Uncertainty Visualization: State of the Art

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Advances in Technology

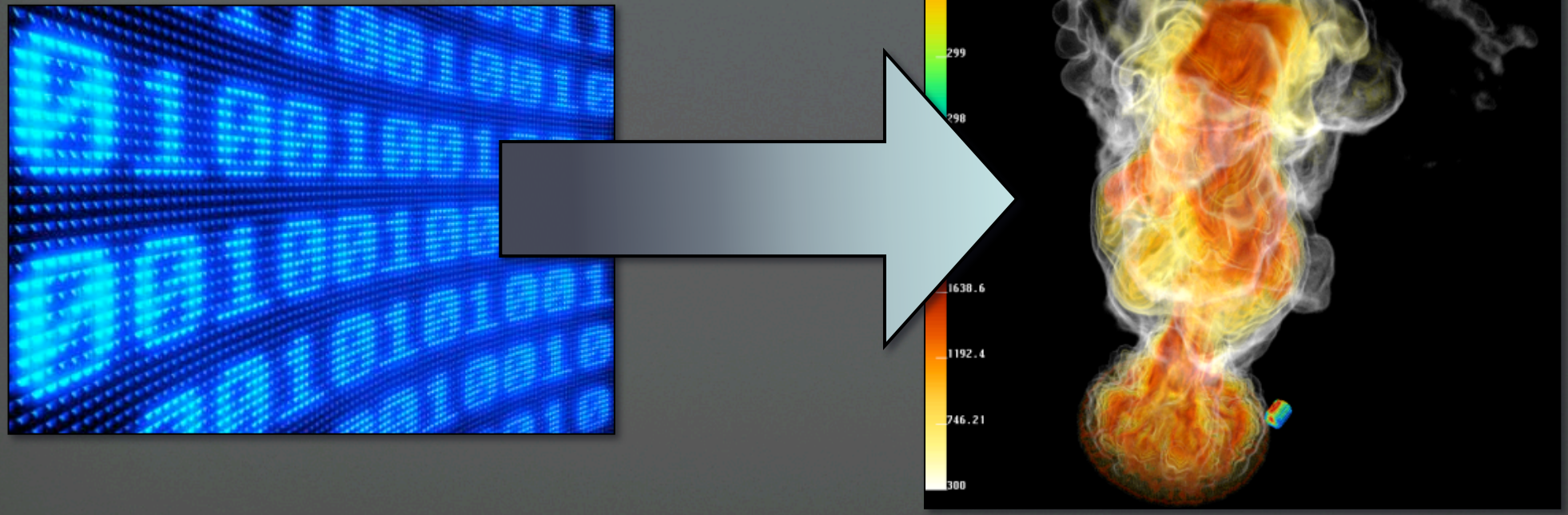
Jaguar Super Computer,
Oak Ridge National Lab

- More bandwidth, storage, & computational power
- Larger data sets:
  - Higher resolutions
  - Longer runs
  - More sophisticated models

All this leads to huge amounts of complex data!

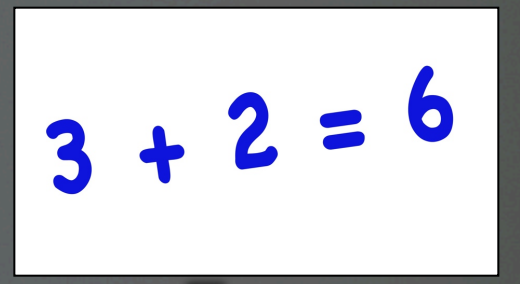
Visualization is Communication

- Translate data into images, “see” the data
- Brings out relationships & features in data
- Lets scientists communicate within their fields and out to others



Scientists need to know about uncertainty too!

- What is the quality of the data?
- How much can the data be trusted?
- What is the risk in making decisions on the data?



Error

Accuracy

Missing Data

Confidence

Motivation

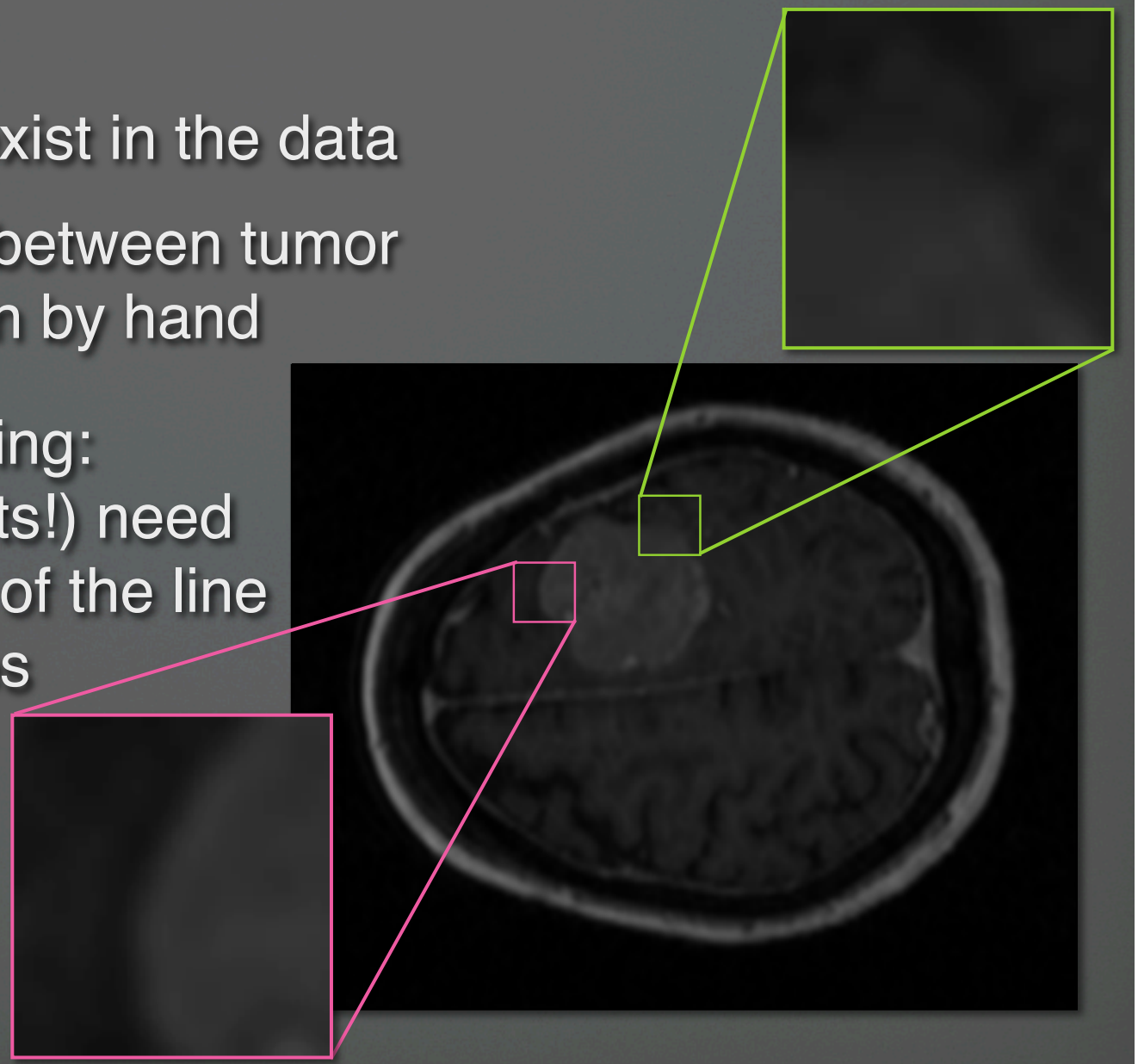
- Most scientific papers are expected to include uncertainty information with data
- Typically 2D graphs with error/uncertainty
  - *If important in publications should be important to visualization*
- Accurate & complete visualization primary goal
- Absent due to difficulty to visually express

C.R. Johnson.
"Top Scientific Visualization Research Problems".
In *IEEE CG&A*, 24(4), 2004.

C.R. Johnson and A.R. Sanderson.
"A Next Step: Visualizing Errors and Uncertainty".
In *IEEE CG&A*, 23(5), 2003.

Example - Brain Tumor Scan

- Fuzzy boundaries exist in the data
- Hard to distinguish between tumor & brain tissue - even by hand
- Pre-operative planning: Doctors (and patients!) need to know confidence of the line between tissue types



Whole Brain Atlas
http://www.med.harvard.edu/AANLIB/home.html

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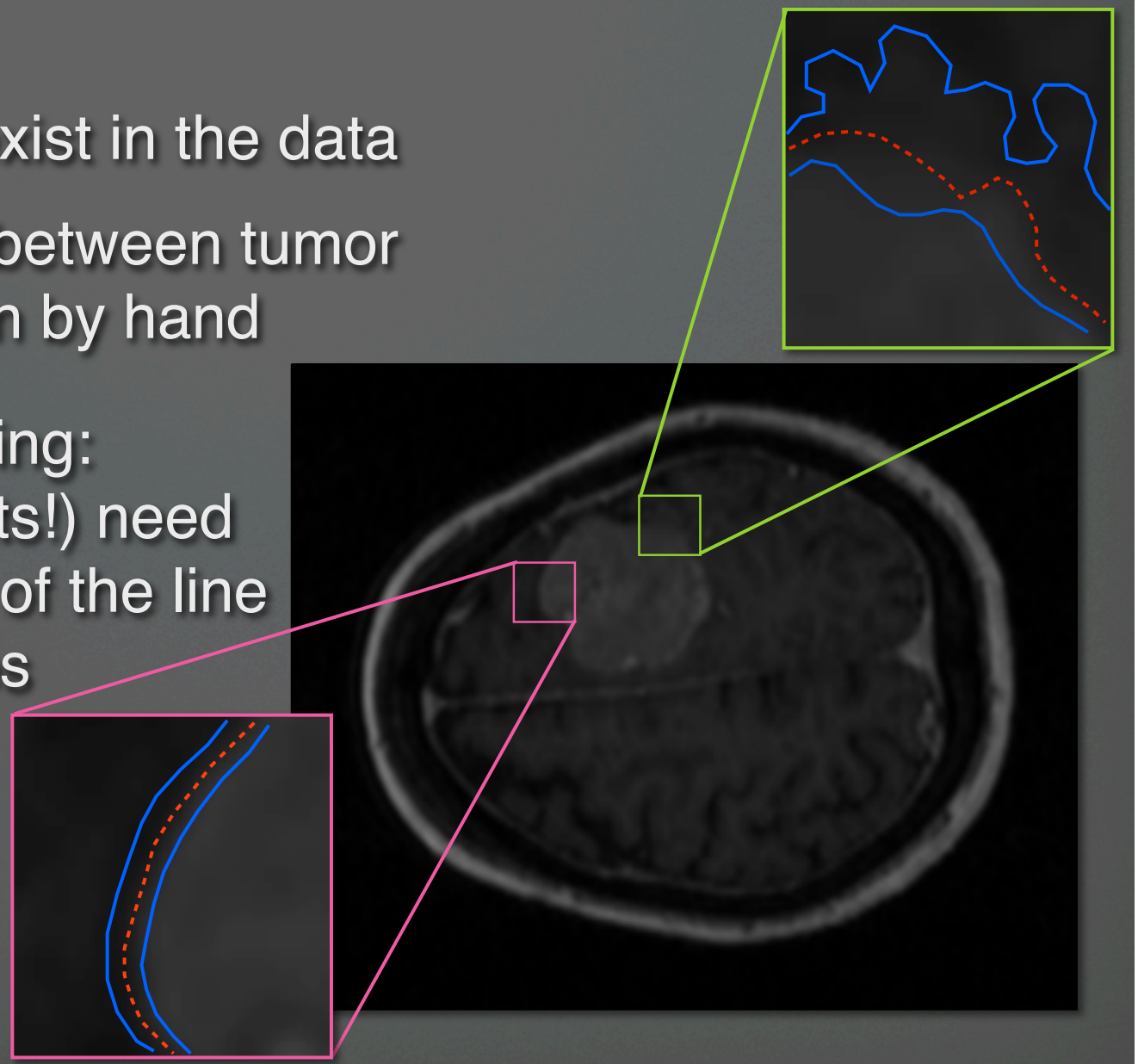

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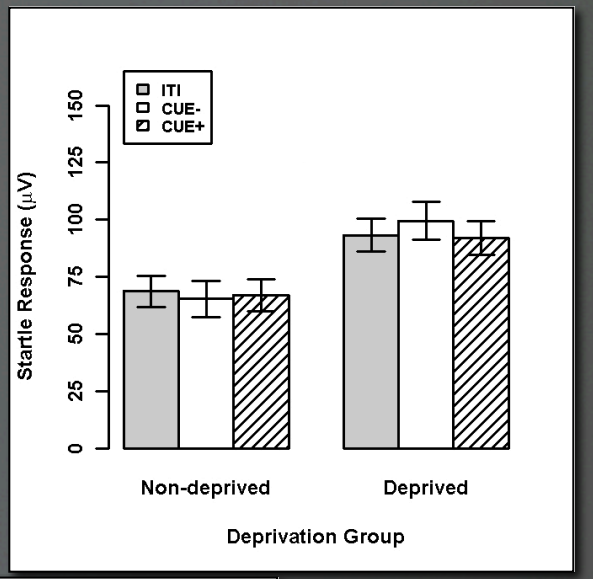
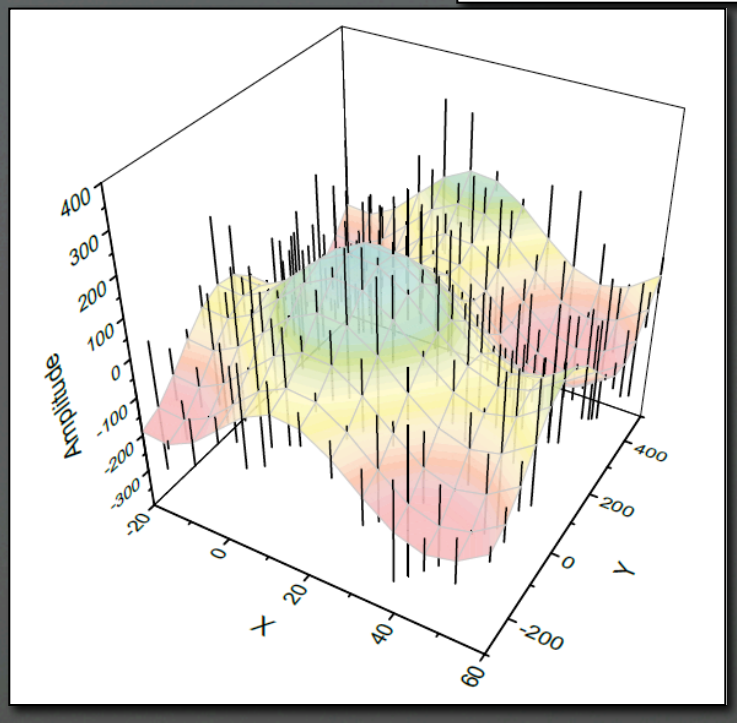
Outline

- Origins
- 2D data
- 3D data
- Architectural Data
- Visualization uncertainties

Traditional Display of Uncertainty

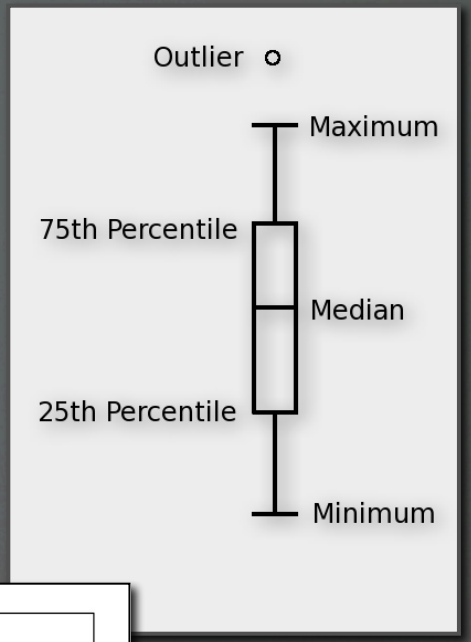
- Error bars
  - convey accuracy by amount of +/- error
  - std dev or std error

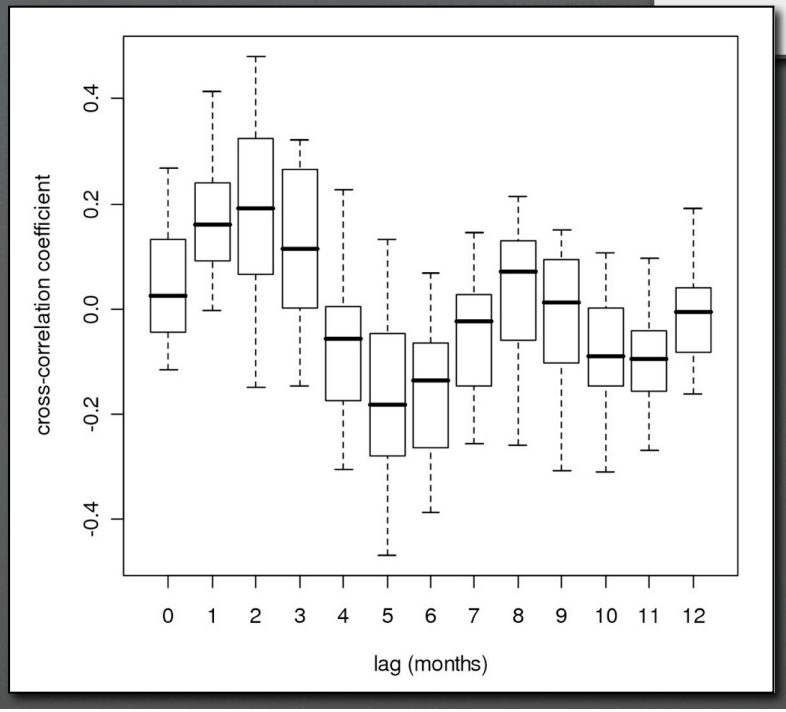




Traditional Display of Uncertainty

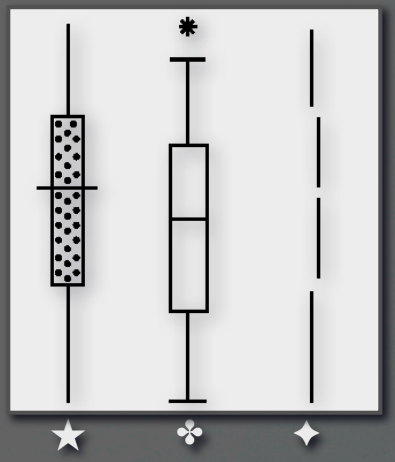
- Boxplots
  - Quartile range including median
  - Outliers
  - Assume gaussian

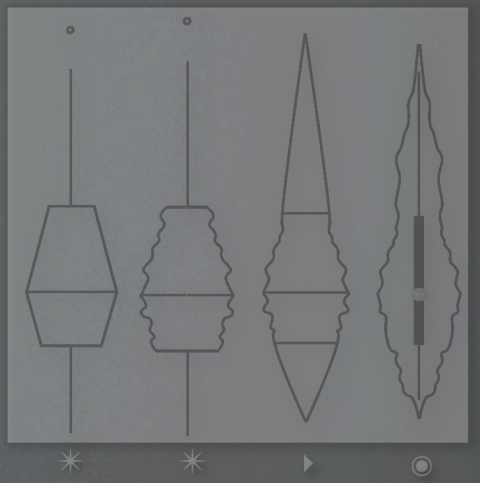


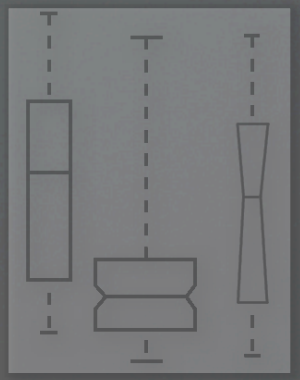


Boxplot Modifications

- Visual Modifications
  - Refinement for aesthetic purposes
- Density indications
  - Use the box sides to encode
- Data Characteristics
  - sample size, confidence levels
- Additional Statistics
  - skew, modality







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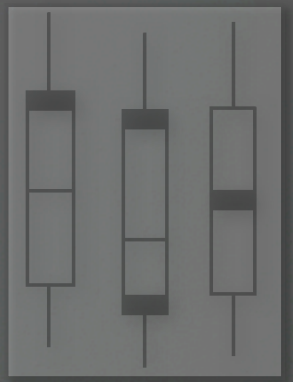

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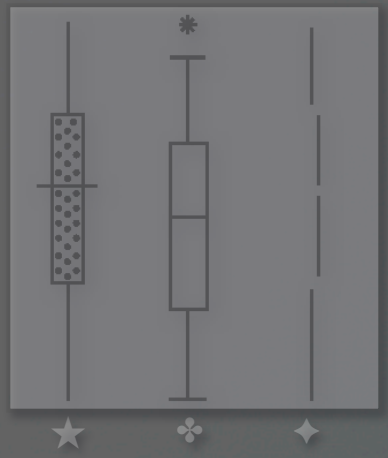

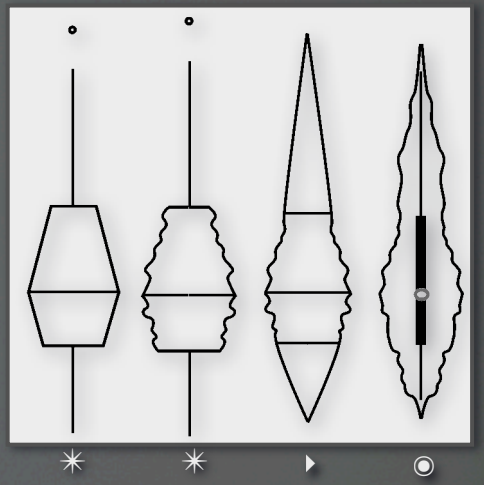

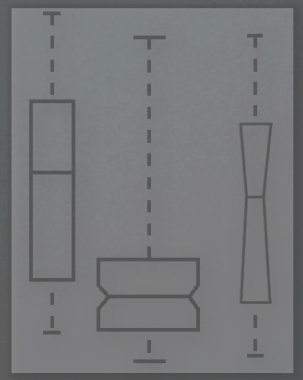

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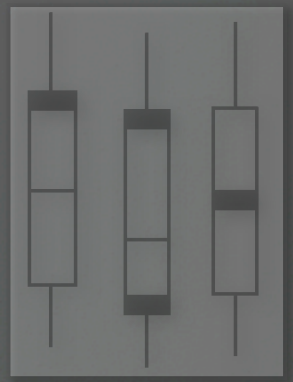

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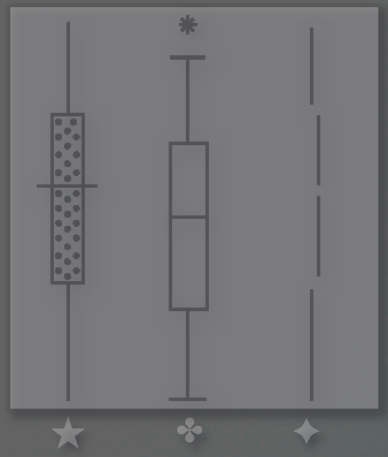

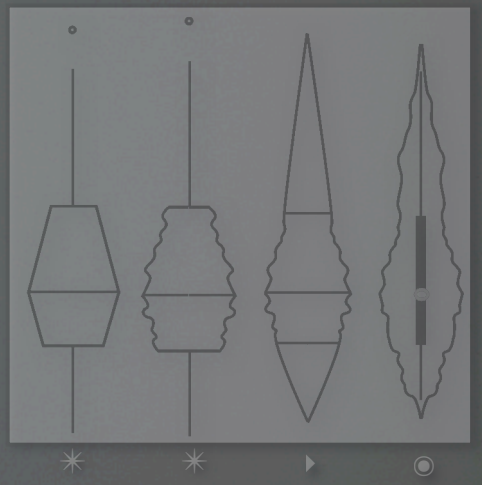

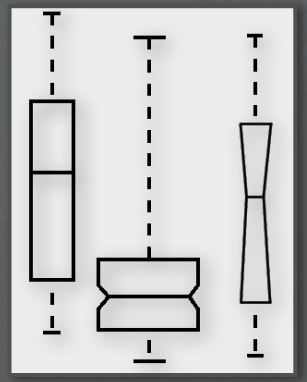

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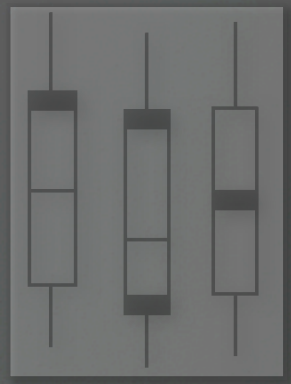

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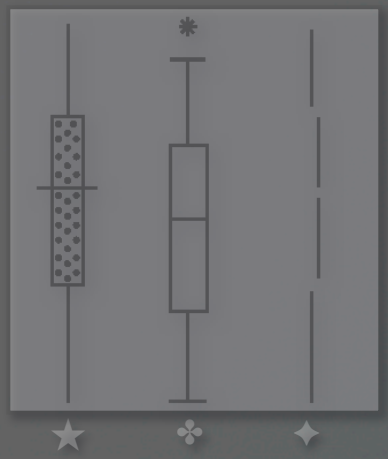

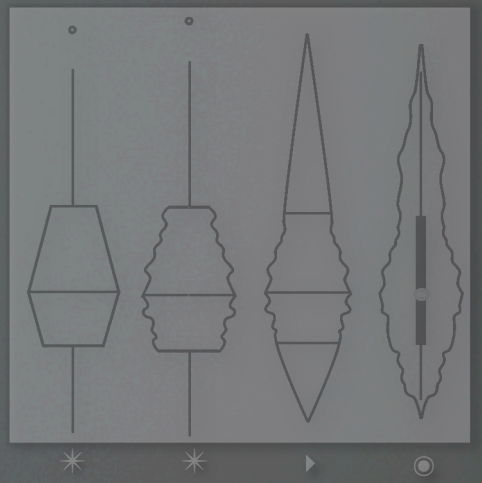

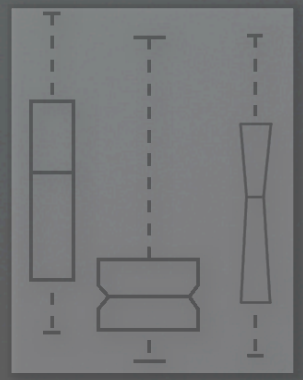

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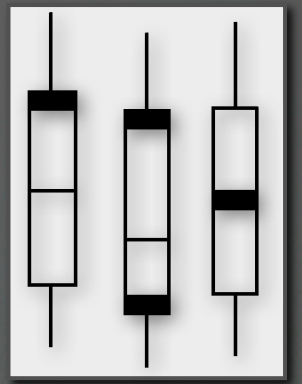

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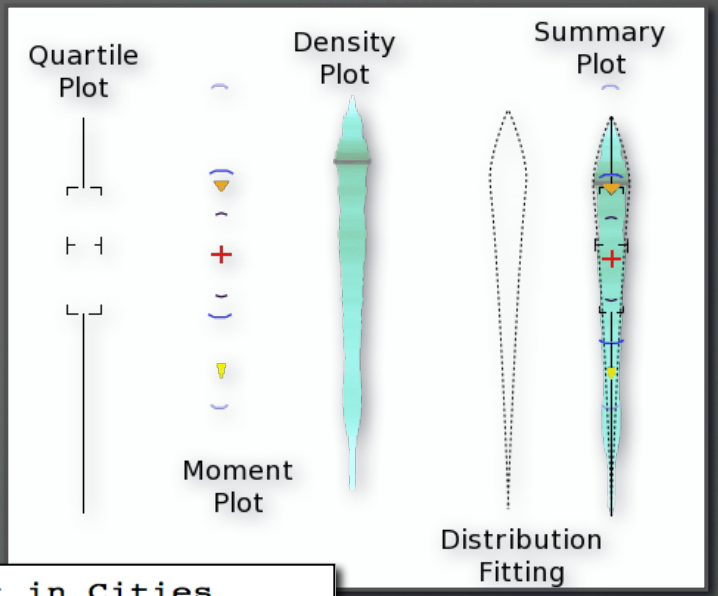
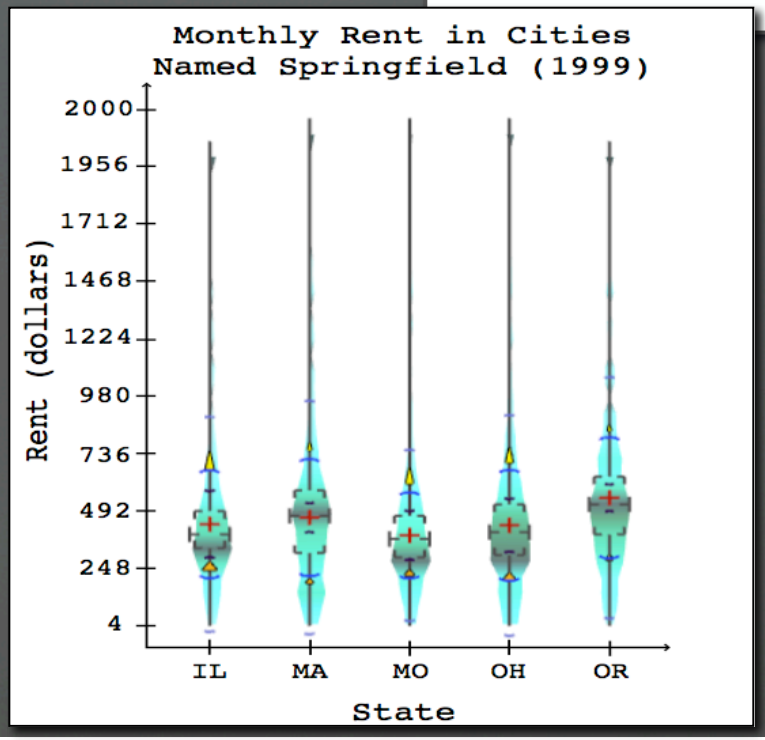
The Summary Plot

- Augment boxplot with numerous display techniques
- Emphasize characteristics other than mean/variance
- Indicate quantity & location of uncertainty

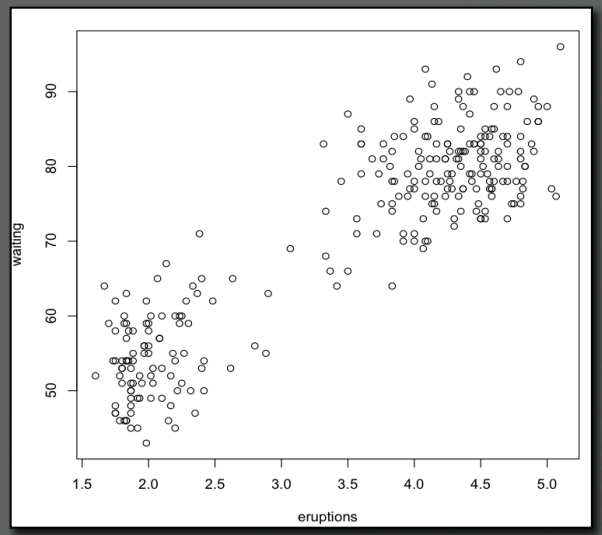




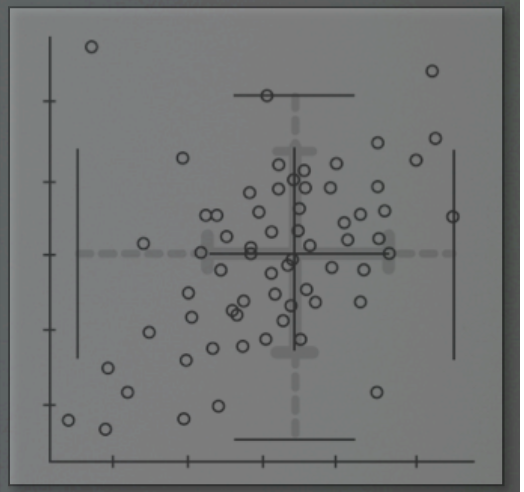
K. Potter, J. Kniss, R. Riesenfeld, C.R. Johnson. "Visualizing Summary Statistics and Uncertainty". In Proc *Eurovis 2010*, 29(3), 2010.

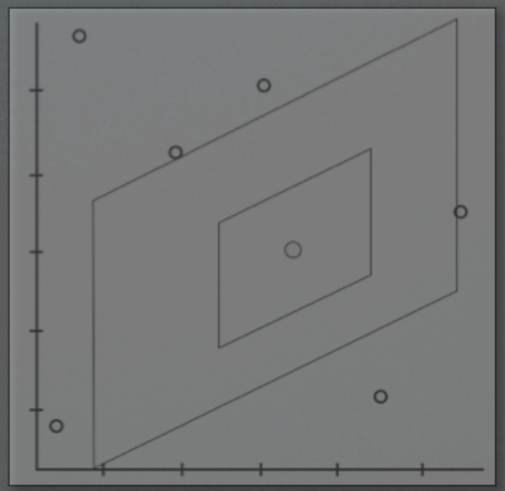
2D Box Plots

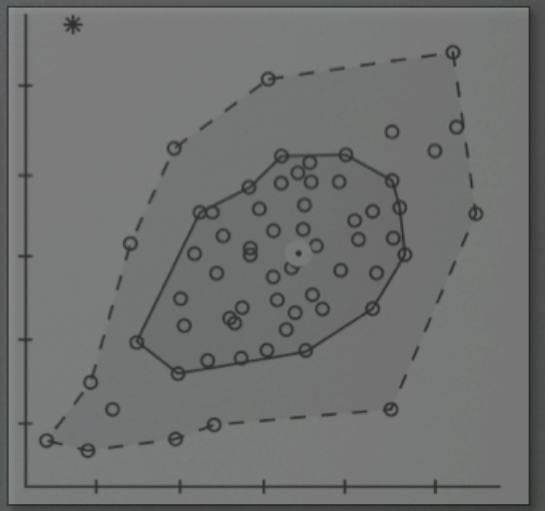
- Scatterplots
  - 2D position of samples
- RangeFinder Plot
  - 1D boxplot per axis
- Two Dimensional Boxplot
  - Robust line partition
- Bagplot
  - Halfspace depth (spatial equiv to quartiles)



Scatterplot


Rangefinder Plot


Two-Dimensional Boxplot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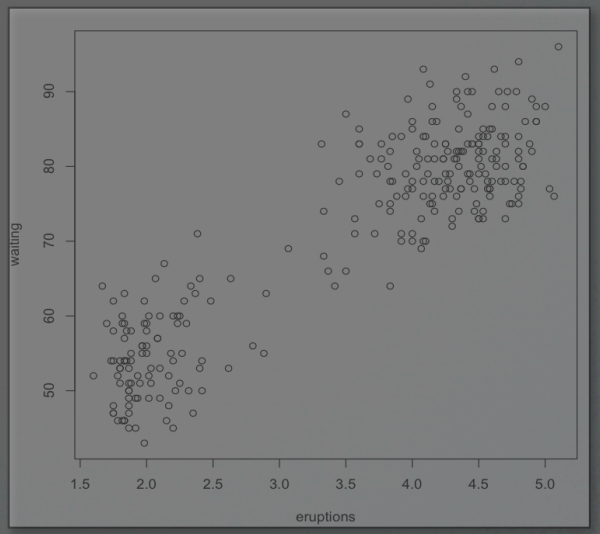

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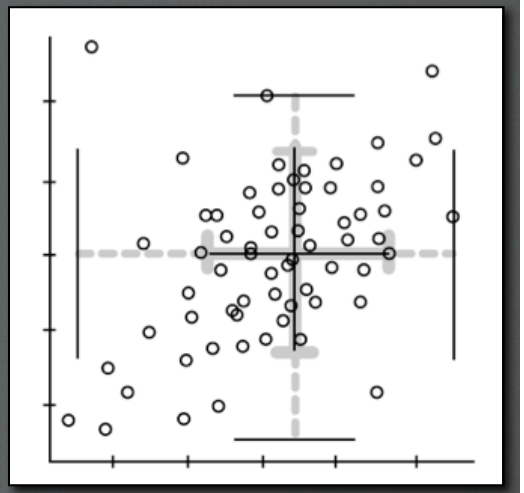
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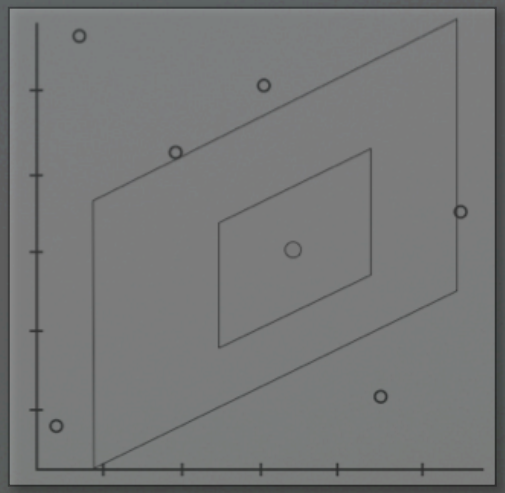
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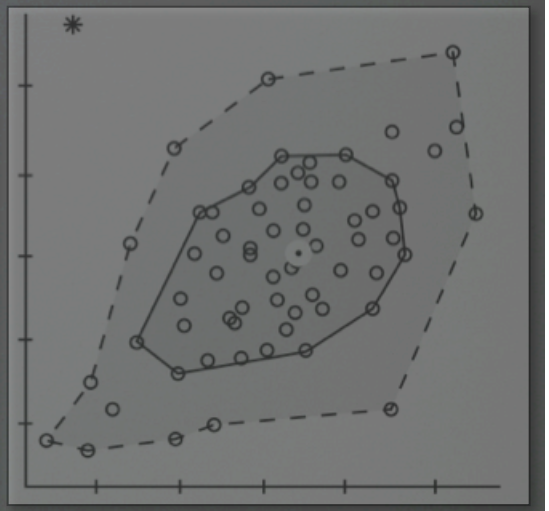
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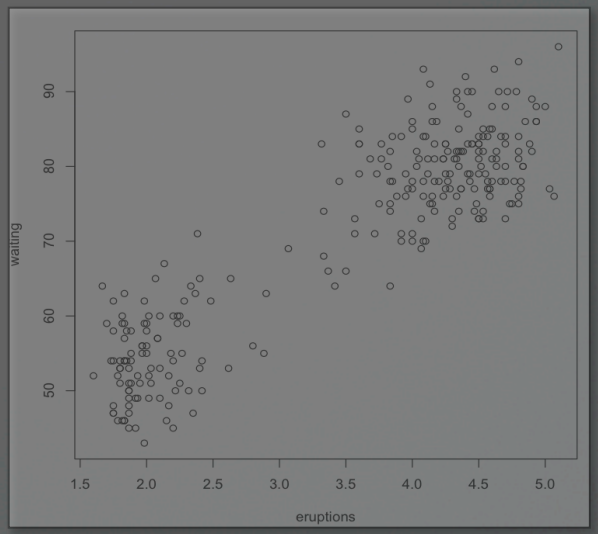

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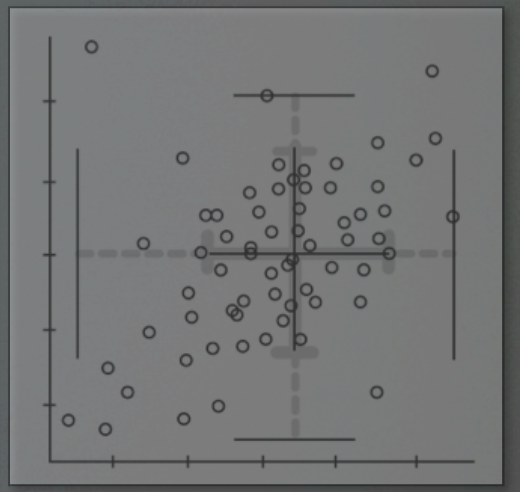
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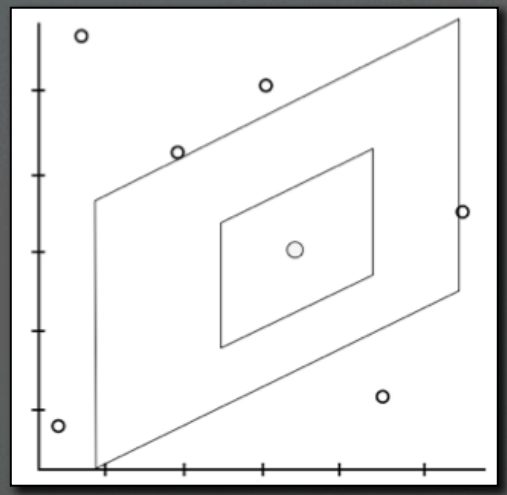
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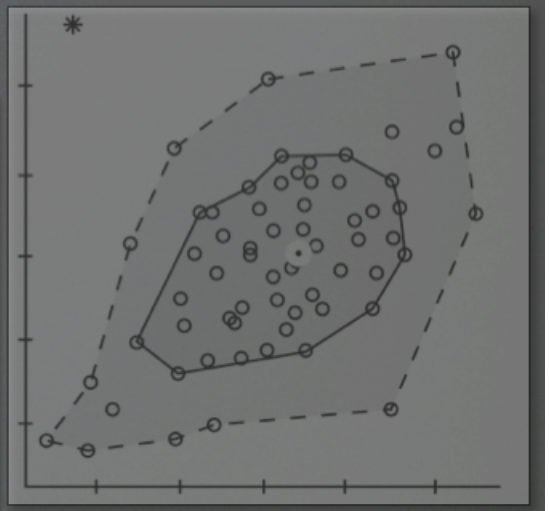
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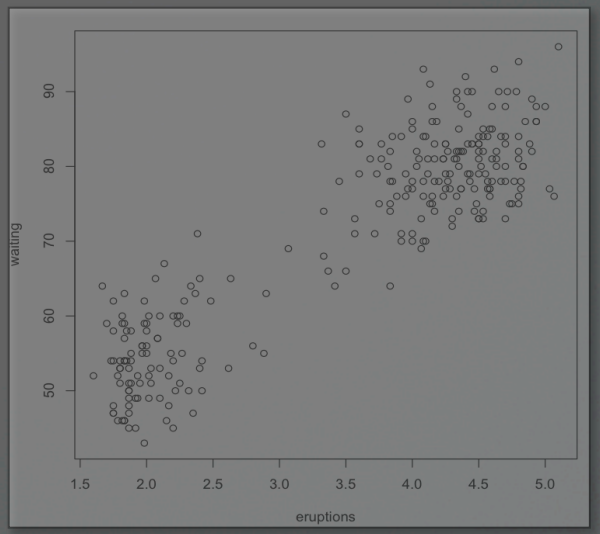

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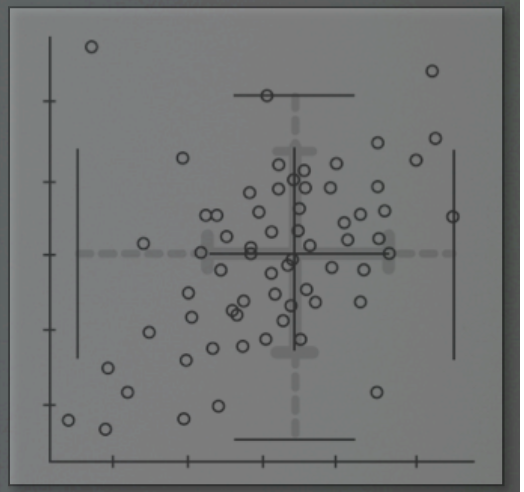

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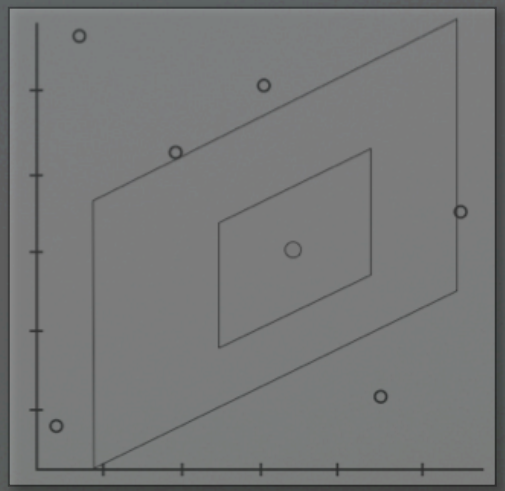

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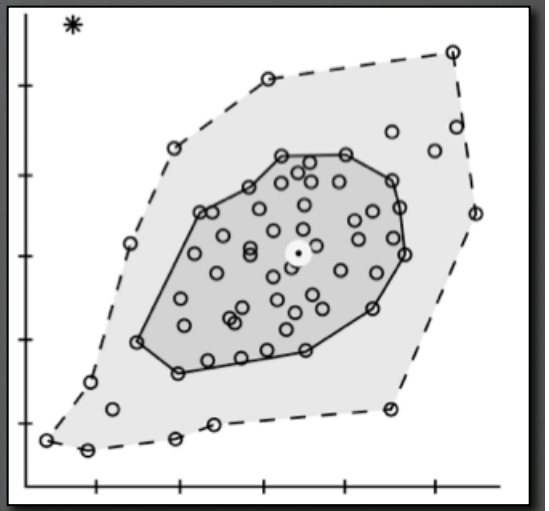


Bagplot

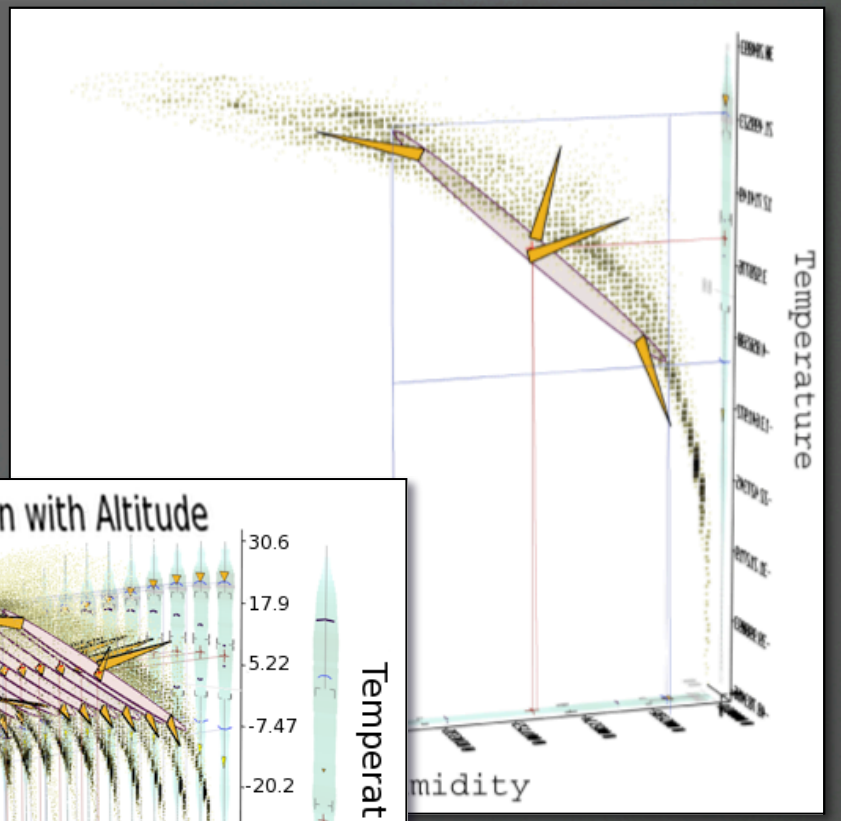
S. Becketti, W. Gould, Rangefinder box plots. TAStatistician 41(2), 1987.

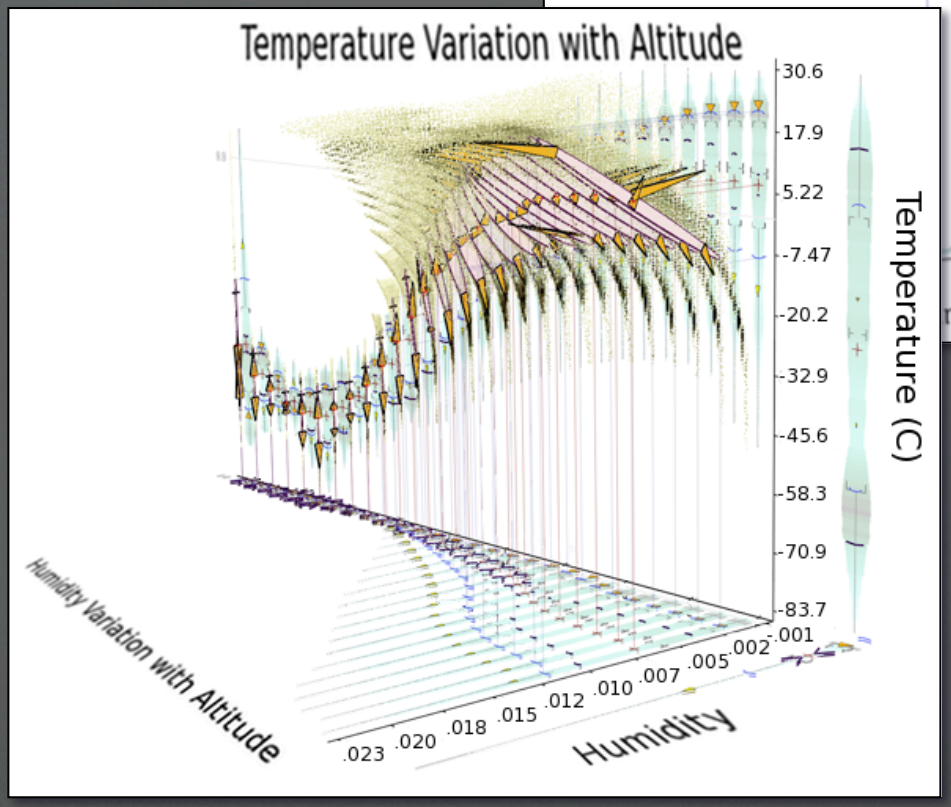
P. Tongkumchum, Two-dimensional box plot. Songklanakarin J Sci Technol, 27(4),2005.

P.J. Rousseeuw, I. Ruts, J. Tukey. The bagplot: A bivariate boxplot. TAS, 53(4), 1999.

2D Summary Plot

- Statistics similar to summary plot
- Highlight correlations

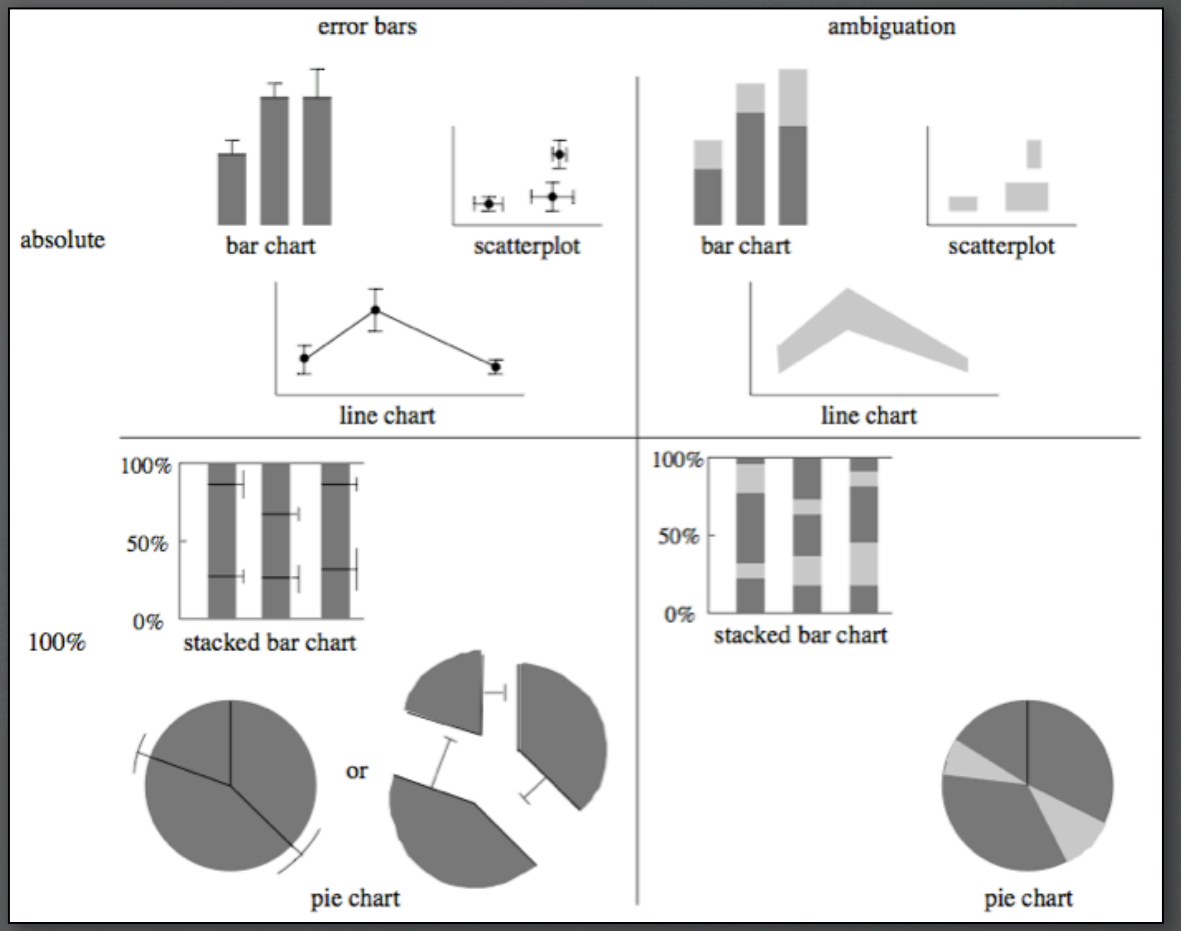




K. Potter, J. Kniss, R. Riesenfeld, C.R. Johnson. "Visualizing Summary Statistics and Uncertainty". In Proc *Eurovis 2010*, 29(3), 2010.

Bounded Uncertainty

- Numeric interval guaranteed to contain data value
- No assumptions about the pdf within the interval



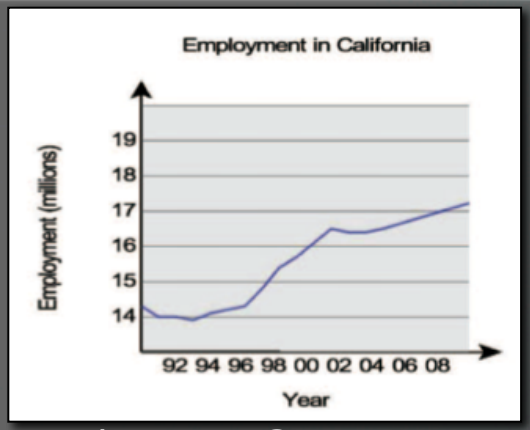
Statistical Uncertainty

Bounded Uncertainty

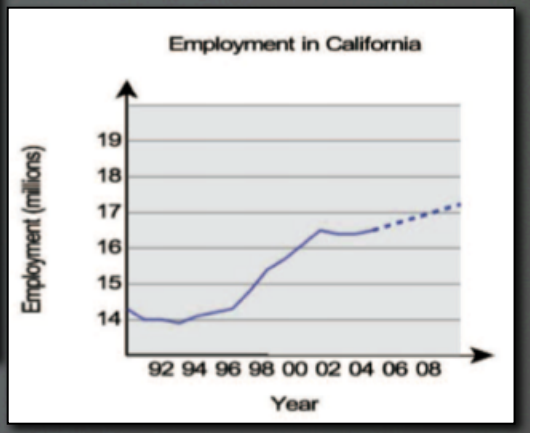
C. Olston, J.D. Mackinlay.
Visualizing Data with Bounded Uncertainty.
In Proc *IEEE InfoVis*, 2002.

Information Uncertainty

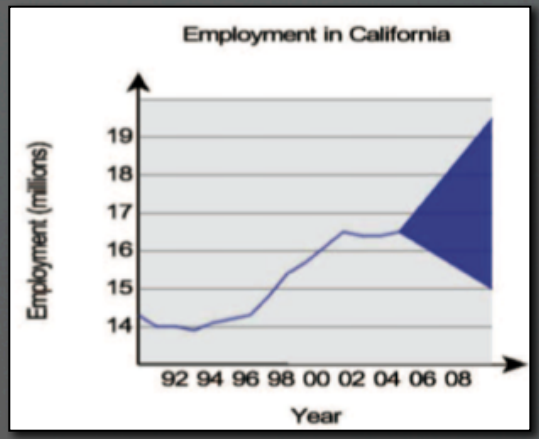
- Indication of how much confidence
- Qualitative rather than quantitative
- Spreadsheet interface characterizes the data



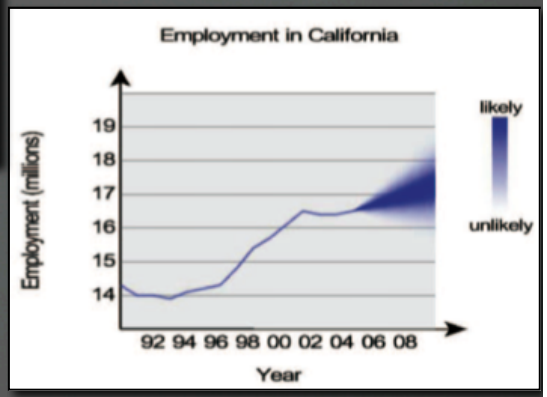
Average Growth



Estimated Growth



Possible Growth

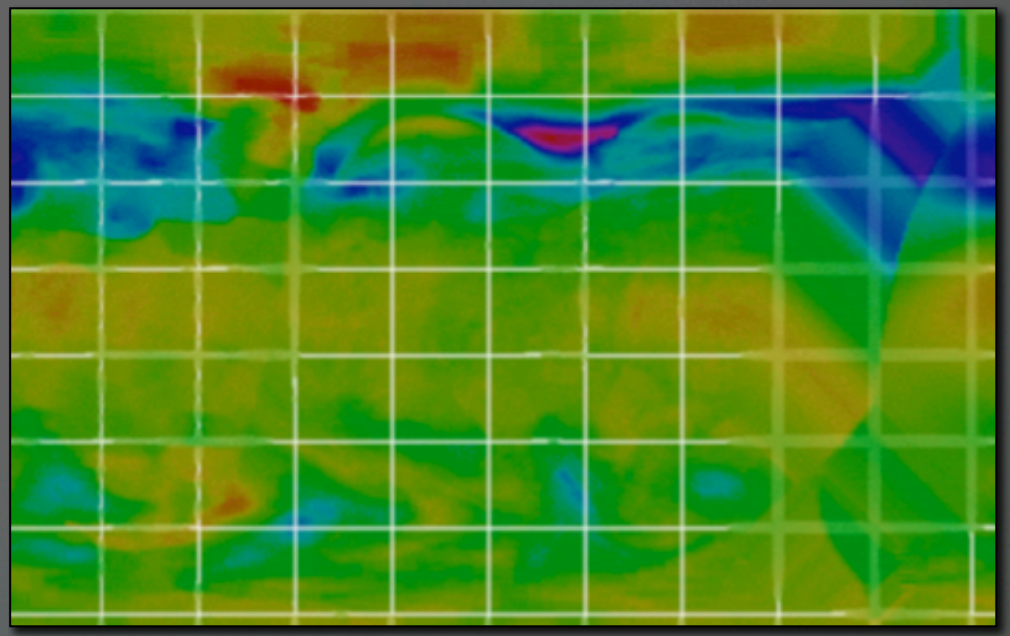


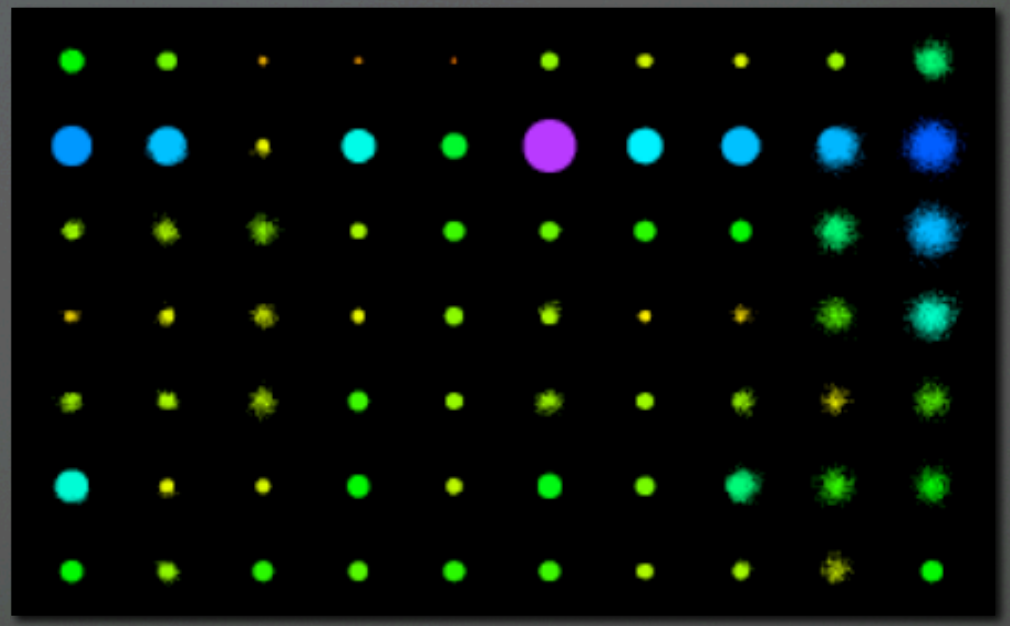
Likely Growth

A. Streit, B. Pham, R. Brown.
A Spreadsheet Approach to Facilitate Visualization of Uncertainty in Information.
In *IEEE TVCG* 14(1), 2008.

2D Annotation

- Modulate annotation lines or glyphs with uncertainty
- Minimal interference
- Uncertainty not emphasized

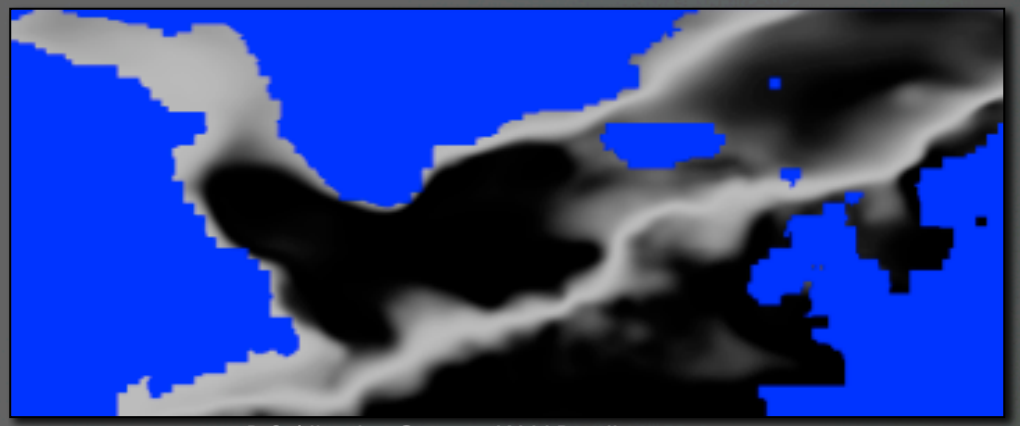




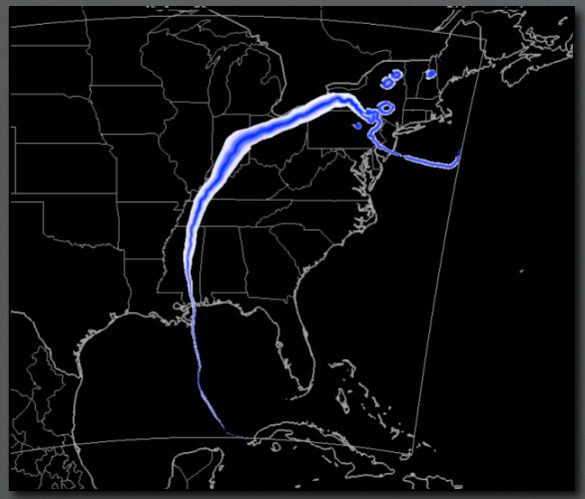
A. Cedilnik, P. Rheingans.
Procedural Annotation of Uncertain Information.
In Proc *IEEE Vis*, 2000.

Contouring

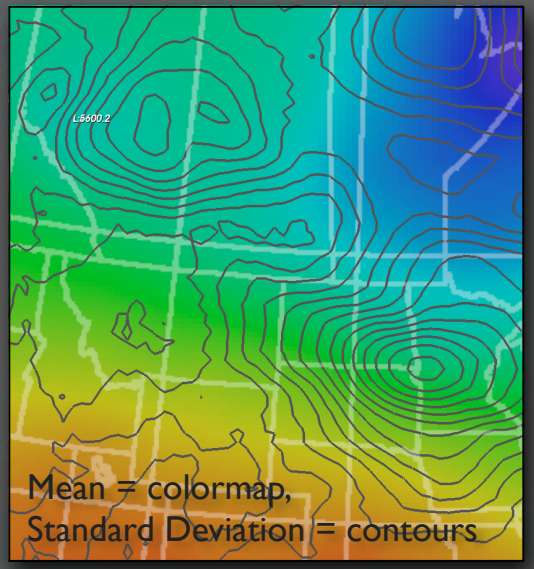
- Contours follow the line of a specific data value (ex. terrain map)
- Standard Deviation
- Fuzzy contours
- Graduated contours



R.S. Allendes Osorio, K.W. Brodlie.
Contouring with Uncertainty.
In *Theory and Practice of Computer Graphics Conf*, 2008.



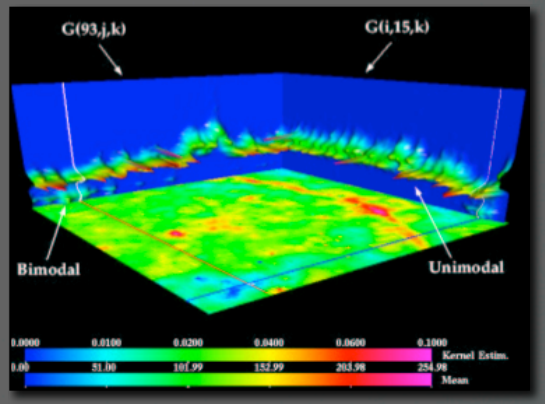
J. Sanyal, S. Zhang, J. Dyer, A. Mercer, P. Amburn.
Noodles: A Tool for Visualization of Numerical Weather Model Ensemble Uncertainty
In Proc *IEEE Vis*, 2010.



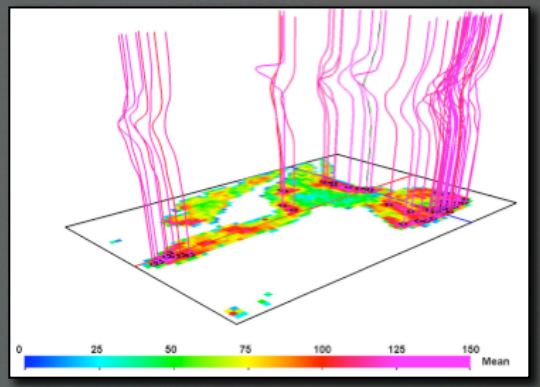
K. Potter, et al.
Ensemble-Vis: A Framework for the Statistical Visualization of Ensemble Data.
In *IEEE ICDM Workshop on Knowledge Discovery from Climate Data: Prediction*, 2009.

Ensembles / 2D Distributions

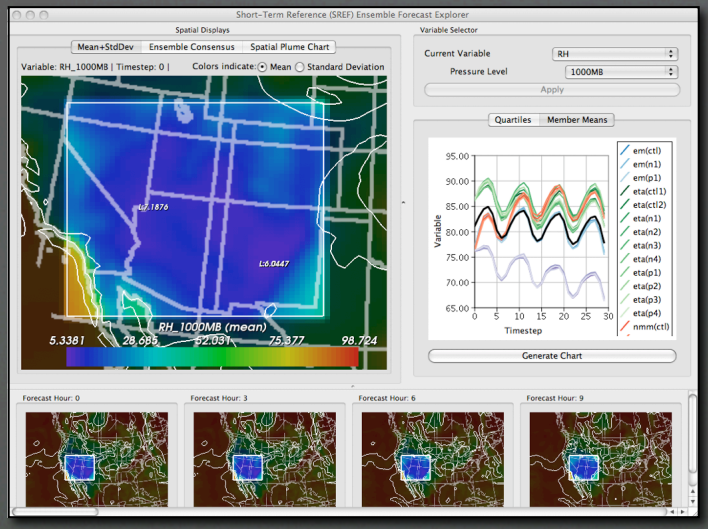
- Multi-run/model simulations
- Distribution of data at every pt
- Mean/std dev may not be appropriate



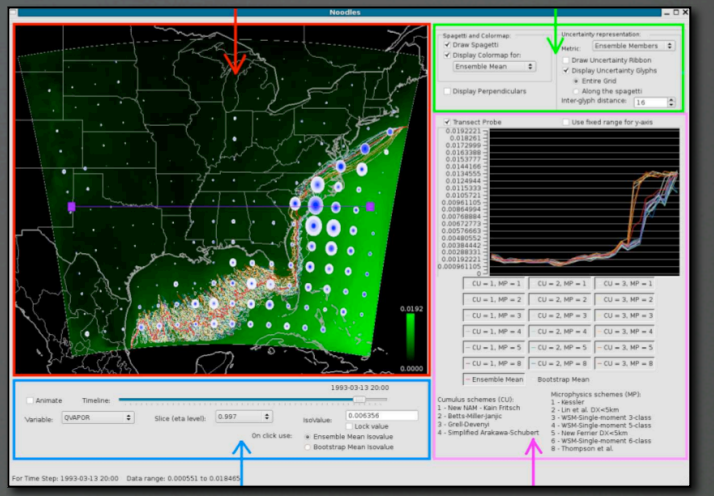
D. Kao, A. Luo, J. Dungan, A. Pang.
Visualizing Spatially Varying Distribution Data.
In Proc *Information Visualisation*, 2002.



D. Kao, M. Kramer, A. Luo, J. Dungan, A. Pang.
Visualizing Distributions from Multi-Return Lidar Data to Understand Forest Structure.
In *The Cartographic Journal*, 42(1), 2005.

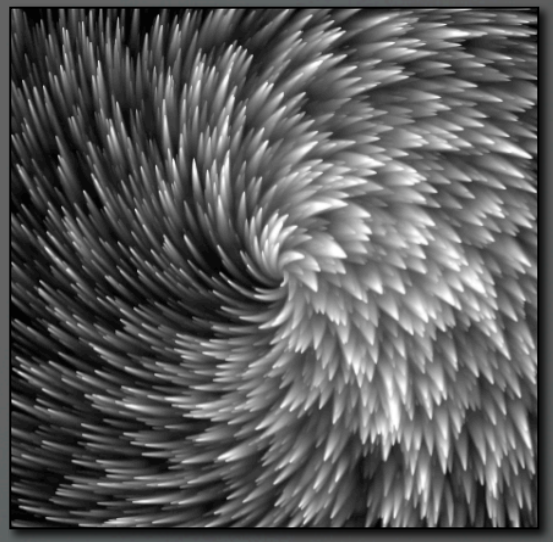


K. Potter, et al.
Ensemble-Vis: A Framework for the Statistical Visualization of Ensemble Data.
In *IEEE ICDM Workshop on Knowledge Discovery from Climate Data: Prediction*, 2009.

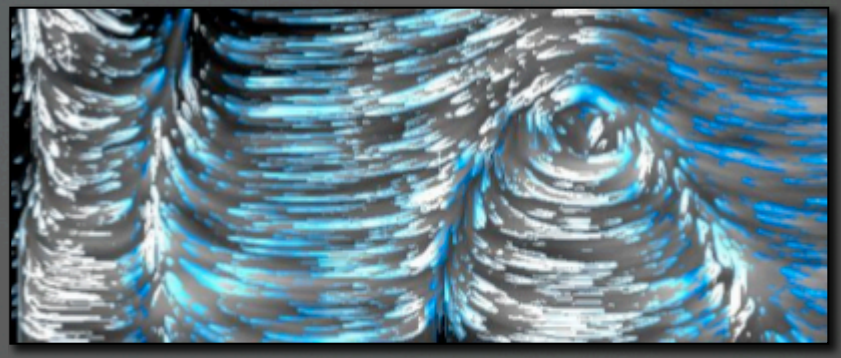


J. Sanyal, S. Zhang, J. Dyer, A. Mercer, P. Amburn.
Noodles: A Tool for Visualization of Numerical Weather Model Ensemble Uncertainty
In Proc *IEEE Vis*, 2010.

2D Vector Fields



- Texture-based
- Particle positions along streamlines
- Measuring errors and their influence on position



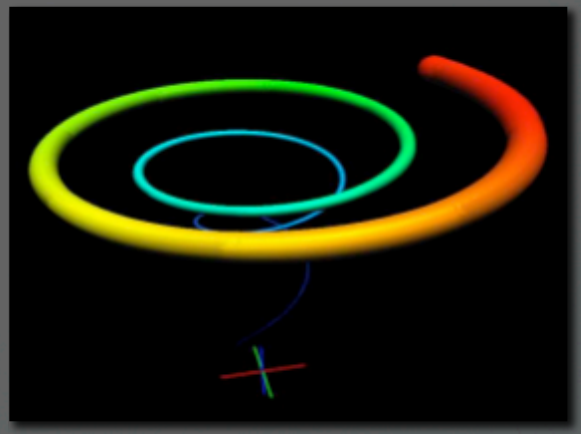
R. Botchen, D. Weiskopf, T. Ertl.
Interactive visualisation of uncertainty in flow fields using texture-based techniques.
In *International Symposium on Flow Visualisation*, 2006.

R. Botchen, D. Weiskop, T. Ertl.
Texture-based visualization of uncertainty in flow fields.
In *IEEE Vis*, 2005.

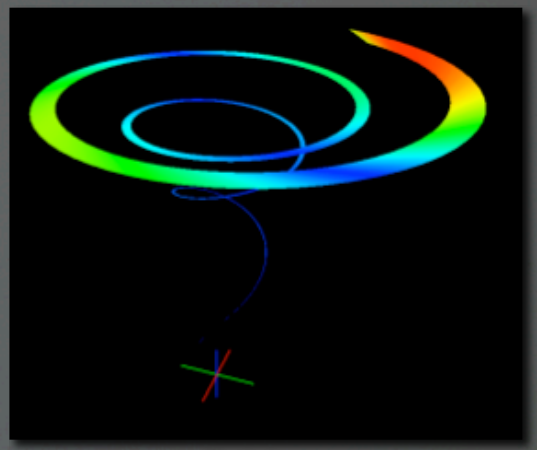
Streamlines

- Uncertainty from numerical algs for particle tracing in fluid flow
- Highlight sensitivity of algorithm choice - particularly near critical pts

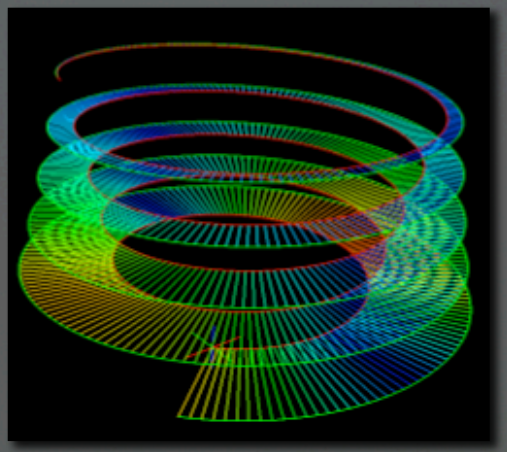
Differences between 2 streamlines



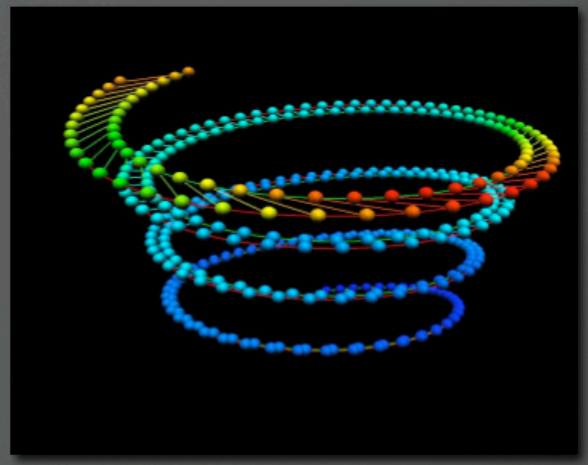
Tube



Ribbon



Lines



Balls + lines

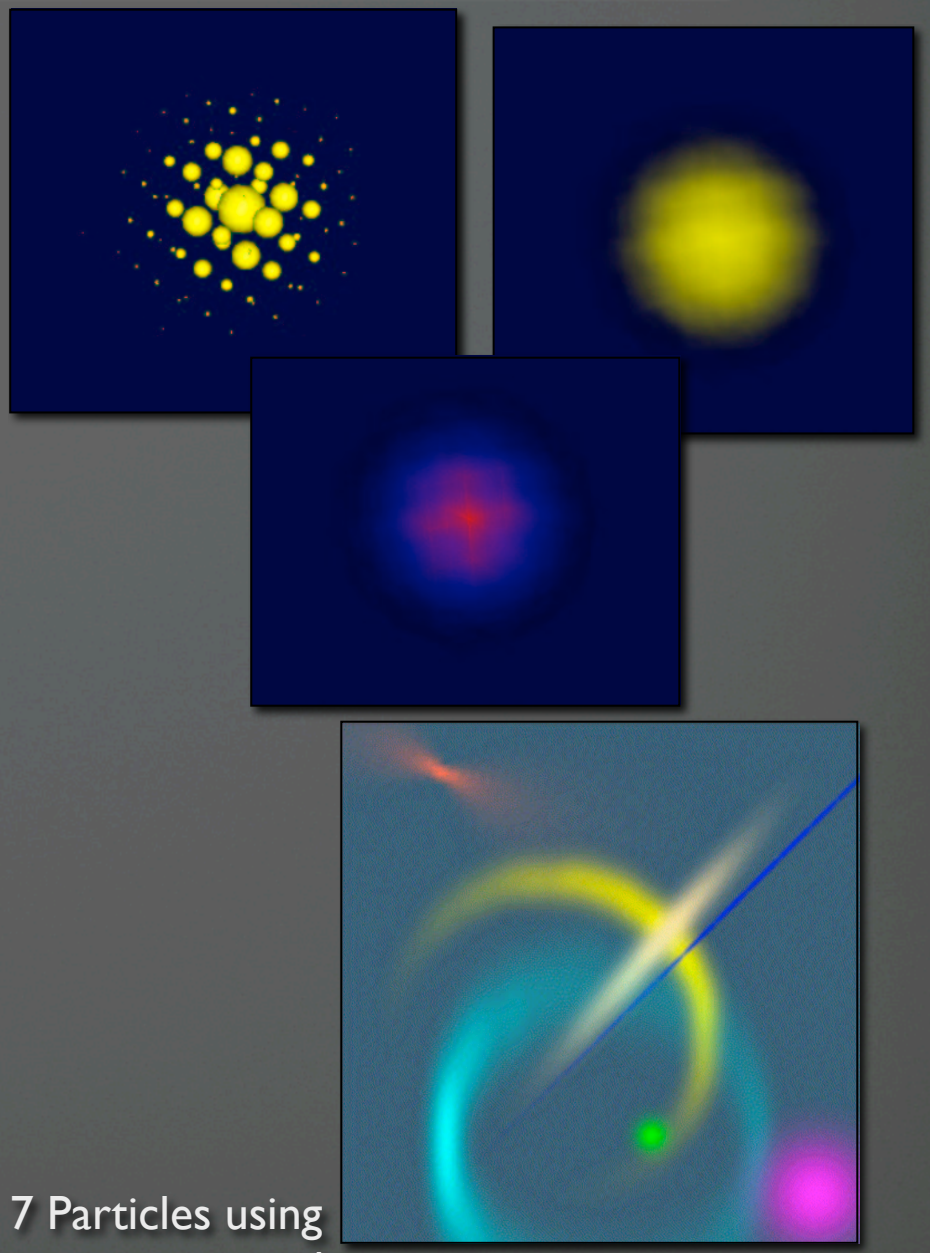
S. Lodha, A. Pang, R. Sheehan, C. Wittenbrink.
UFLOW: visualizing uncertainty in fluid flow.
In Proc *IEEE Vis*, 1996.

* S. Lodha, C. Wilson, R. Sheehan.
"LISTEN: sounding uncertainty visualization".
In Proceedings Visualization '96, pp. 189--195, 1996.

* Modulate pitch based on uncertainty.

Particle Movement

- PDFs to characterize initial position, speed, and direction of the particle
- Location of particle position visualized as a probability cloud
- Sphere glyphs, transparency, and transparency + color

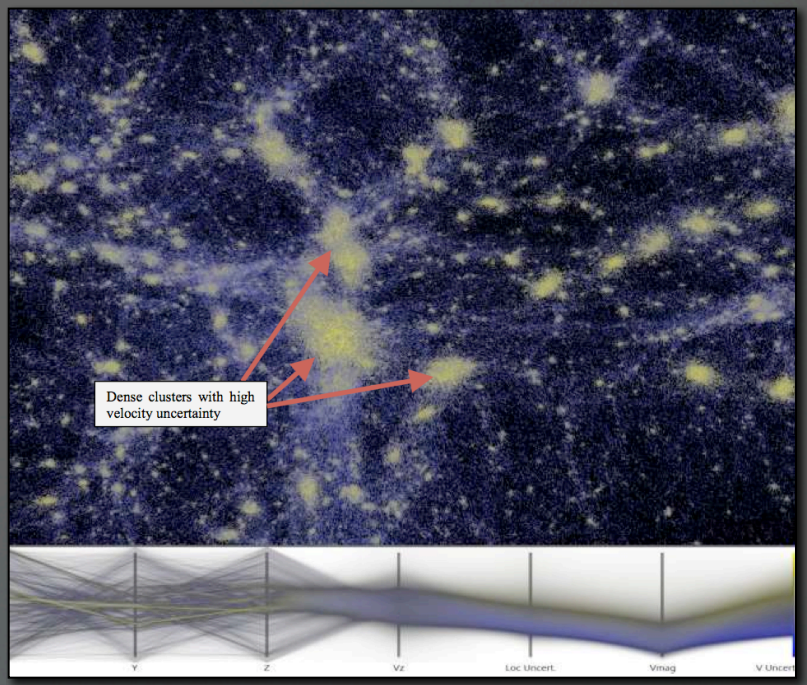


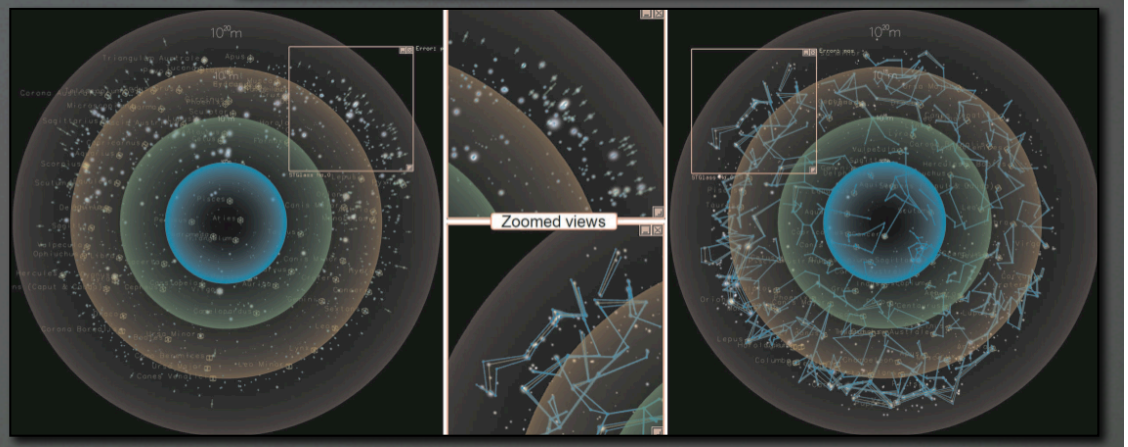
7 Particles using transparency + color

S. Lodha, N. Faaland, A. Charaniya.
Visualization of uncertain particle movement.
In Proc *Computer Graphics and Imaging*, 2002.

Astrophysical/Cosmological

- Huge datasets
- Trajectory uncertainty
- Observational uncertainties



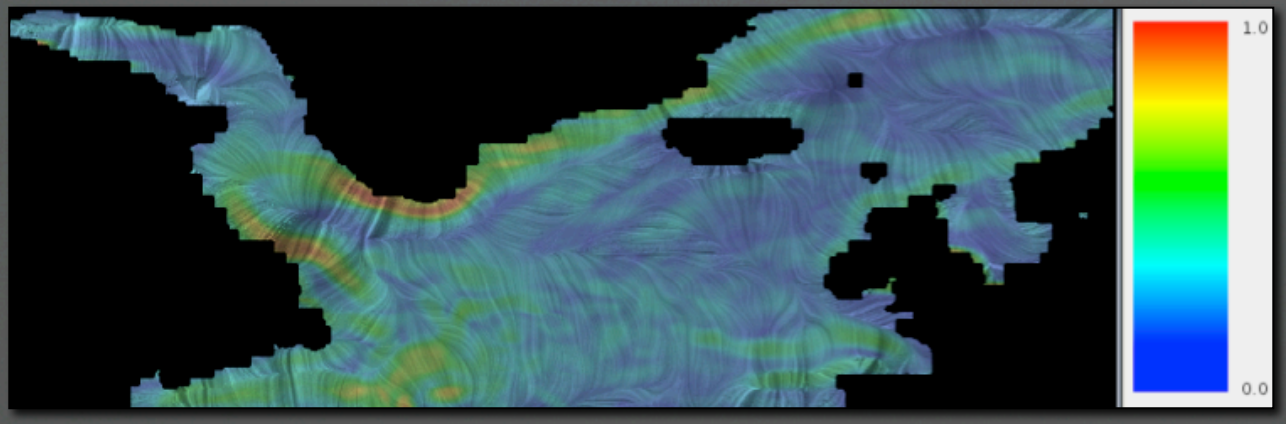


S. Haroz, K. Ma, K. Heitmann.
Multiple Uncertainties in Time-Variant Cosmological Particle Data.
In *IEEE Pacific Visualization Symposium*, 2008.

H. Li, C. Fu, Y. Li, A. Hanson.
Visualizing Large-Scale Uncertainty in Astrophysical Data.
In *IEEE TVCG*, 13(6), 2007.

2D Vector Fields - LIC

- Line Interval Convolution
- 2D steady flow
- PDF describes the magnitude & direction of each vector in the field
- LIC representation of the gradient field, color encodes magnitude of uncertainty



R. S. Allendes Osorio, K. W. Brodlie.
Uncertain Flow Visualization using LIC.
In *Theory and Practice of Computer Graphics*, 2009.

2D Vector Fields - Topology

- Transport of local uncertainty in the flow
- Vector field as a density distribution function
- Uncertain topological skeleton

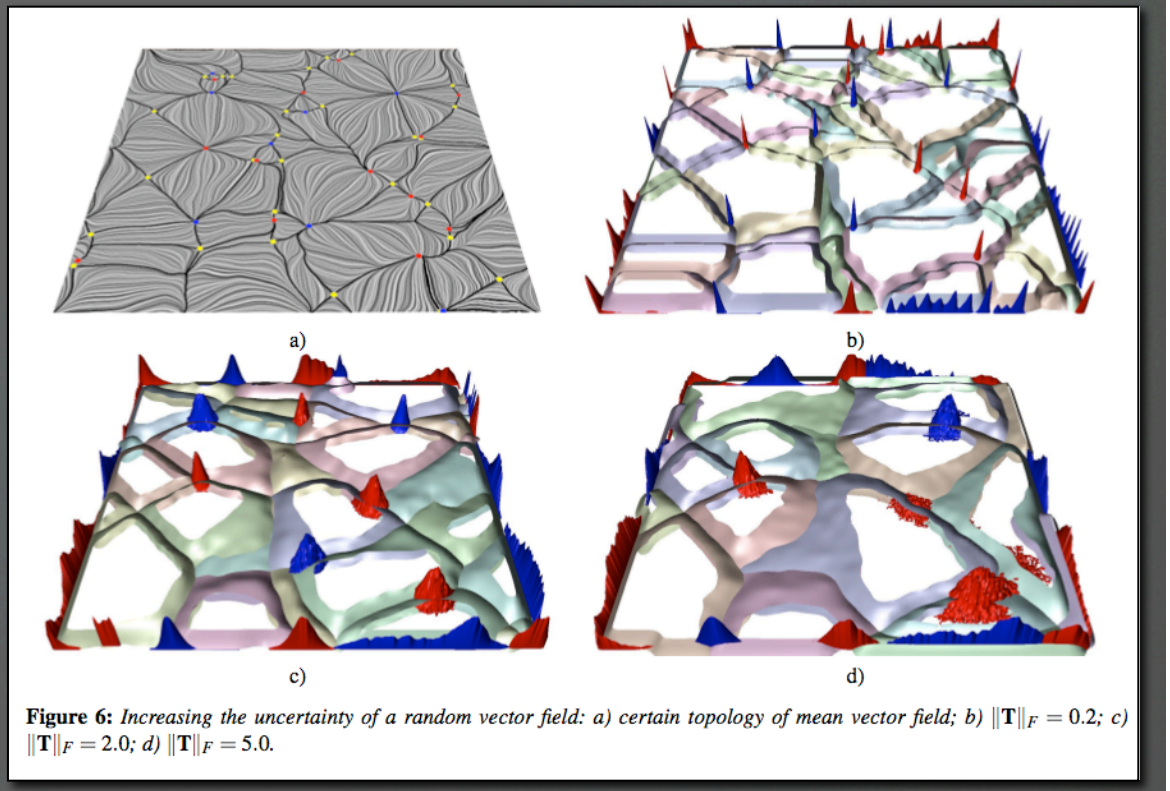
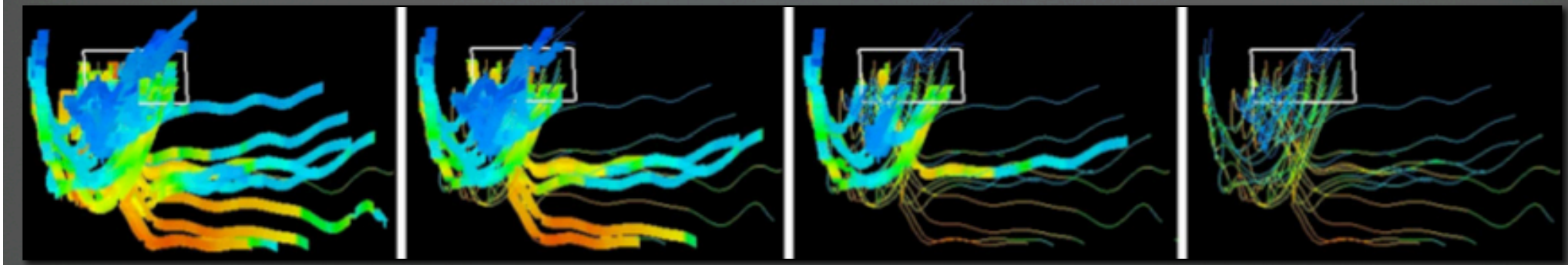


Figure 6: *Increasing the uncertainty of a random vector field: a) certain topology of mean vector field; b)* $\|\mathbf{T}\|_F = 0.2$*; c)* $\|\mathbf{T}\|_F = 2.0$*; d)* $\|\mathbf{T}\|_F = 5.0$.

M. Otto, T. Germer, H.C. Hege, H. Theisel.
Uncertain 2D Vector Field Topology.
In *CGF*, 29(2), 2010.

3D Meteorological Trajectory

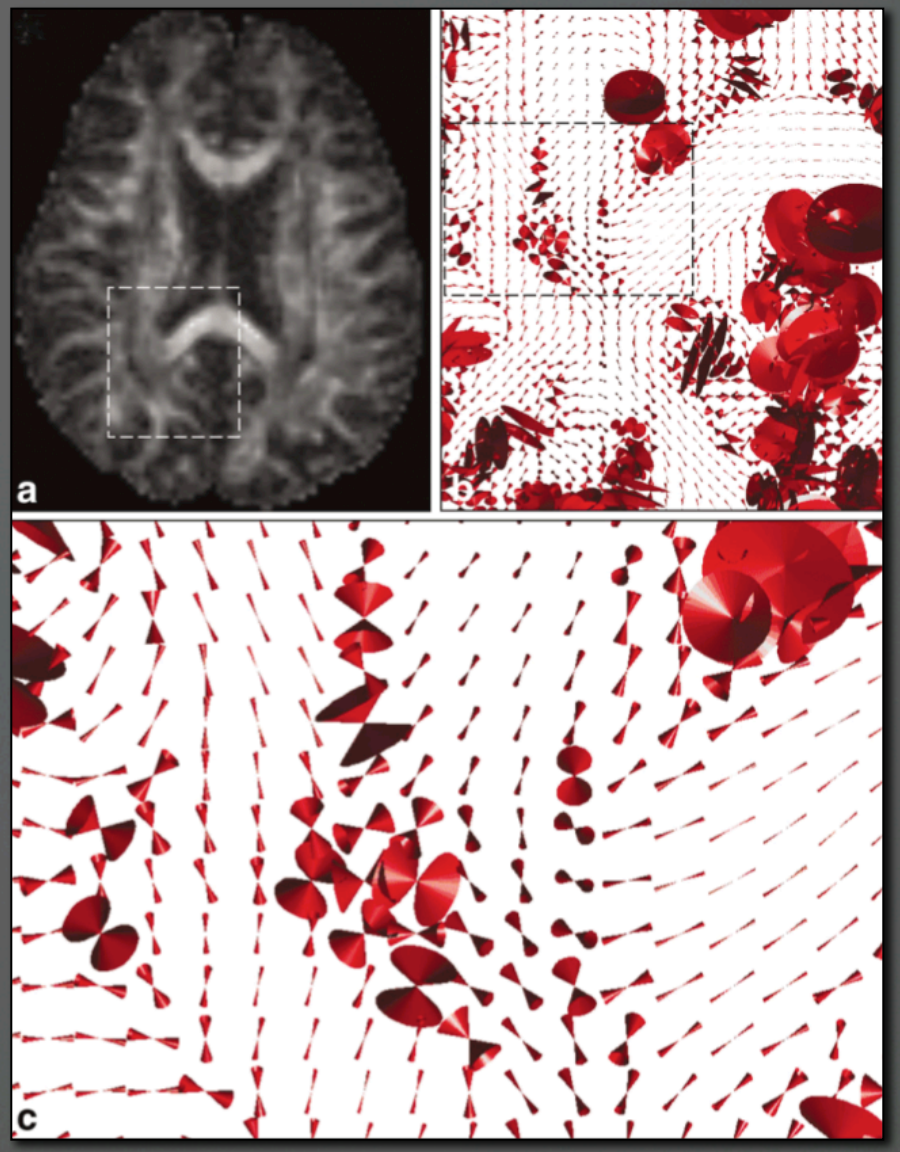
- Estimate uncertainty due to interpolation
- User seeded trajectories
- Prune trajectories with high uncertainty



R. Boller, S. Braun, J. Miles, D. Laidlaw.
Application of Uncertainty Visualization Methods to Meteorological Trajectories.
In *Earth Science Informatics*, 3(1-2), 2010.

DT-MRI

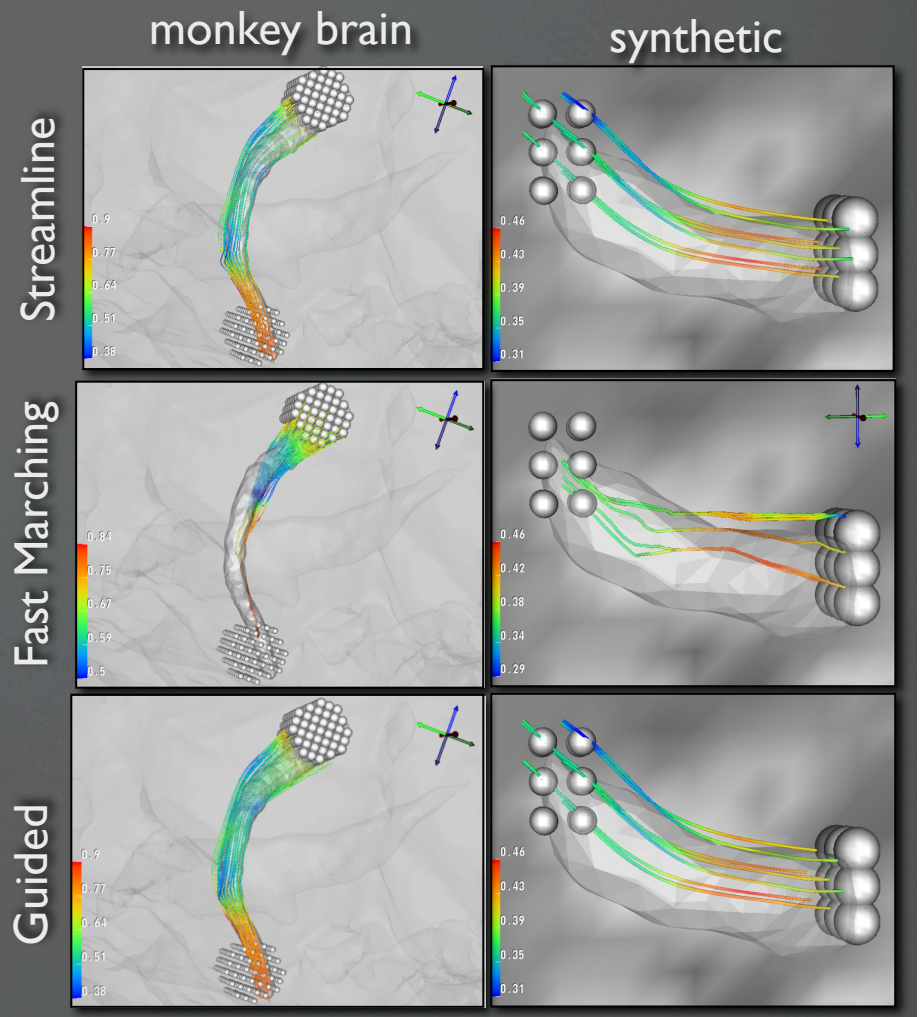
- Determine dominant orientation of structured tissue
- Uncertainty associate with eigenvector estimation
- Confidence intervals on fiber orientation
- “Cones of uncertainty” show orientation & uncertainty



D. Jones.
Determining and visualizing uncertainty in estimates of fiber orientation from diffusion tensor MRI.
In *Magnetic Resonance in Medicine*, vol. 49, 2003.

DTI Fiber Tracking

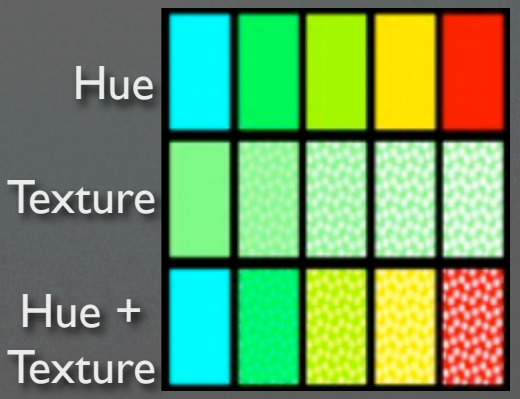
- Metrics to quantify differences in fiber tracking algorithms
  - Area between corresponding fibers of each bundle
  - Earth Mover's Distance between two fiber bundle volumes
  - Current distance between two fiber bundle volumes

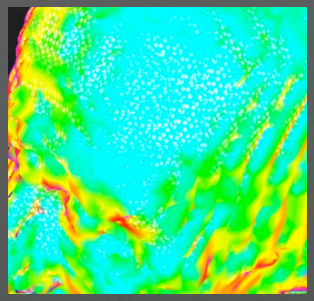


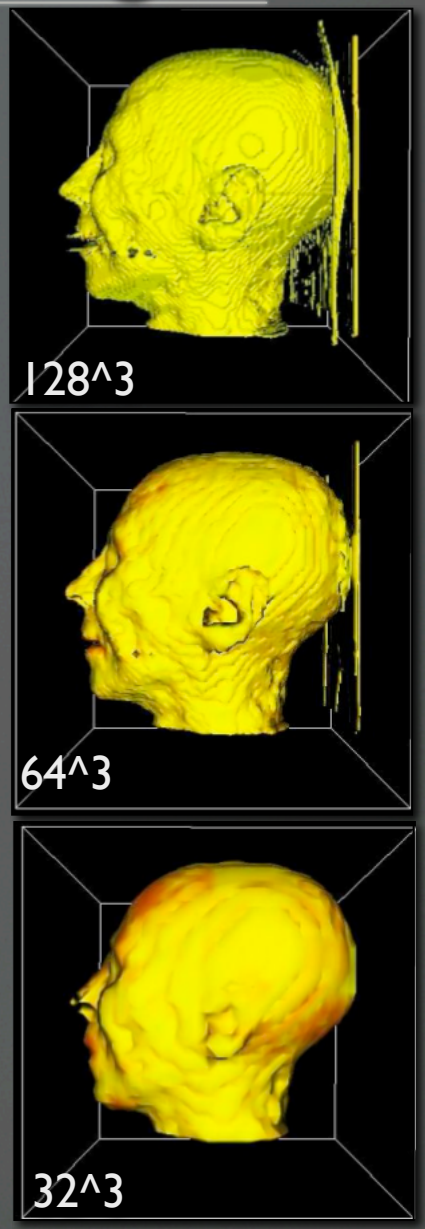
F. Jiao, J. Phillips, J. Stinstra, J. Krueger, R. Varma, E. Hsu, J. Korenberg, C,R. Johnson
Metrics for Uncertainty Analysis and Visualization of Diffusion Tensor Images
Medical Imaging and Augmented Reality, 2010.

Volumetric Data - Isosurfacing

- Isosurfaces show where a volumetric data value lies in space
- Map uncertainties to:
  - hue, saturation, brightness
  - texture mapping
- Isovalue eases display





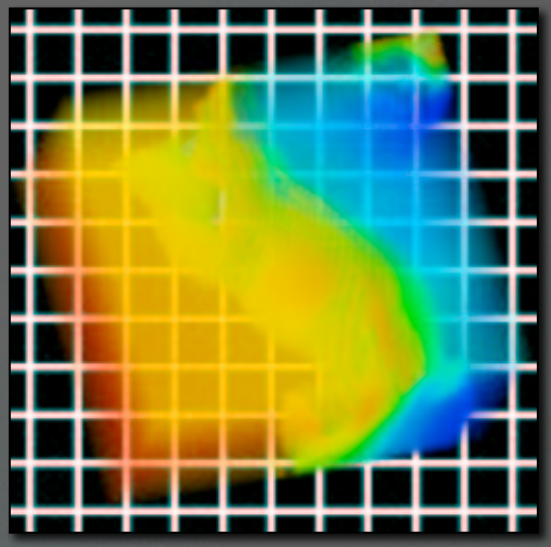


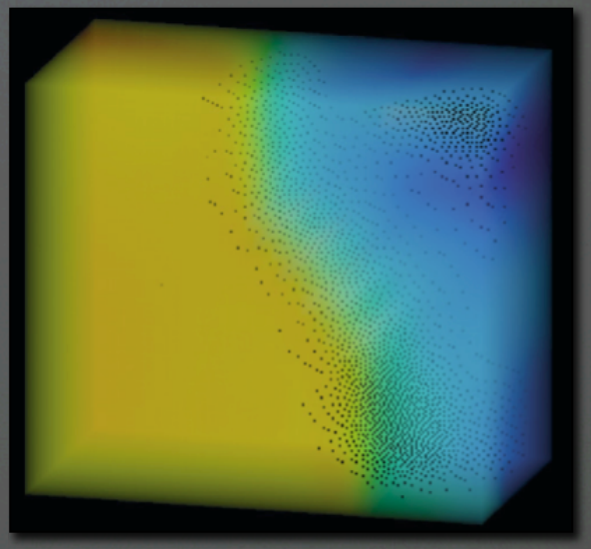
Isovalue = 0.185

P. Rhodes, R. Laramee, R.D. Bergeron, T. Sparr.
Uncertainty Visualization Methods in Isosurface Rendering.
In *EUROGRAPHICS 2003 Short Papers*, 2003.

Volumetric Data-Volume Rendering

- Show data with high or low uncertainty
- Map data to color & uncertainty to opacity
- Add discontinuities to regions of high uncertainty (speckles, noise)

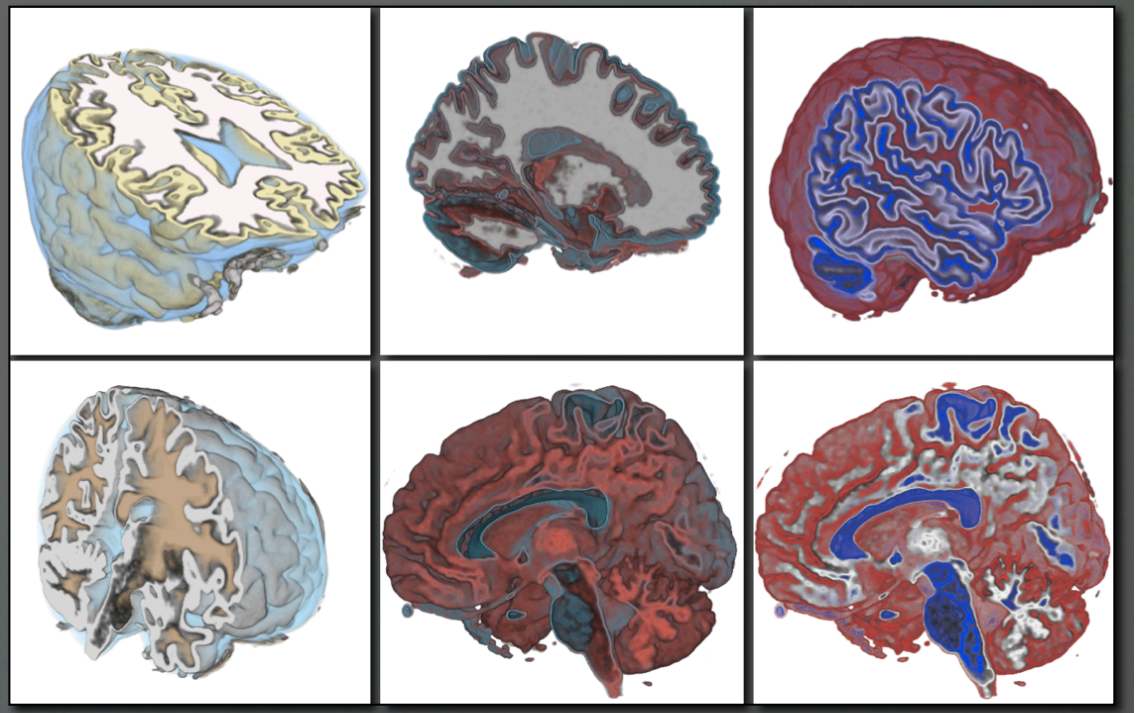




S. Djurcilov, K. Kim, P. Lermusiaux, A. Pang.
Visualizing Scalar Volumetric Data with Uncertainty.
In *Computers and Graphics*, vol. 26, 2002.

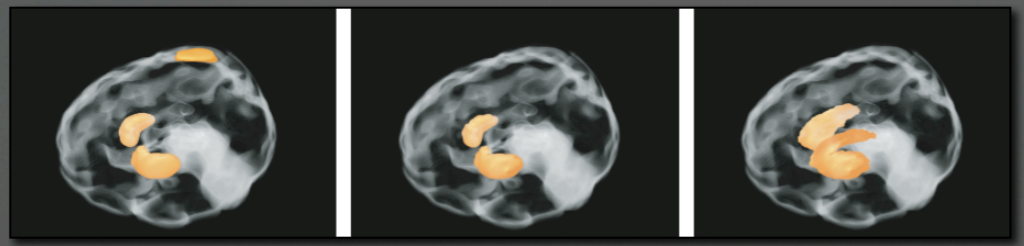
Fuzzy Classification - Volume Rendering

- Transfer function to segment data into structures
- Border between structures is fuzzy
- Statistically quantify this boundary



Fuzzy Sensitivity Confidence

J. Kniss, R. Van Uitert, A. Stephens, G. Li, T. Tasdizen, C. Hansen.
Statistically Quantitative Volume Visualization".
In *IEEE Vis*, 2005.

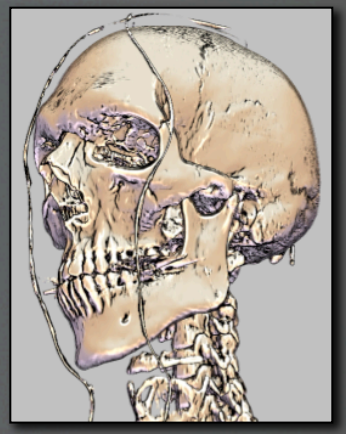


Tissue segmentation based on shape prior

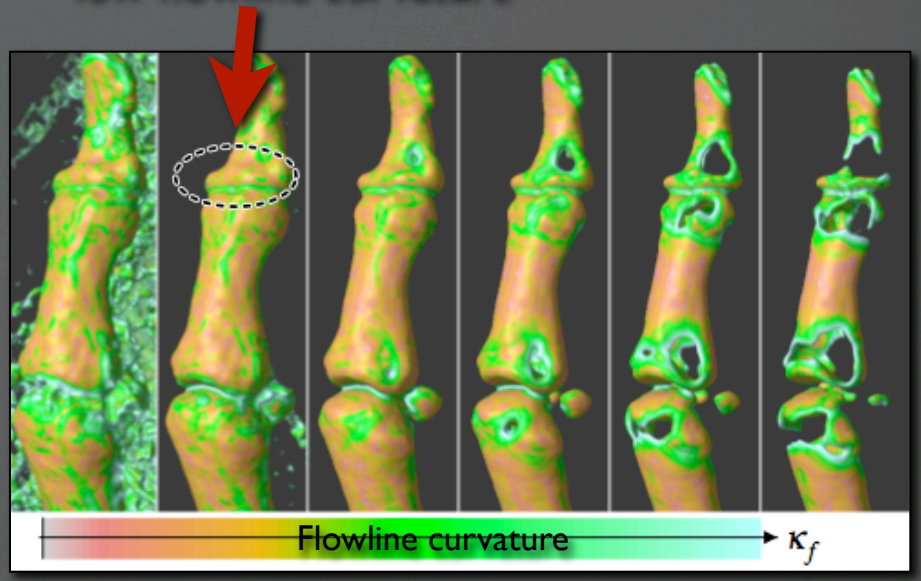
A. Saad, G. Hamarneh, T. Möller
Exploration and Visualization of Segmentation Uncertainty Using Shape and Appearance Prior Information
In *IEEE Vis*, 2010.

Curvature-Based Transfer Functions

- Flowline curvature measures the curvature of the path of the flow, viewed from above.
- Indicates sensitivity of isosurface to small changes
- Use as a measure of uncertainty - stability of isosurface



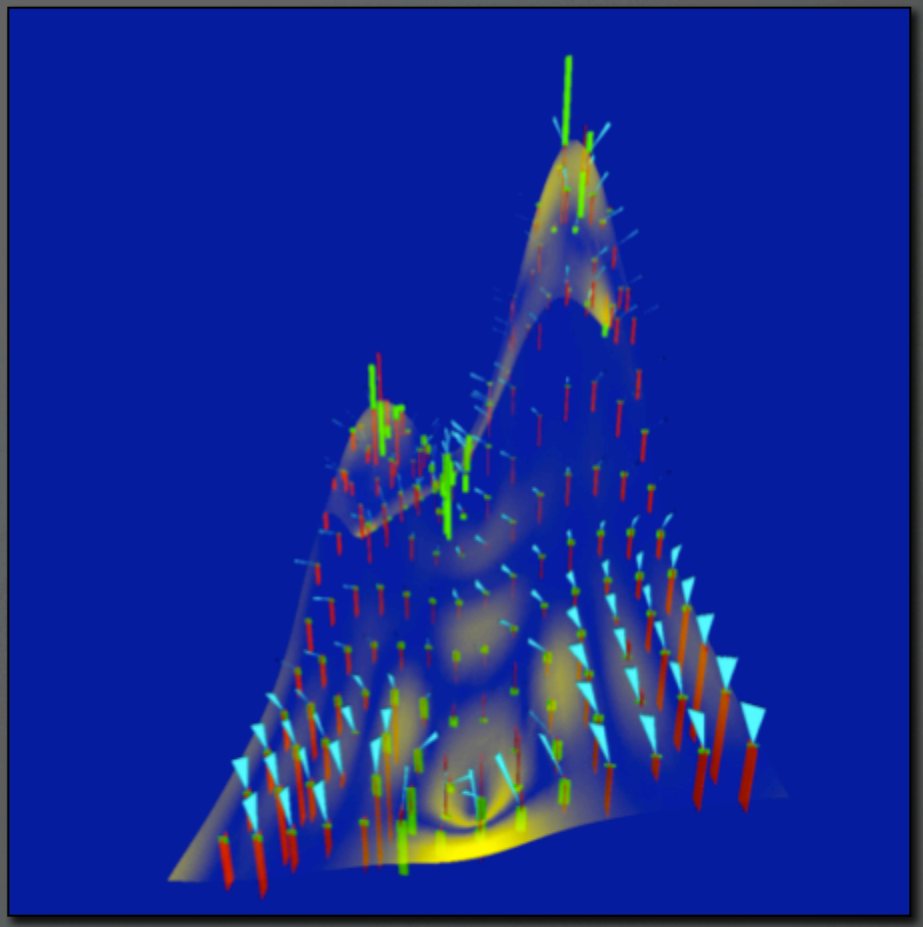
High surface curvature & low flowline curvature



G. Kindlmann, R. Whitaker, T. Tasdizen, T. Moller.
Curvature-Based Transfer Functions for Direct Volume Rendering: Methods and Applications.
In *IEEE Vis*, 2003.

Surface Interpolants

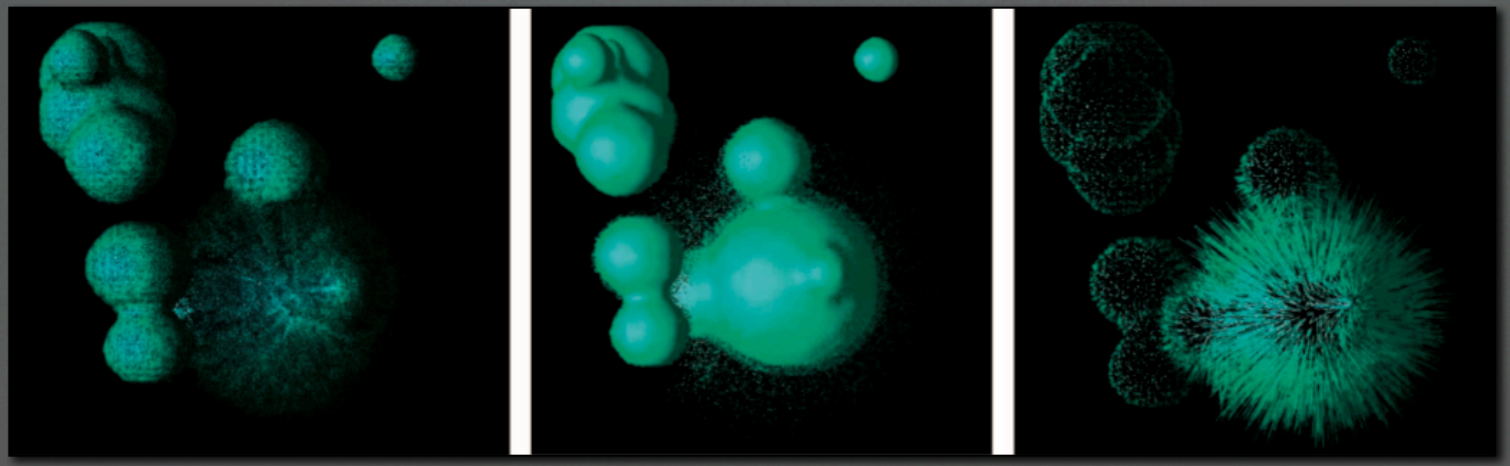
- Geometric uncertainty/ interpolation error
  - differences in position, normals, principal curvatures and directions, and mean and Gaussian curvatures
  - bilinear, C0 linear, C2 bicubic B-spline, multiquadrics, inverse multiquadrics and thin plate splines



S. Lodha, B. Sheehan, A. Pang, C. Wittenbrink.
Visualizing geometric uncertainty of surface interpolants.
In Proc *Conference on Graphics Interface*, 1996.

Surface Uncertainty

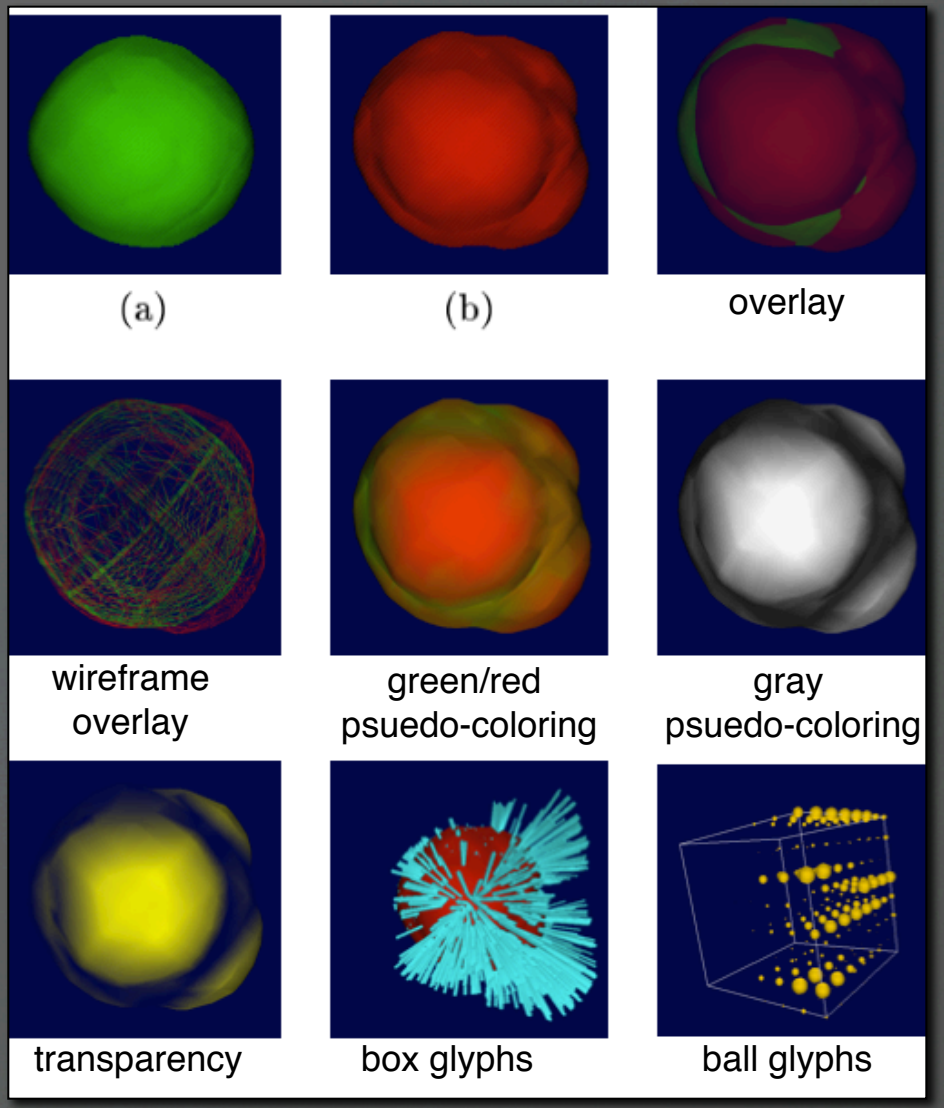
- Uncertainty in position of data
- Points as primitives, displace pts with uncertainty



Gevorg Grigoryan and Penny Rheingans.
Point-Based Probabilistic Surfaces to Show Surface Uncertainty.
In *IEEE TVCG*, 10(5), 2004.

Visualization Uncertainties - Isosurfaces

- Uncertainty from differences in isosurface creation
- Compare
  - marching cubes & marching cubes with ambiguous cell correction
  - interpolation schemes



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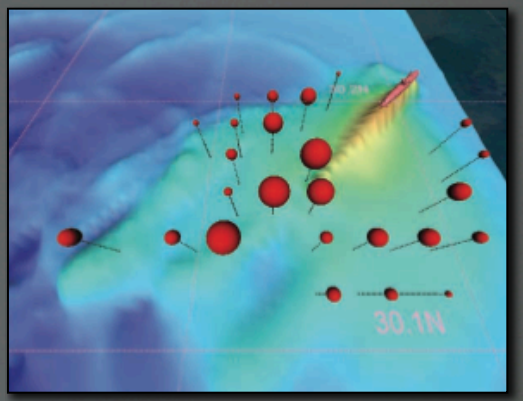
In Proc *Computer Graphics and Imaging*, 1999.

N-Dimensional Uncertainty

- Multidimensional & multivariate
- Often found in earth sciences data



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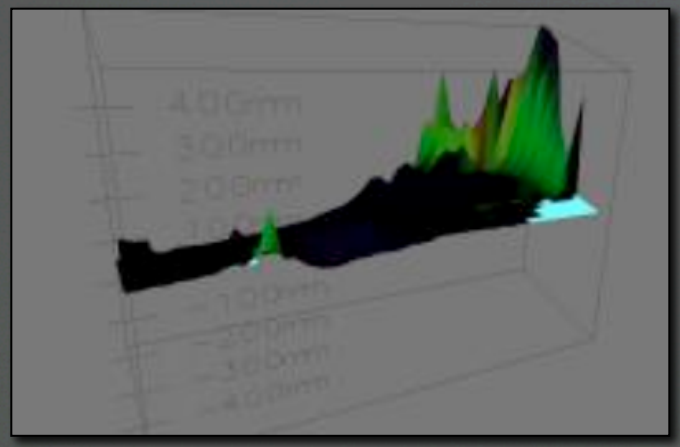
Multidimensional visual representations for underwater environmental uncertainty.
In *IEEE CG&A*, 24(5), 2004.



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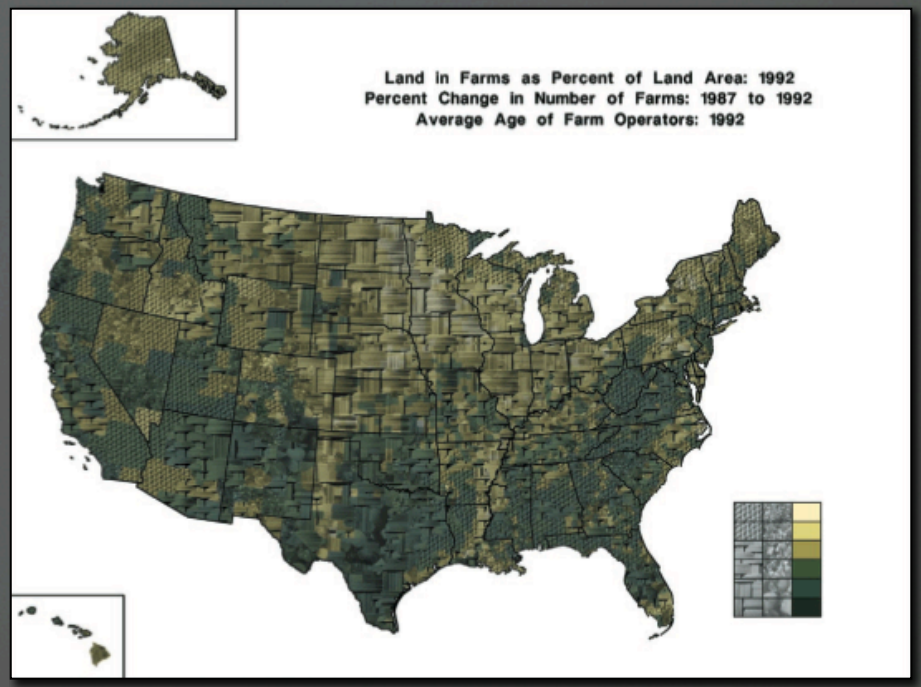
Modeling and Visualizing Uncertainty in a Global Water Balance Model.
In *Proceedings of the 2003 ACM symposium on Applied computing*, 2003.



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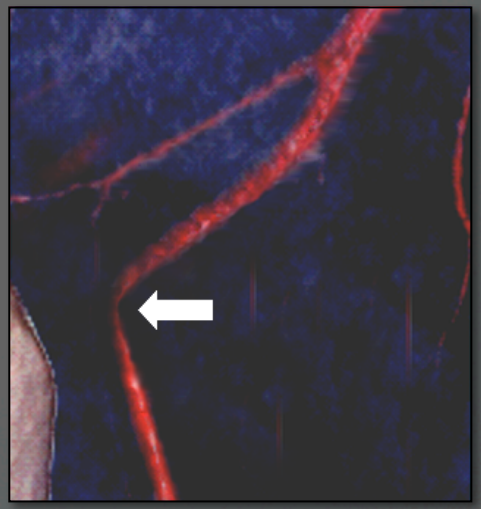
In *IEEE CG&A*, 20(6), 2000.

Visualization Uncertainties - DVR

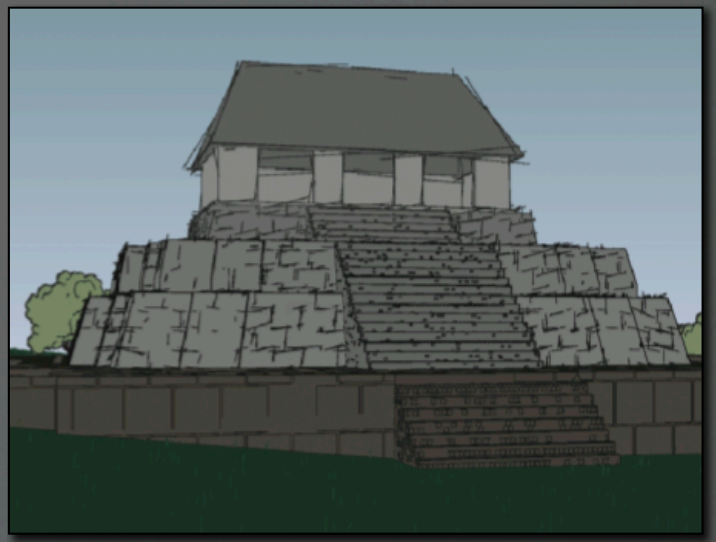
- Direct volume rendering
  - transfer function defines mapping to color & opacity
- Sensitivity lens
  - UI animates region of interest



C. Lundstrom, P. Ljung, A. Persson, A. Ynnerman.
Uncertainty Visualization in Medical Volume Rendering Using Probabilistic Animation.
In *IEEE TVCG*, 13(6), 2007.

Architectural Uncertainties

- Confidence of 3D reconstruction
- Position, materials, arch details are unknowns
- Shiny pictures, solid lines indicate truth



K. Potter, et al.
Resolution Independent NPR-Style 3D Line Textures.
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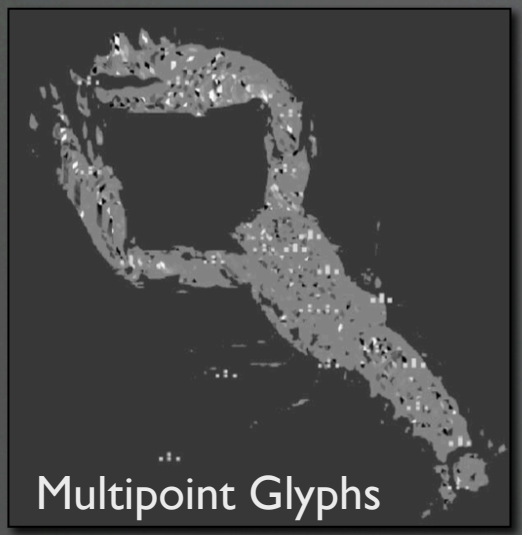
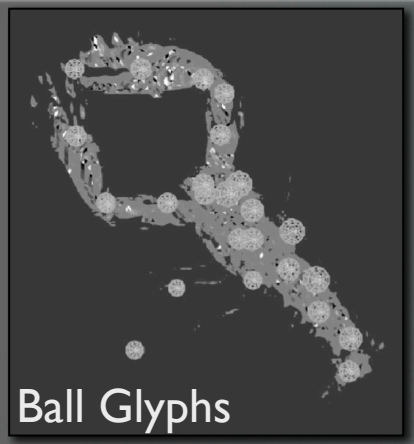
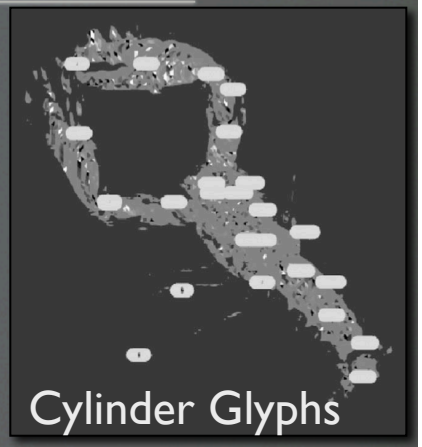
T. Strothotte, M. Masuch, T. Isenberg.
Visualizing Knowledge about Virtual Reconstructions of Ancient Architecture.
In Proc *of Computer Graphics International*, 1999.

Evaluation - 1

- Usability study
- Glyphs: arrow, ball, cone, cylinder, multipoint
- Other: colormapping, transparency, aliasing



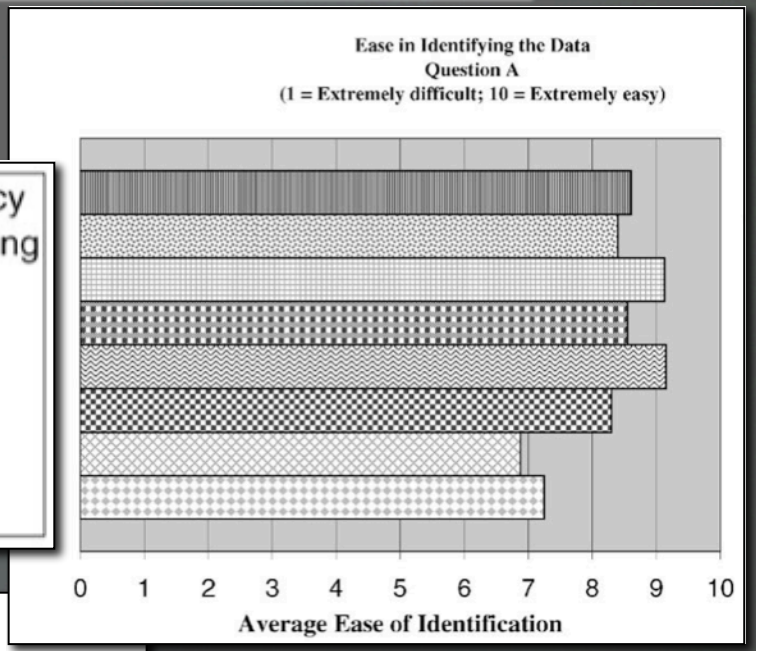


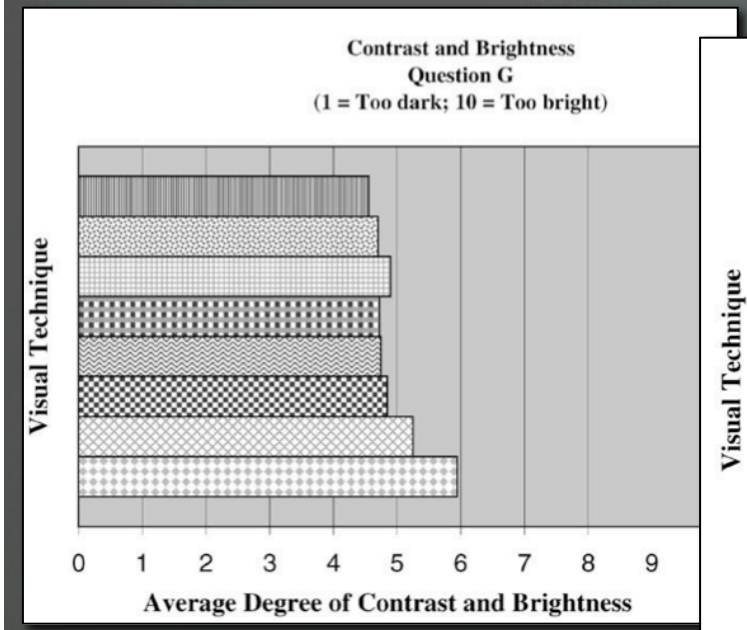


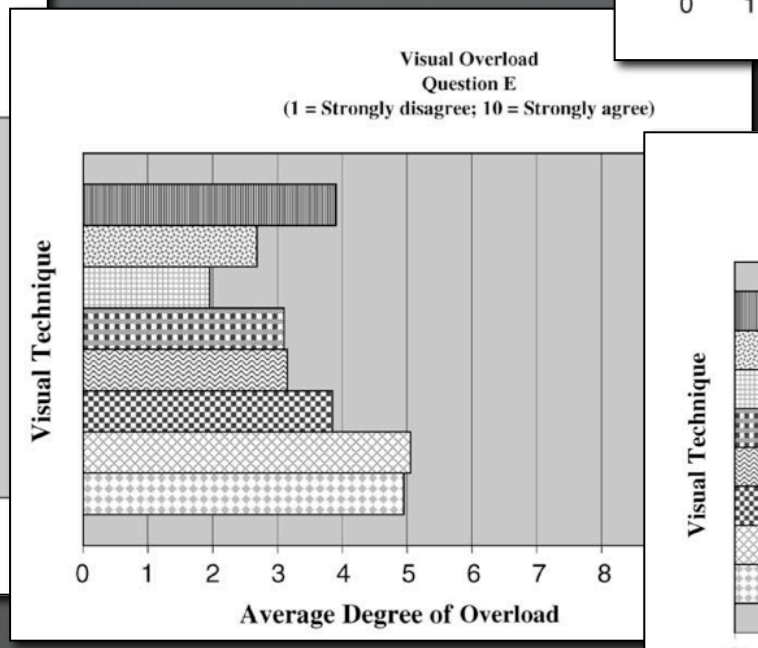
T. Newman, W. Lee.
On visualizing uncertainty in volumetric data: techniques and their evaluation.
In *Journal of Visual Languages and Computing*, 15, 2004.

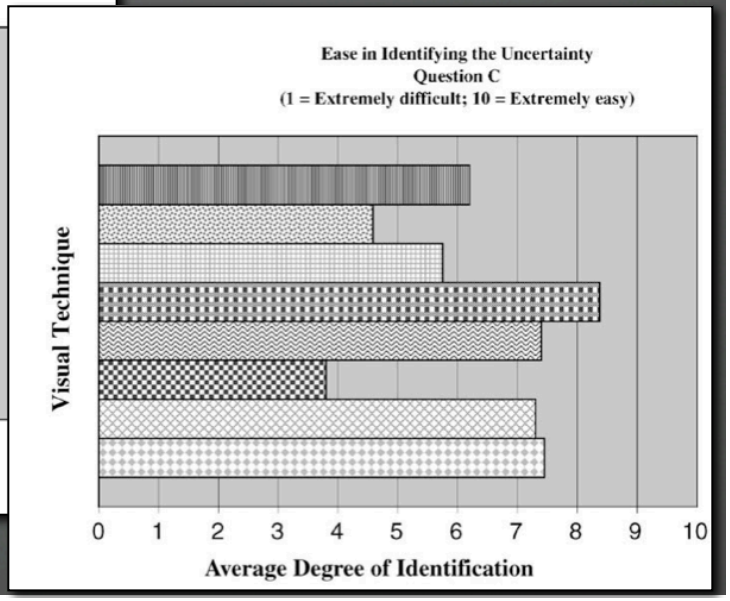
Evaluation - 1

- Evaluate effectiveness:
  - identify the presence and degree of uncertainty
  - identify the underlying phenomena in the data





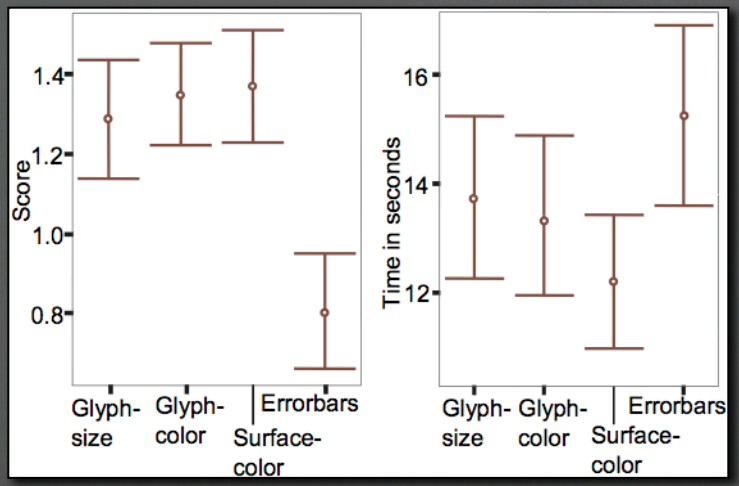


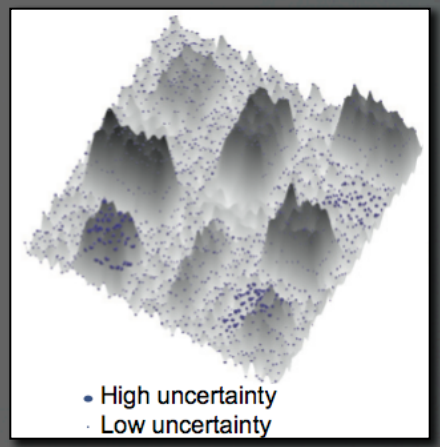


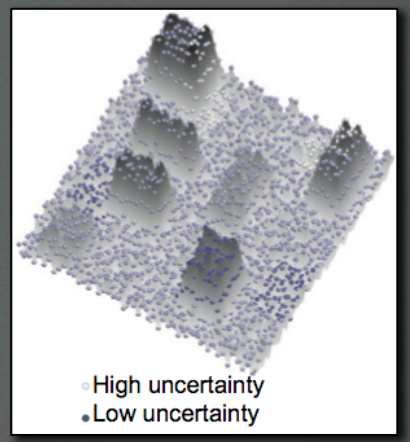
T. Newman, W. Lee.
On visualizing uncertainty in volumetric data: techniques and their evaluation.
In *Journal of Visual Languages and Computing*, 15, 2004.

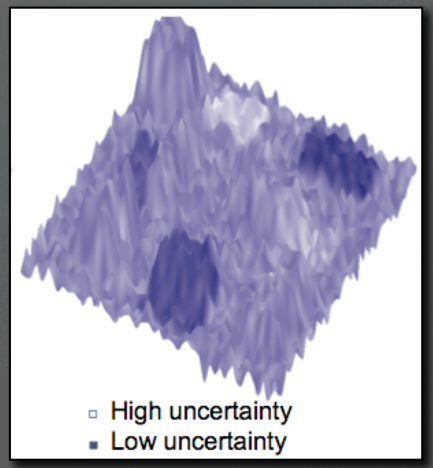
Evaluation - 2

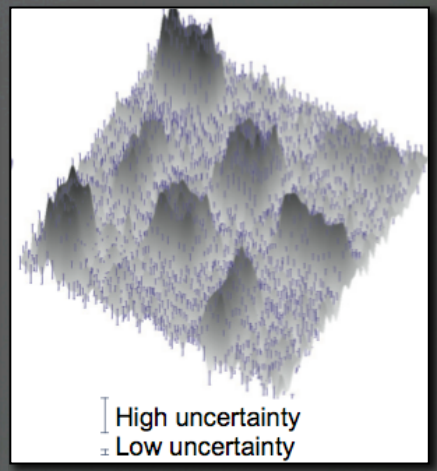
- Evaluate effectiveness of:
  - size of glyphs
  - color of glyphs
  - color of data
  - errorbars
- Found errorbars to perform worse











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Summary/Conclusion

- Uncertainty vis becoming a hot topic in the vis community
- Many (most?) obvious solutions have been tried
- Increasing visual complexity main challenge

Thanks!

Surveys

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In The Visual Computer, 13(8), p. 370-390, 1997.

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In Cartography and Geographic Information Science, 32(3), p. 139-160, 2005.