Performance of Dynamic Load Balancing Algorithms for Unstructured Mesh Calculations

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Introduction

- Load balancing for structured mesh is relatively easy
- How to achieve load balancing for unstructured meshes?
Introduction

• Load balancing means:
  – Each processor has approximately equal work
  – Communication overhead is minimized
  – Communication occurs in large messages

• Strategy classification:
  – By Inspection
  – Static
  – Quasi-Dynamic (↔ paper discusses this type)
  – Dynamic
Introduction

• Paper discusses quasi-dynamic algorithms
  – Application to scalable codes
  – Require parallel implementation to avoid sequential bottleneck

• Proposed methods:
  – Simulated Annealing (SA)
  – Orthogonal Recursive Bisection (ORB)
  – Eigenvector Recursive Bisection (ERB)
The Optimization Problem

• Define cost function that measures “cost” of running problem.
• Cost function is minimized when the total running time is minimized
• Cost function independent of algorithm details.
• Cost function: \( H = H_{\text{calc}} + \mu H_{\text{comm}} \)
Graph Coloring Analogy
Simulated Annealing

• Analogous to cooling of physical system
• Metropolis algorithm:
  – Propose random changes
  – Accept unconditionally when \( dH < 0 \) (\( H \) decreases)
  – If \( dH > 0 \) (\( H \) increases) except with probability \( \exp(-dH/T) \)
• Requirement for proposed changes is reachability.
Simulated Annealing

Low Temperature

High Temperature
Orthogonal Recursive Bisection

- Reduce problem to a number of smaller problems
- Natural parallelism

**Figure 8:** Load balancing by ORB for four processors. The elements (left) are reduced to points at their centers of mass (middle), then split into two vertically, then each half split into two horizontally. The result (right) shows the assignment of elements to processors.
Eigenvalue Recursive Bisection

- Separate along eigenvectors using adjacency matrix.
- Specifically, define Laplacian of graph
  \[ Q = D - A, \]
  where \( A \) is adjacency matrix and \( D \) diagonal containing degree of nodes.
- Split based on smallest (non-zero) eigenvector of \( Q \)
Testing Method

- Coarse unstructured triangular mesh of 280 elements.
- Laplace equation with Dirichlet boundary conditions.
- Solved using Jacobi iteration.
- Adaptive mesh refinement. Final mesh had 5772 elements.
Results

Machine independent measurements

Figure 12. Machine-independent measures of load balancing performance. Left, percentage load imbalance, lower left, total amount of communication, below, total number of messages.
Results

Machine dependent measurements

Figure 13: Machine-dependent measures of load balancing performance. Left, running time per Jacobi iteration in units of the time for a floating-point operation (flop); right, time spent doing local communication in flops.
Results
Measurements for dynamic load balancing

Figure 14: Percentage of elements migrated during each load balancing stage. The percentage may be greater than 100 because the recursive bisection methods may cause the same element to be migrated several times.
Conclusions

• Results apply to range of applications
• ORB is cheap and fast but does not produce optimal performance
• SA gives best run times, but is costly and parameter sensitive
• ERB good compromise: quality solution at slightly higher cost than ORB
Thank You!