles

The subject of triangles as taught in high school constructed them from angles, edge lengths, and trigonometric functions. Typically, we dealt with *one* triangle. In computing and visualization, we also encounter triangles; however now they exist in rather large numbers. Some objects consist of up to 1,000,000,000 triangles.

We present a brief review of triangle facts. Everything in this review holds for both 2D and 3D triangles.

Triangle. A *triangle* is a 2D or 3D object defined by three vertices $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$, where the \mathbf{v}_i are points in 2D or 3D.