administrivia...
- assignment 8 due Thursday

- assignment 9 out tomorrow… due in 2.5 weeks
assignment 6 scores

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last time...
binary search trees (BSTs)
- A binary search tree is a binary tree with a restriction on the ordering of nodes,
  - all items in the left subtree of a node are less than the item in the node,
  - all items in the right subtree of a node are greater than or equal to the item in the node.

- BSTs allow for fast searching of nodes.
IS THIS A BST?
A) yes
B) no
insertion
insertion & searching

- **average case:** \( O(\log N) \)
  - inserted in random order

- **worst case:** \( O(N) \)
  - inserted in ascending or descending order

- **best case:** \( O(\log N) \)

- how does this compare to a sorted array?
deletion
-since we must maintain the properties of a tree structure, deletion is more complicated than with an array or linked-list

-there are three different cases:
  1. deleting a leaf node
  2. deleting a node with one child subtree
  3. deleting a node with two children subtrees

-first step of deletion is to find the node to delete
  -just a regular BST search
  -BUT, stop at the parent of the node to be deleted
deletion performance

What is the cost of deleting a node from a BST?

-first, find the node we want to delete: \(O(\log N)\)

cost of:

-case 1 (delete leaf):
  set a single reference to null: \(O(1)\)

-case 2 (delete node with 1 child):
  bypass a reference: \(O(1)\)

-case 3 (delete node with 2 children):
  find the successor: \(O(\log N)\)
  delete the duplicate successor: \(O(1)\)
today...
-graphs
-paths
-depth-first search
-breadth-first search
graphs
-trees are a \textit{subset} of graphs

-a \textbf{graph} is a set of \textit{nodes} connected by \textit{edges}
  -an edge is just a link between two nodes
  -nodes don’t have a parent-child relationship
  -links can be bi-directional

-graphs are used \textbf{EXTENSIVELY} throughout CS
Nodes are cities, edges are flights.
some definitions

A directed graph
An undirected graph
Weighted
Unconnected
Node degrees

A cycle
An acyclic graph
A connected acyclic graph, a.k.a. a tree
A rooted tree or hierarchy
Node depths
WHAT MAKES THIS A GRAPH AND NOT A TREE?
- graphs have no root; must store all nodes

```java
class Graph<E> {
    List<Node> nodes;
    ...
}
```

- implementation is more general than a tree

```java
class Node{
    E Data;
    List<Node> neighbors;
    ...
}
```

- the order in which neighbors appear in the list is unspecified
  - a different order still make the same graph!
paths
- A **path** is a sequence of nodes with a start-point and an end-point such that the end-point can be reached through a series of nodes from the start-point.

- In this example, there is a path from SLC to DFW:
  - SLC — IAD — ATL — DFW

- There is *not* a path from DFW to SLC.
pathfinding

- there may be more than one path from one node to another

- we are often interested in the path length

- finding the shortest (or cheapest) path between two nodes is a common graph operation
cycles

-a cycle in a graph is a path from a node back to itself
- B — E — D — B

-while traversing a graph, special care must be taken to avoid cycles, otherwise what?

-can trees have cycles?
-any problem with a starting state, a goal state, and options as to which direction to take for each step can be represented with a graph

-and solved with pathfinding!
example

-in games, moving a character around a space

-character finds the shortest path from its current location to the destination
  -not always a straight line

-terrain is represented as a graph
  -every non-obstacle spot on the terrain is a node
  -nodes are connected to adjacent nodes

-navigating a maze…
- depth-first search (just like a tree) — DFS

- breadth-first search — BFS

- if there exists a path from one node to another these algorithms will find it
  - the nodes on this path are the steps to take to get from point A to point B

- if multiple such paths exist, the algorithms may find different ones
WE WANT TO FIND A PATH FROM A TO C
depth-first search
- look at the first edge going out of the start node
- recursively search from the new node
- upon returning, take the next edge
- if no more edges, return

- when visiting a node, mark it as visited so we don’t get stuck in a cycle
  - skip already visited nodes during traversal

- for each node visited, save a reference to the node where we came from to reconstruct the path
WE WANT TO FIND A PATH FROM A TO C

SO... START FROM A, TRAVERSE ITS FIRST EDGE, SAVE WHERE WE CAME FROM, AND RECURSE

A.visited = true
B.cameFrom = A
TRVERSE THE FIRST UNVISITED NODE IN THE EDGE LIST RECURSIVELY, SAVE WHERE WE CAME FROM

B.visited = true
E.cameFrom = B
TRAVERE THE FIRST UNVISITED NODE IN THE EDGE LIST RECURSIVELY, SAVE WHERE WE CAME FROM

E.visited = true

LOOK AT THE FIRST EDGE; NODE A HAS ALREADY BEEN VISITED, SO SKIP
LOOK AT NEXT EDGE; C HAS NOT BEEN VISITED YET

C.cameFrom = E
NODE C IS OUR GOAL. WE ARE DONE!

C.visited = true

FOLLOW EACH NODE’S `cameFrom` TO RECONSTRUCT THE PATH

C.cameFrom = E, E.cameFrom = B, B.cameFrom = A

PATH: A — B — E — C
IS THERE A BETTER (SHORTER) PATH FROM A TO C?

WHAT DETERMINES WHICH PATH DFS FINDS?

DFS IS NOT GUARANTEED TO FIND THE SHORTEST PATH, JUST A PATH.
DFS(Node curr, Node goal) {
    curr.visited = true

    if(curr.equals(goal))
        return

    for(Node next : curr.neighbors)
        if(!next.visited)
            {
                next.cameFrom = curr
                DFS(next, goal)
            }
}

// path is now saved in nodes’ .cameFrom
breadth-first search
-instead of visiting deeper nodes first, visit shallower nodes first
  -visit nodes closets to the start point first, gradually get further away

-create an empty queue
-put the starting node in the queue
-while the queue is not empty
  -dequeue the current node
  -for each unvisited neighbor of the the current node
    -mark the neighbor as visited
    -put the neighbor into the queue

-notice it is not recursive… it just runs until the queue is empty!
We want to find a path from A to C

Mark and enqueue the start node A

A.visited = true

Queue: [A, , , , ]
DEQUEUE THE FIRST NODE IN THE QUEUE (**A**)

MARK AND ENQUEUE **A**’S UNVISITED NEIGHBORS

B. `cameFrom = A`
D. `cameFrom = A`
B. `visited = true`
D. `visited = true`

```
queue: B D _ _ _
```

```
VISITED O
UNVISITED O
```
DEQUEUE THE FIRST NODE IN THE QUEUE (B)

MARK AND ENQUEUE B’S UNVISITED NEIGHBORS

E.cameFrom = B
E.visited = true
DEQUEUE THE FIRST NODE IN THE QUEUE (D)

MARK AND ENQUEUE D’S UNVISITED NEIGHBORS

C.cameFrom = D
C.visited = true
DEQUEUE THE FIRST NODE IN THE QUEUE (E)

MARK AND ENQUEUE E’S UNVISITED NEIGHBORS

(NO UNVISITED NEIGHBORS!)

queue: C
Deque the first node in the queue (C)

C is the goal! Reconstruct the path with `cameFrom` references

C.cameFrom = D,
D.cameFrom = A

Path: A — D — C
IS THIS THE SHORTEST PATH?
PATH: **A — D — C**

BFS VISITS NODES CLOSEST TO THE START-POINT FIRST

THEREFORE, THE FIRST PATH FOUND IS THE SHORTEST PATH (CLOSEST TO THE START NODE)
BFS(Node start, Node goal)
{
    start.visited = true
    Q.enqueue(start)

    while(!Q.empty())
    {
        Node curr = Q.dequeue()
        if(curr.equals(goal))
            return

        for(Node next : curr.neighbors)
            if(!next.visited)
            {
                next.visited = true
                next.cameFrom = curr
                Q.enqueue(next)
            }
    }
}
What path will BFS find from B to C?
A) B E C
B) B E A D C
C) B E D C
D) none
 WHAT PATH WILL DFS FIND FROM A TO D?

A) A B E D
B) A D
C) none
D) this is a trick question
WHAT IS TRUE OF DFS, SEARCHING FROM A START NODE TO A GOAL NODE?

A) if a path exists, it will find it
B) it is guaranteed to find the shortest path
C) it is guaranteed to not find the shortest path
D) it must be careful about cycles
E) a, b, and d
F) a, c, and d
G) a and d
WHAT IS TRUE OF BFS, SEARCHING FROM A START NODE TO A GOAL NODE?

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E) a, b, and d
F) a, c, and d
G) a and d
topological sort
- the **indegree** of a node is the number of edges it has incoming

- this can be saved as part of the `Node` class, and can be easily computed as the graph is constructed

- any time a node adds another node as a neighbor, increase the neighbor’s indegree
topological sort

- consider a graph with no cycles

- a topological sort orders nodes such that...
  - if there is a path from node A to node B, then A appears before B in the sorted order

example: scheduling tasks

- represent the tasks in a graph
- if task A must be completed before task B, then A has an edge to B
1. step through each node in the graph
   - if any node has indegree 0, add it to a queue

2. while the queue is not empty
   - dequeue the first node in the queue and add to the sorted list
   - visit that node’s neighbors and decrease their indegree by 1
   - if a neighbor’s new indegree is 0, add it to the queue
queue:

sorted list:
ENQUEUE ANY NODES WITH INDEGREE 0

queue: CS1410 MATH2250

sorted list:
queue: MATH2250
sorted list: CS1410

WHILE QUEUE NOT EMPTY:
1. DEQUEUE NODE, ADD IT TO THE SORTED LIST
WHILE QUEUE NOT EMPTY:
1. DEQUEUE NODE, ADD IT TO THE SORTED LIST
2. DECREASE NEIGHBORS’ INDEGREE
WHILE QUEUE NOT EMPTY:
1. DEQUEUE NODE, ADD IT TO THE SORTED LIST
2. DECREASE NEIGHBORS’ INDEGREE
3. IF NEIGHBORS’ NEW INDEGREE IS 0, ADD IT TO THE QUEUE

queue: MATH2250  CS2100  CS2420
sorted list: CS1410
WHILE QUEUE NOT EMPTY:
1. DEQUEUE NODE, ADD IT TO THE SORTED LIST
2. DECREASE NEIGHBORS’ INDEGREE
3. IF NEIGHBORS’ NEW INDEGREE IS 0, ADD IT TO THE QUEUE

queue: CS2100 CS2420
sorted list: CS1410 MATH2250
while queue not empty:
  1. dequeue node, add it to the sorted list
  2. decrease neighbors’ indegree
  3. if neighbors’ new indegree is 0, add it to the queue
WHILE QUEUE NOT EMPTY:
1. DEQUEUE NODE, ADD IT TO THE SORTED LIST
2. DECREASE NEIGHBORS’ INDEGREE
3. IF NEIGHBORS’ NEW INDEGREE IS 0, ADD IT TO THE QUEUE
while queue not empty:
1. dequeue node, add it to the sorted list
2. decrease neighbors' indegree
3. if neighbors' new indegree is 0, add it to the queue

queue: CS4150, CS3500, CS3810, CS3200
sorted list: CS1410, MATH2250, CS2100, CS2420, CS3100
queue: \[ \text{CS3500, CS3810, CS3200} \]

sorted list: \[ \text{CS1410, MATH2250, CS2100, CS2420, CS3100, CS4150} \]

WHILE QUEUE NOT EMPTY:
1. DEQUEUE NODE, ADD IT TO THE SORTED LIST
2. DECREASE NEIGHBORS’ INDEGREE
3. IF NEIGHBORS’ NEW INDEGREE IS 0, ADD IT TO THE QUEUE
queue: CS3810 CS3200 CS3505

sorted list: CS1410 ... CS4150 CS3500

while queue not empty:
  1. dequeue node, add it to the sorted list
  2. decrease neighbors' indegree
  3. if neighbors' new indegree is 0, add it to the queue
While queue not empty:
1. Dequeue node, add it to the sorted list
2. Decrease neighbors’ indegree
3. If neighbors’ new indegree is 0, add it to the queue
WHILE QUEUE NOT EMPTY:
1. DEQUEUE NODE, ADD IT TO THE SORTED LIST
2. DECREASE NEIGHBORS’ INDEGREE
3. IF NEIGHBORS’ NEW INDEGREE IS 0, ADD IT TO THE QUEUE
WHILE QUEUE NOT EMPTY:
1. DEQUEUE NODE, ADD IT TO THE SORTED LIST
2. DECREASE NEIGHBORS’ INDEGREE
3. IF NEIGHBORS’ NEW INDEGREE IS 0, ADD IT TO THE QUEUE

queue: CS4500
sorted list: CS1410  ...  CS3810  CS3200  CS3505  CS4400
WHILE QUEUE NOT EMPTY:
1. DEQUEUE NODE, ADD IT TO THE SORTED LIST
2. DECREASE NEIGHBORS’ INDEGREE
3. IF NEIGHBORS’ NEW INDEGREE IS Ø, ADD IT TO THE QUEUE

queue:
sorted list: CS1410 ... CS3810 CS3200 CS3505 CS4400 CS4500
sorted list: `[CS1410, MATH2250, CS2100, CS2420, CS3100, CS4150, CS3500, CS3810, CS3200, CS3505, CS4400, CS4500]`
WHICH OF THE FOLLOWING IS A VALID TOPOLOGICAL ORDERING?

A) A D C E B
B) C D E B A
C) A B E D C
D) A B C D E
next time...
-reading
  - chapter 14 in book

-homework
  - assignment 8 due Thursday
  - assignment 9 out tomorrow