

Uncertainty and Parameter Space Analysis in Visualization | Oct 15, 2012

<http://tinyurl.com/8hwtzrz>

STATISTICAL UNCERTAINTY

From Quantification to Visualization

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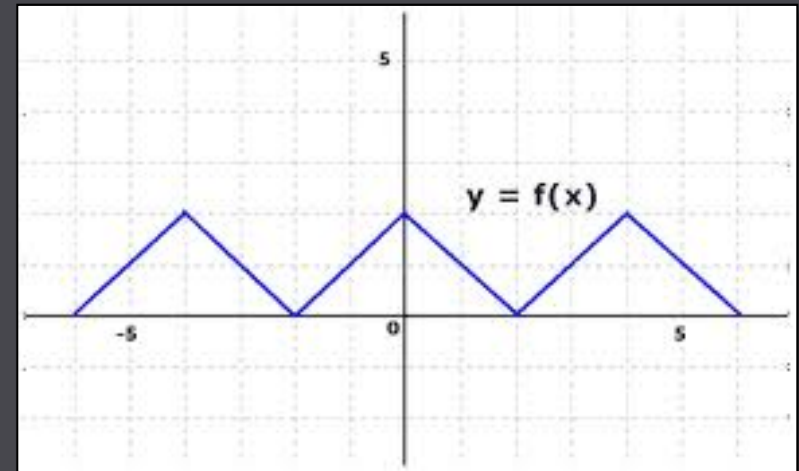
University of Utah

VisWeek 2012 Tutorial

Statistical Uncertainties

random fluctuations of measurement

Sources of Uncertainty



Sources

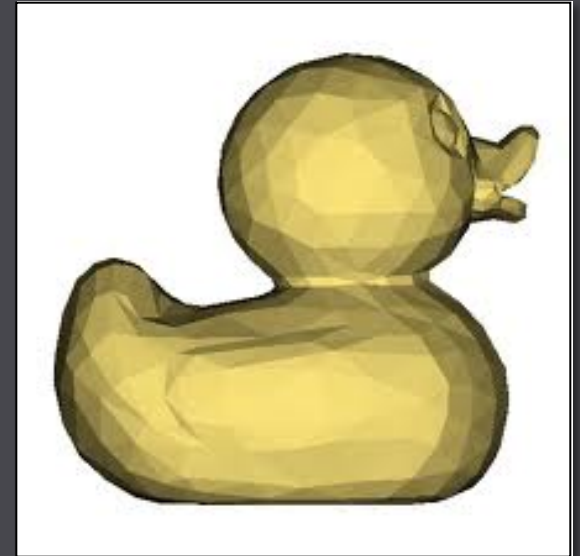
PARAMETER UNCERTAINTY

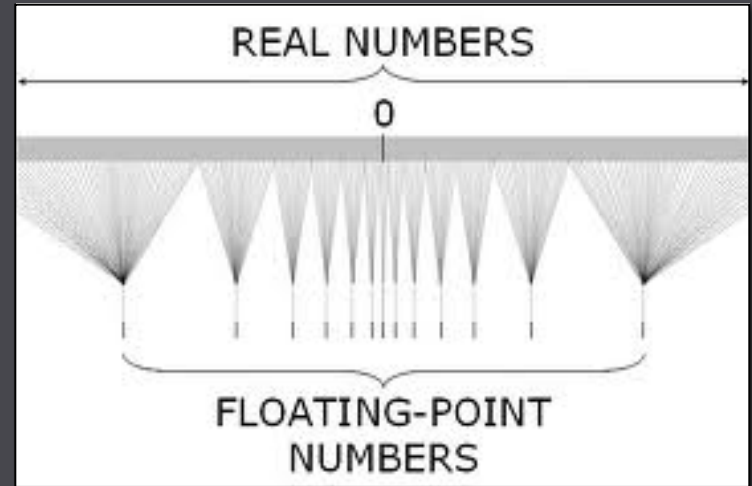
- *influence of model parameters*
- *(exact, correct, best) values unknown*
- *cannot be controlled by experiment*

Sources

MODEL INADEQUACY

- *aka model bias, model discrepancy*
- *model is an approximation*
- *lack of knowledge of the underlying problem*
- *accuracy discrepancy of the model to reality*





Sources

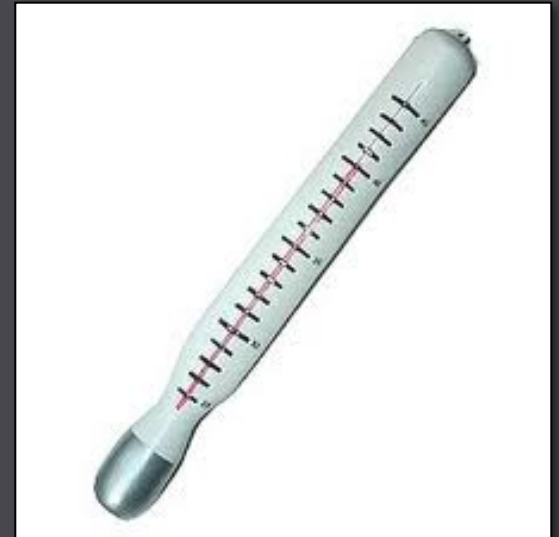
ALGORITHMIC UNCERTAINTY

- *aka numerical uncertainty*
- *numerical errors, approximations*
- *translation of mathematical model to the computer*

Sources

EXPERIMENTAL UNCERTAINTY

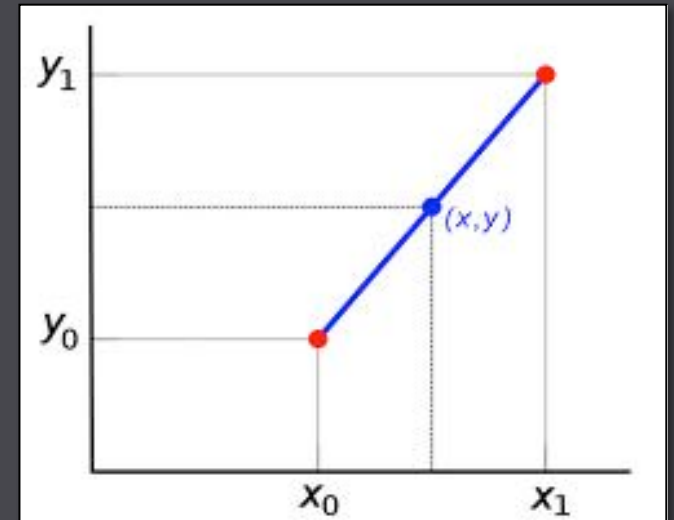
- *aka observational error*
- *variability of experimental measurements*
- *non-determinism*



Sources

INTERPOLATION UNCERTAINTY

- *lack of available data*
- *interpolate/extrapolate for desired response*
- *choice of interpolation method*



Categories



Categories

ALEATORIC UNCERTAINTY

- aka *statistical* uncertainty
- *unknowns that differ on each run*
- *i.e. throwing dice*

Irreducible: *cannot be eliminated through improvements in models or measurements*

Quantification

*reduce epistemic uncertainties to aleatoric
get to a relatively straightforward quantification*

use statistics for quantification

Statistical Uncertainties

common representations for visualization

Probability Distribution Functions (PDFs)

approximate outcome through a probability function

Probability Density

continuous random variables
frequency of outcome values

Probability Distribution Functions (PDFs)

approximate outcome through a probability function

Categorical Distribution
discrete random variables
finite set of outcome values

Statistics on PDFs

MEAN

expected value, arithmetic mean

Statistics on PDFs

MEDIAN

middlemost value

Statistics on PDFs

STANDARD DEVIATION

spread of values

Statistics on PDFs

MODE

most frequently occurring value

Statistics on PDFs

SUPPORT/RANGE

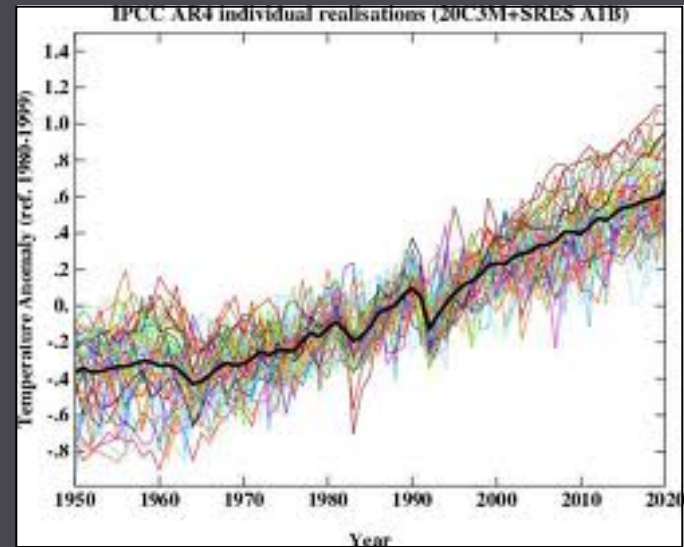
interval where value probability is not zero

Uncertainty in Data

Ensembles

MULTI-RUN SIMULATIONS

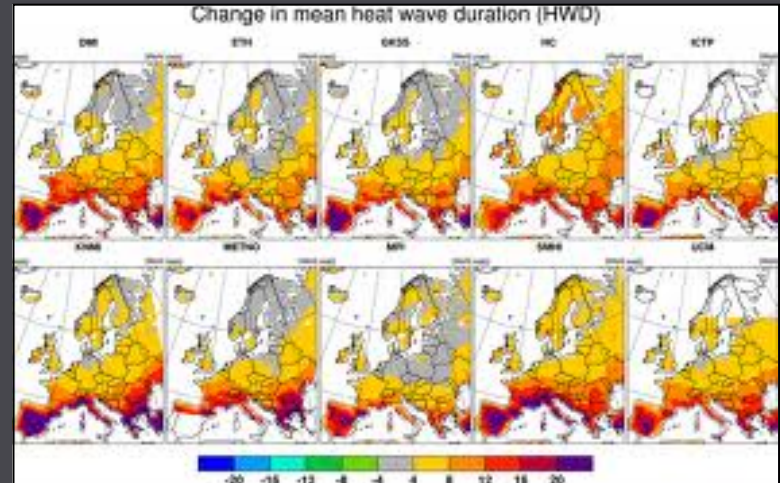
- *explore space of parameters*
- *mitigate model error*
- *cover range of initial conditions / outcomes*
- *combine multiple models*



Ensembles

COLLECTION OF DATASETS

- *members, realizations*
- *full simulation run for each parameter set/input condition*



Ensembles

MULTI-

- *dimensional*

spatial domain & time

- *variate*

simulate over many variables

- *valued*

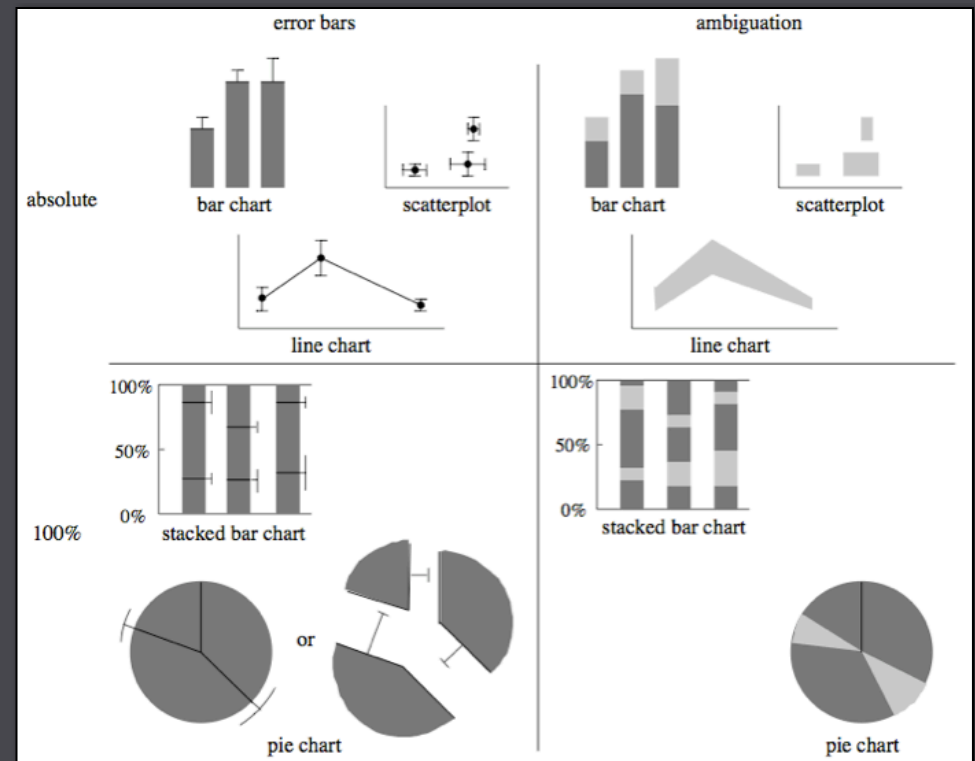
many vales for each variable/location

Graphical Data Analysis

visual indication of pdf

Ambiguation

- *numeric interval guaranteed to contain data value*
- *no assumptions about the pdf within the interval*



Statistical Uncertainty

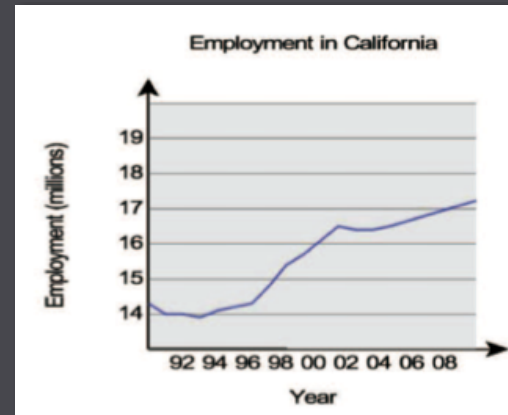
Bounded Uncertainty

Visualizing Data with Bounded Uncertainty.

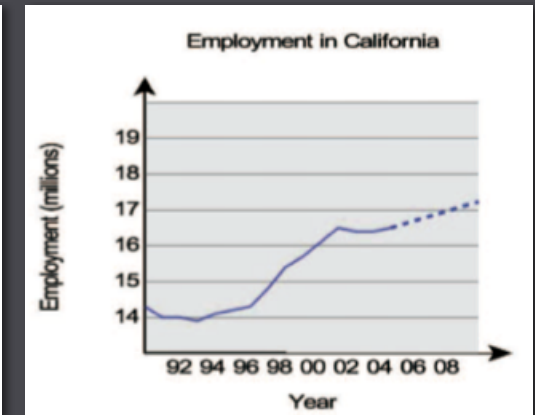
C. Olston, J.D. Mackinlay. InfoVis, 2002.

Information Uncertainty

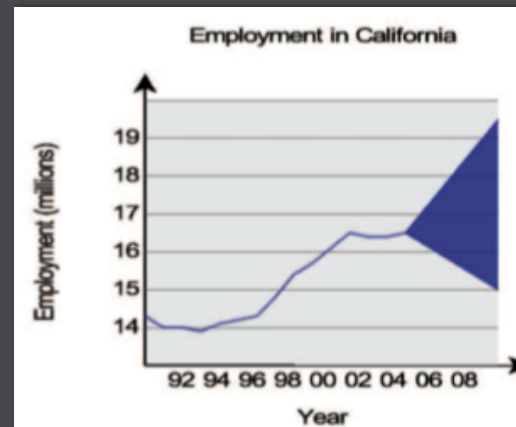
- *indication of known information*
- *qualitative rather than quantitative*
- *spreadsheet interface characterizes the data*



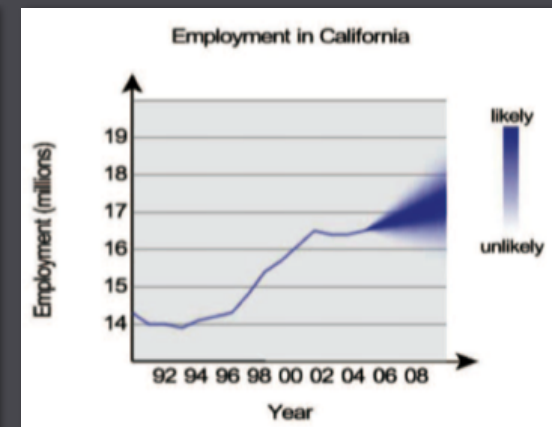
Average Growth



Estimated Growth



Possible Growth



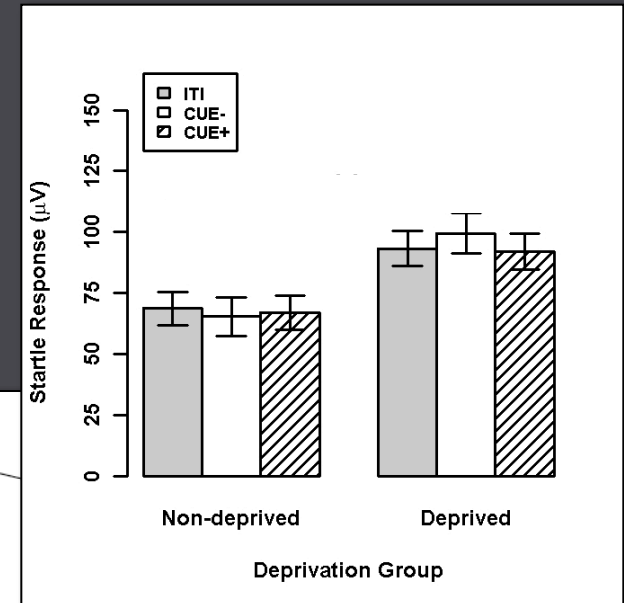
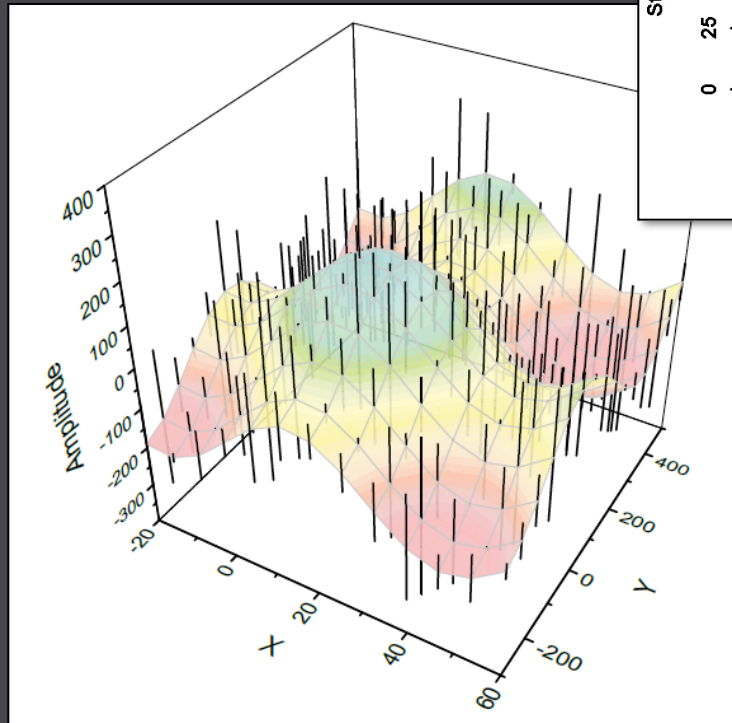
Likely Growth

A Spreadsheet Approach to Facilitate Visualization of Uncertainty in Information.

A. Streit, B. Pham, R. Brown. TVCG 14(1), 2008.

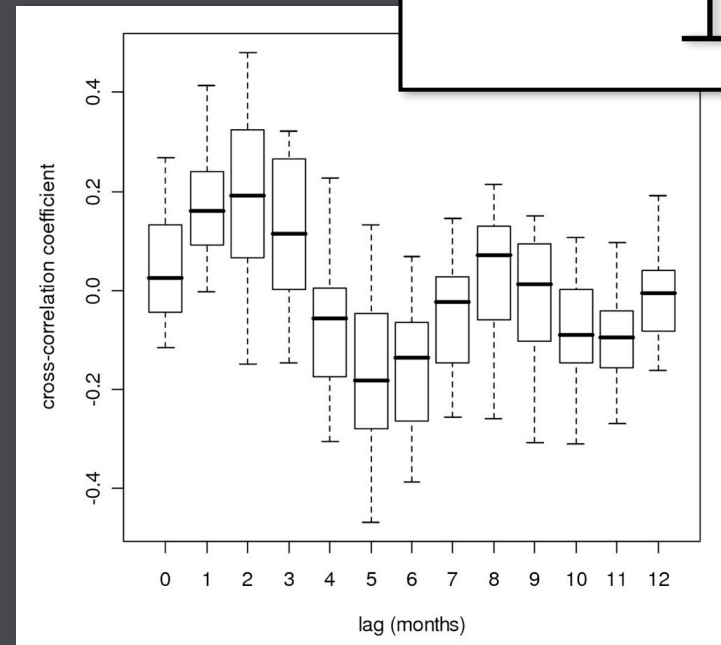
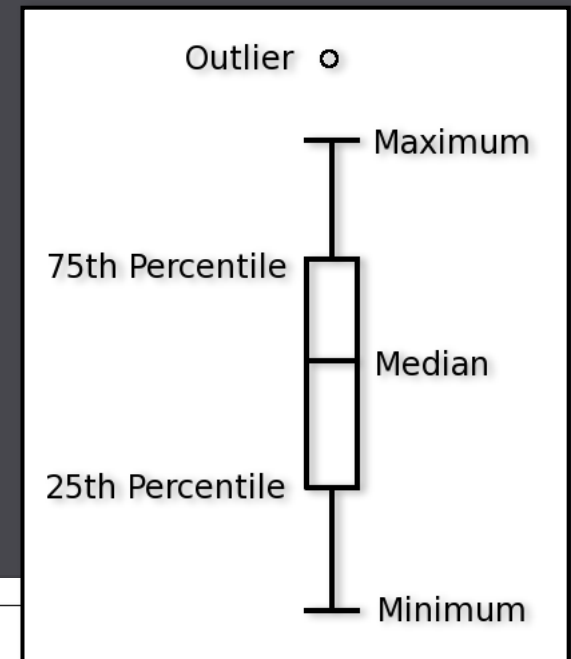
Error Bars

- convey accuracy by amount of \pm error
- std dev or std error



Boxplots

- *quartile range including median, outliers*
- *assume Gaussian*

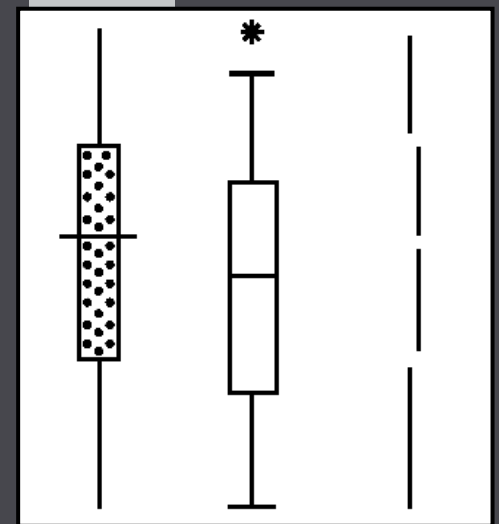


Boxplot Modifications

VISUAL MODIFICATIONS

refinement for aesthetic purposes

range bar



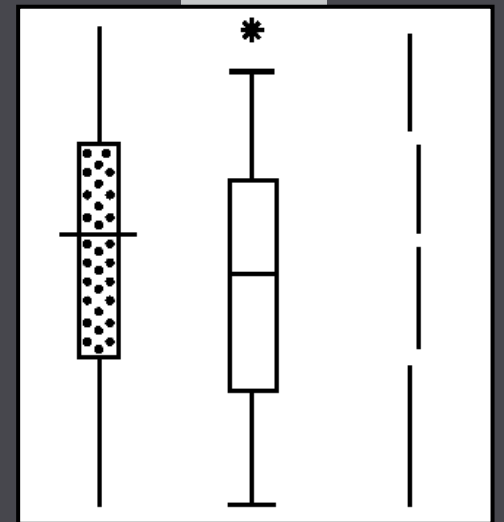
Charting Statistics.

Mary Eleanor Spear. McGraw-Hill, 1952.

Boxplot Modifications

VISUAL MODIFICATIONS
refinement for aesthetic purposes

boxplot



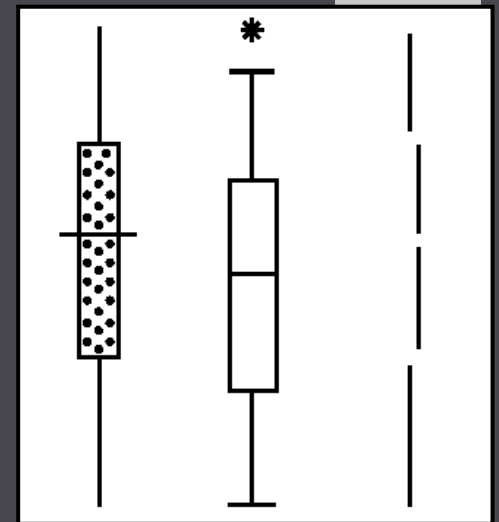
Exploratory Data Analysis.

John W. Tukey. Addison-Wesley, 1977.

Boxplot Modifications

VISUAL MODIFICATIONS
refinement for aesthetic purposes

inner-quartile plot

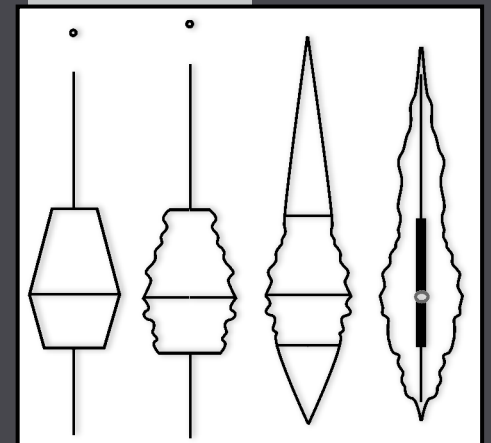


The Visual Display of Quantitative Information.
Edward Tufte. Graphics Press, 1983.

Boxplot Modifications

DENSITY MODIFICATIONS

add indication of value prevalence



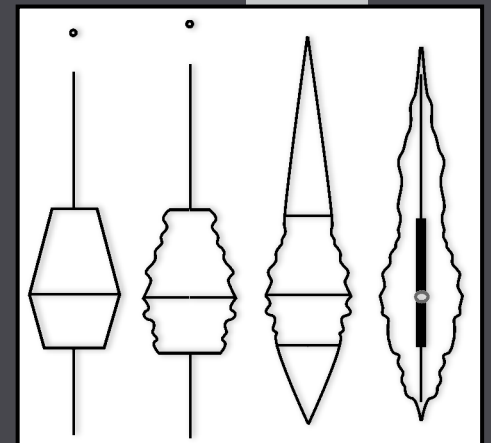
Opening the Box of a Boxplot.

Y. Benjamini. *The American Statistician*, 42(4), 1988.

Boxplot Modifications

DENSITY MODIFICATIONS

add indication of value prevalence



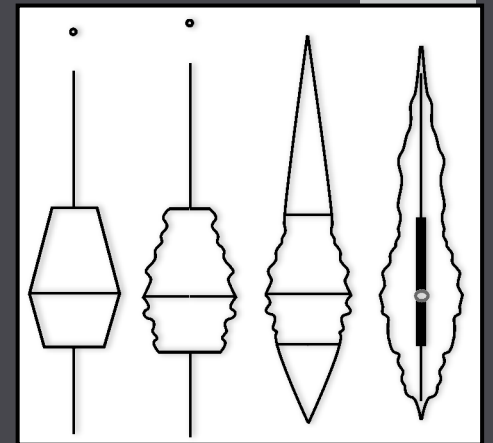
The Box-Percentile Plot.

W. Esty, J. Banfield. Journal of Statistics Software, 8(17), 2003.

Boxplot Modifications

DENSITY MODIFICATIONS

add indication of value prevalence



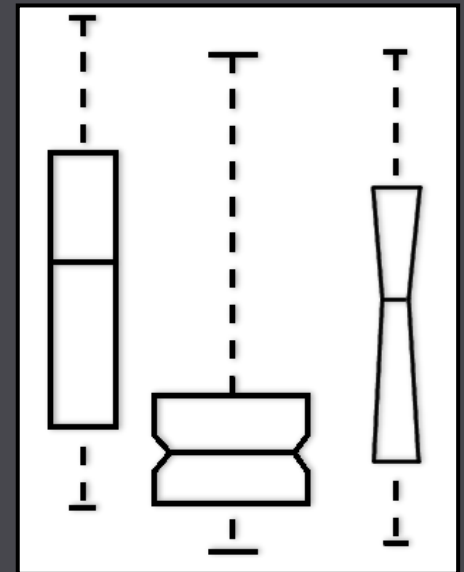
Violin Plots.

J. Hintze, R. Nelson. *The American Statistician*, 52(2), 1998.

Boxplot Modifications

DATA CHARACTERISTICS

sample size, confidence levels



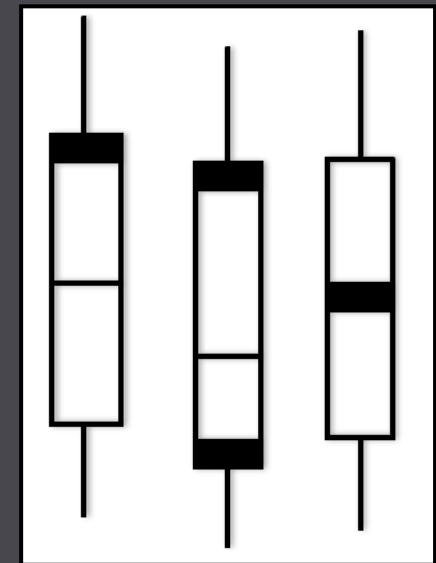
Variations of Box Plots.

R. McGill, J.W. Tukey, W.A. Larsen. *The American Statistician*, 32(1), 1978.

Boxplot Modifications

ADDITIONAL STATISTICS

• *moments, modality*



Can the Box Plot Be Improved?

C. Choonpradub, D. McNeil. Songklanakarin J Sci Technol, 27(3), 2005,

Boxplot Modifications

SUMMARY PLOT

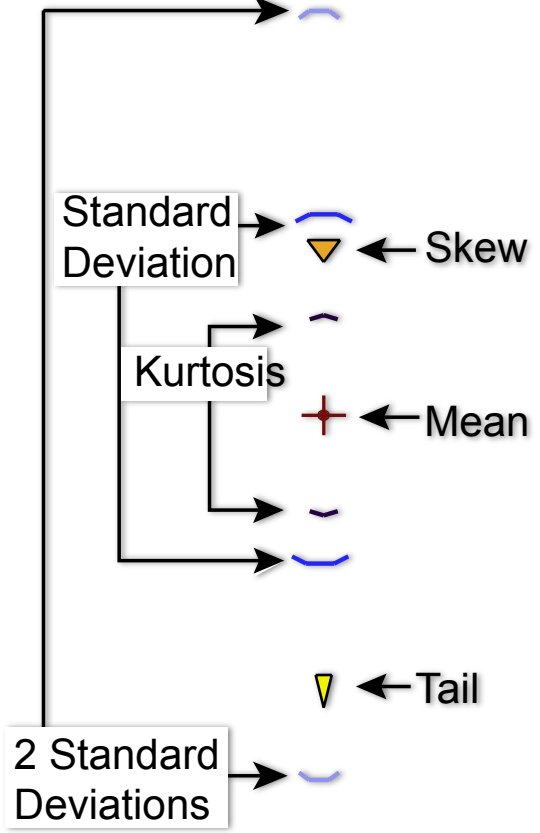
combine 4 plots into one

augment with more descriptive statistics
indicate quantity & location of uncertainty

Visualizing Summary Statistics and Uncertainty.

K. Potter, J. Kniss, R. Riesenfeld, C.R. Johnson. CGF 29(3), 2010.

**Abbrev.
Box Plot**

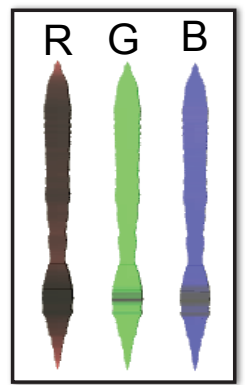


Moment Plot

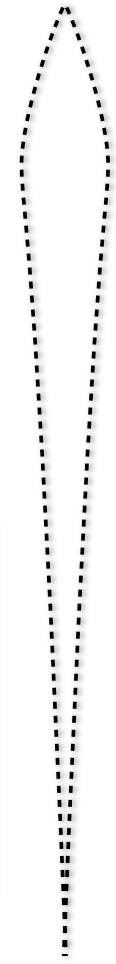
Histogram



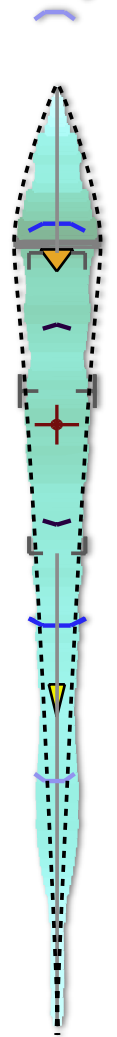
Density



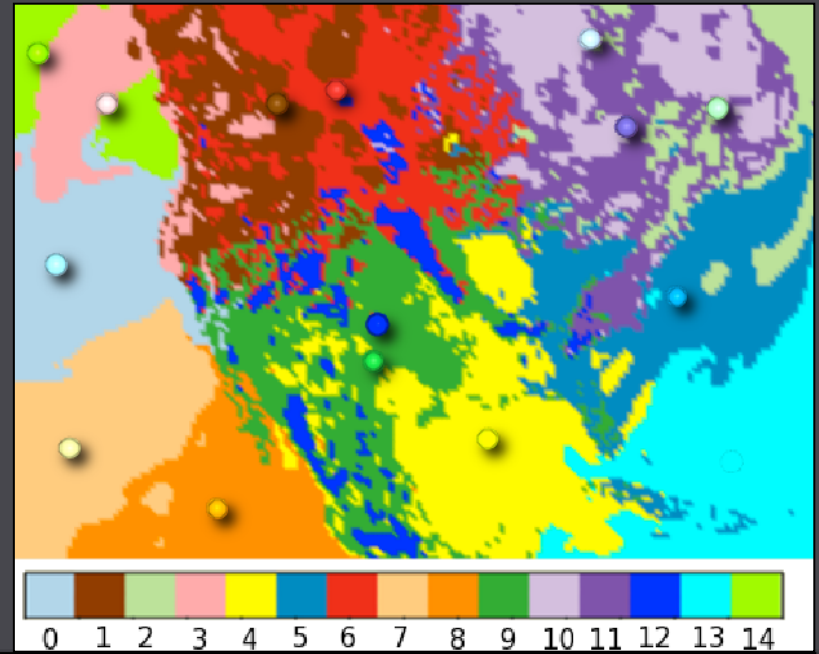
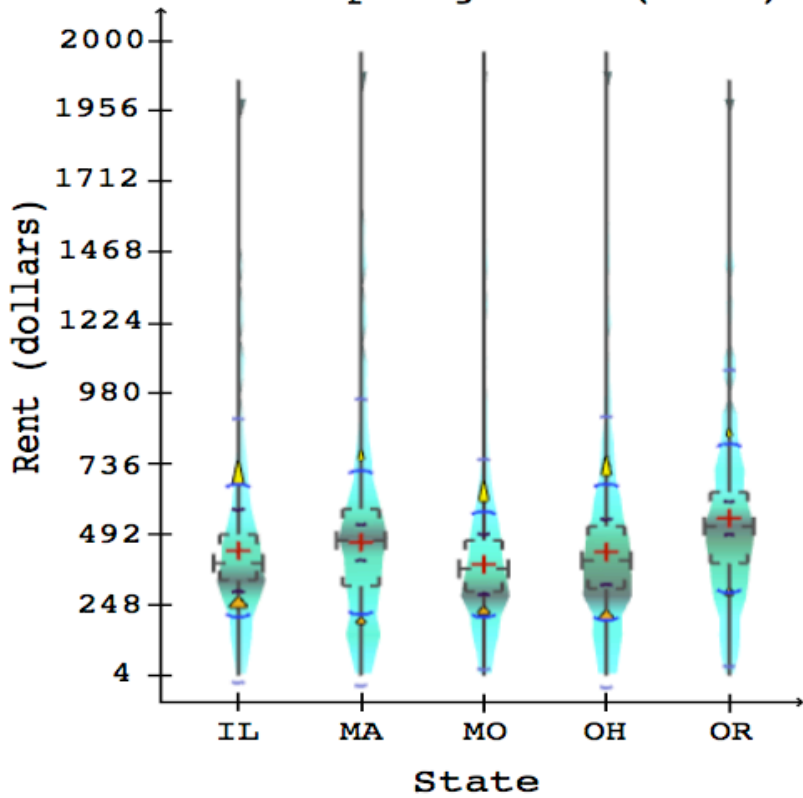
Distribution Fitting



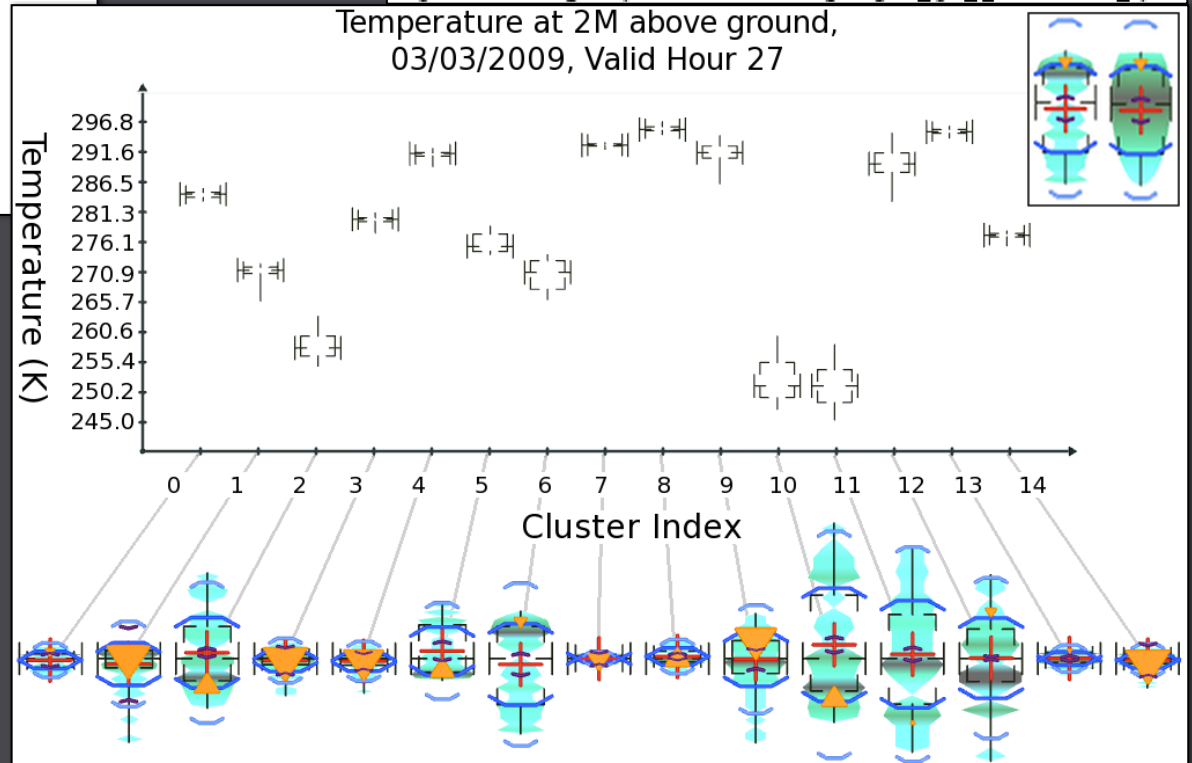
Summary Plot



Monthly Rent in Cities Named Springfield (1999)

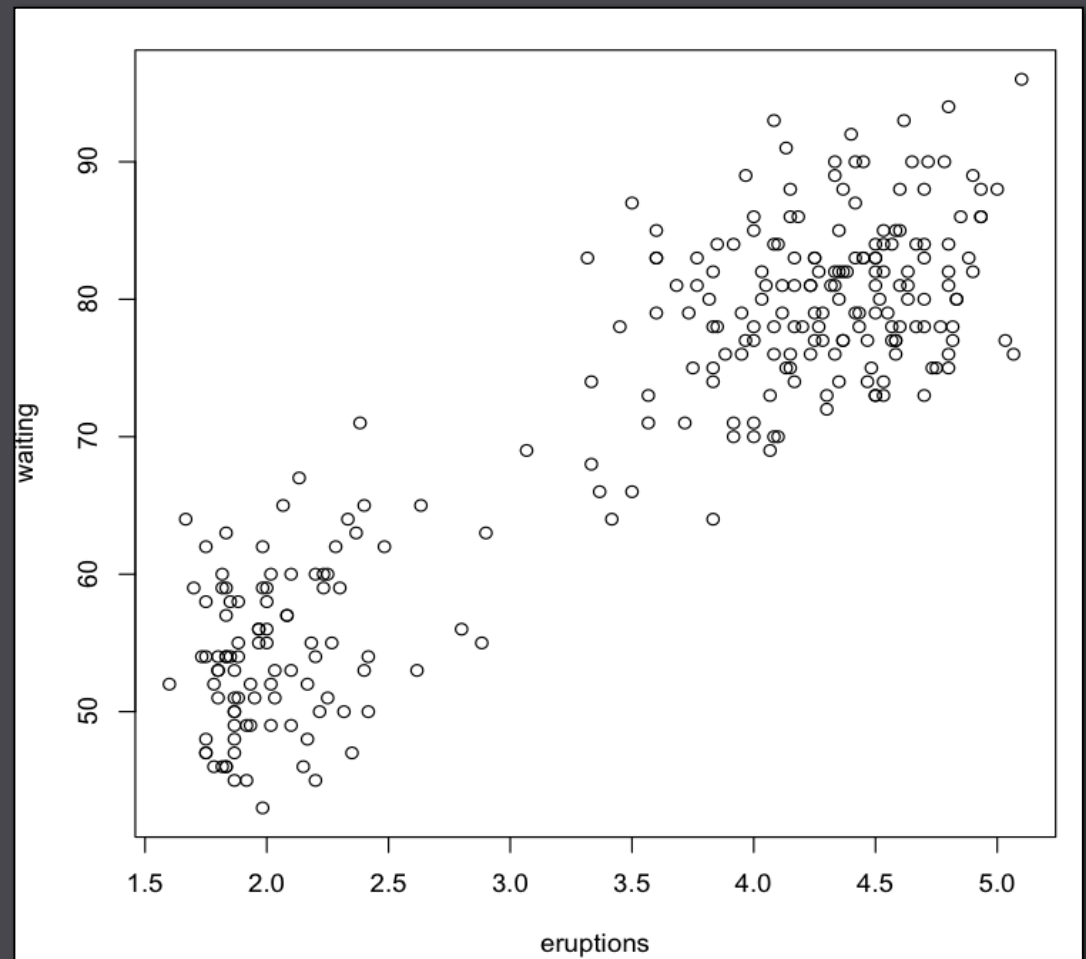


Temperature at 2M above ground, 03/03/2009, Valid Hour 27



2D Data

- *Scatter plot of 2D position of samples*
- *William Playfair (1759–1823)*
 - pie charts
 - line graphs
 - bar charts



The Early Origins and Development of the Scatterplot

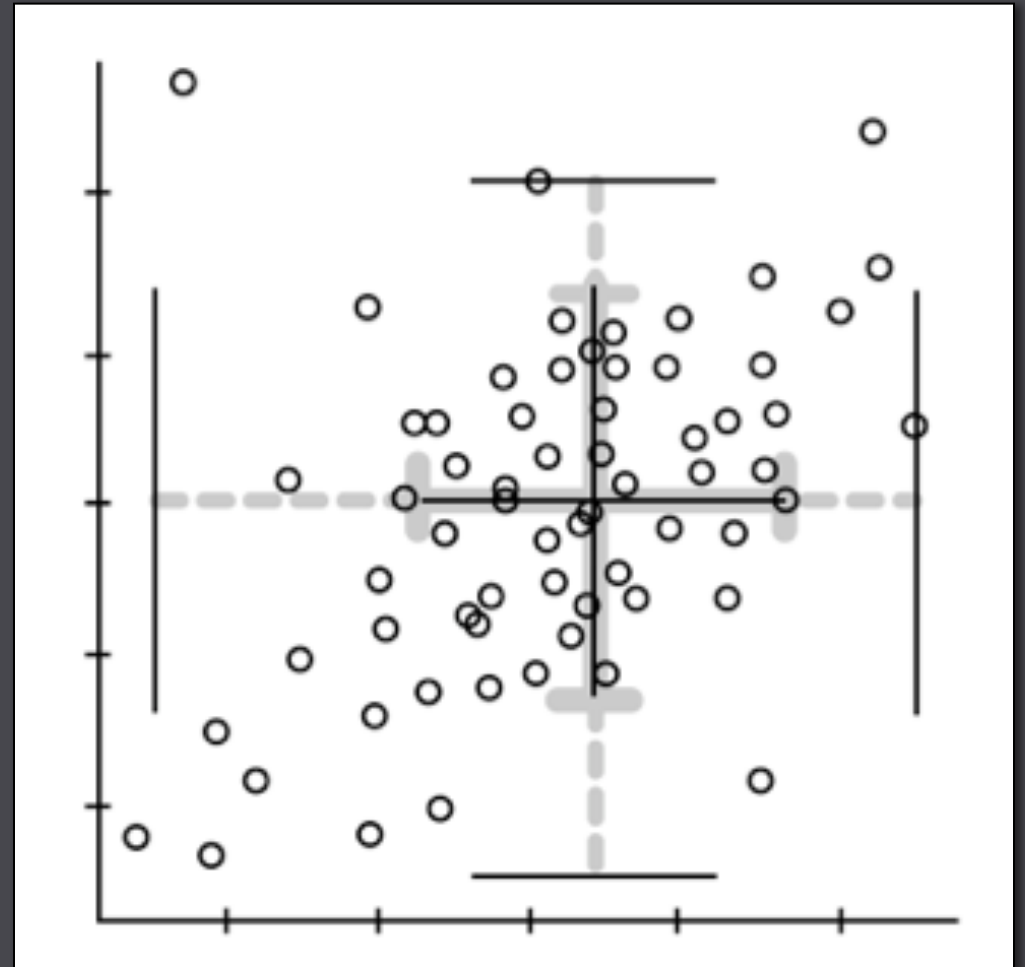
Michael Friendly and Daniel Denis

Journal of the History of the Behavioral Sciences, Vol. 41 (2), 103–130 Spring 2005

2D Box Plots

RangeFinder Plot

1D boxplot per axis

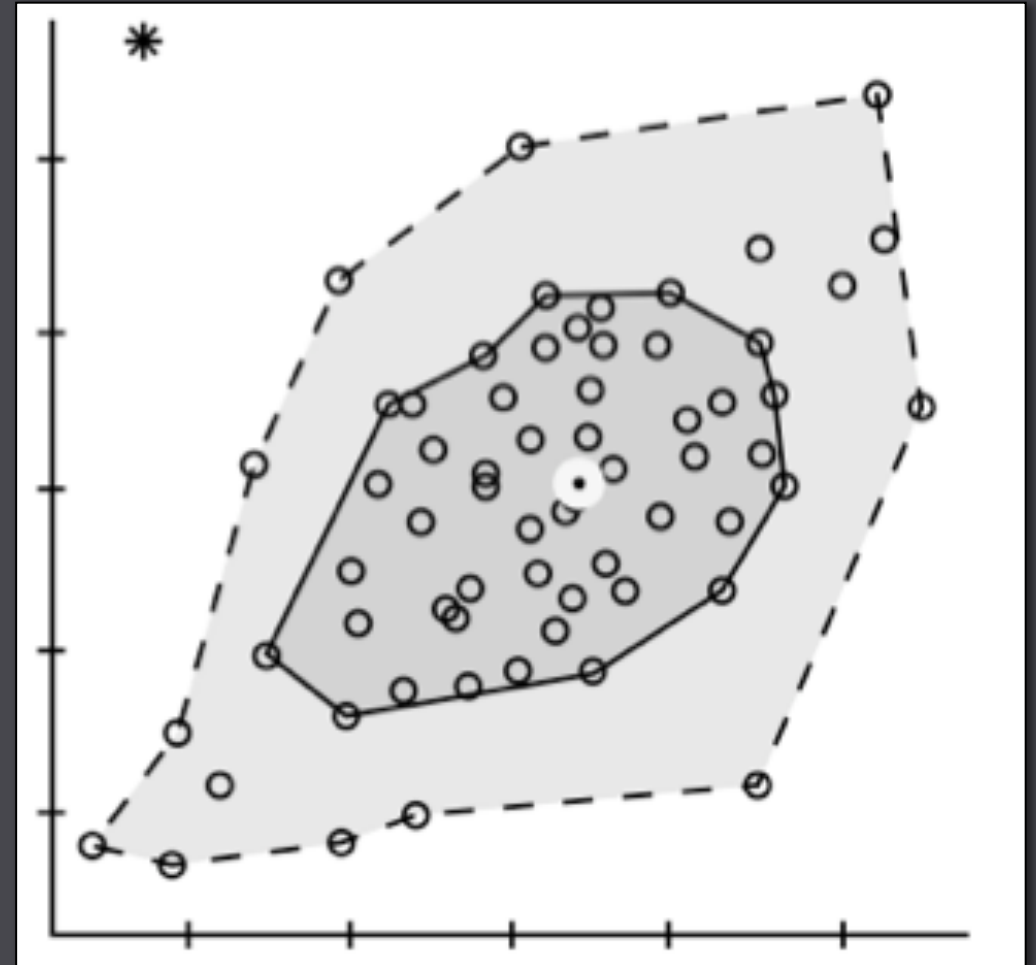


RangeFinder box plots.

S. Beckett, W. Gould, TAS 41 (2), 1987.

2D Box Plots

Two-Dimensional Boxplot
Robust line partition



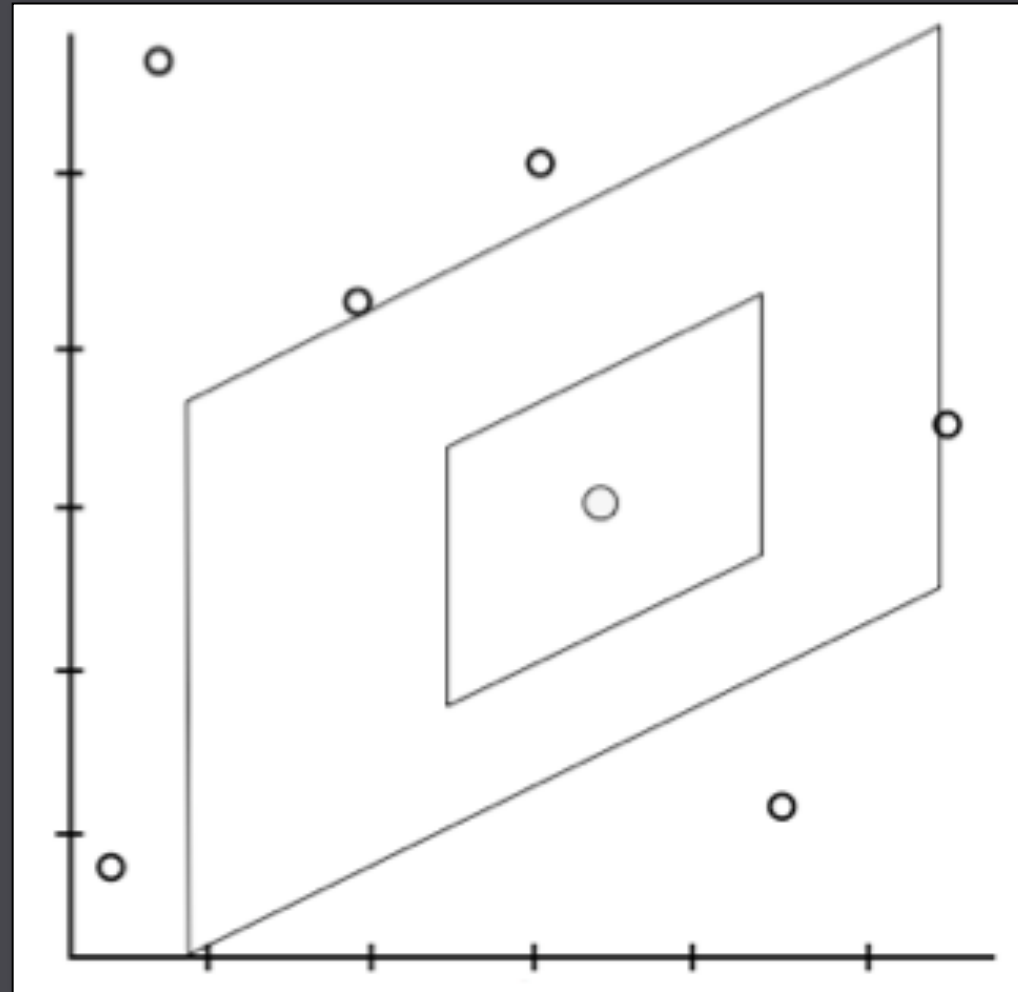
Two-dimensional box plot.

P.Tongkumchum, Songklanakarin J Sci Technol, 27(4),2005.

2D Box Plots

Bagplot

Halfspace depth (spatial quartiles)



The Bagplot: A Bivariate Boxplot.

P.J. Rousseeuw, I. Ruts, J. Tukey. *TAS*, 53(4), 1999.

Functional Box Plot

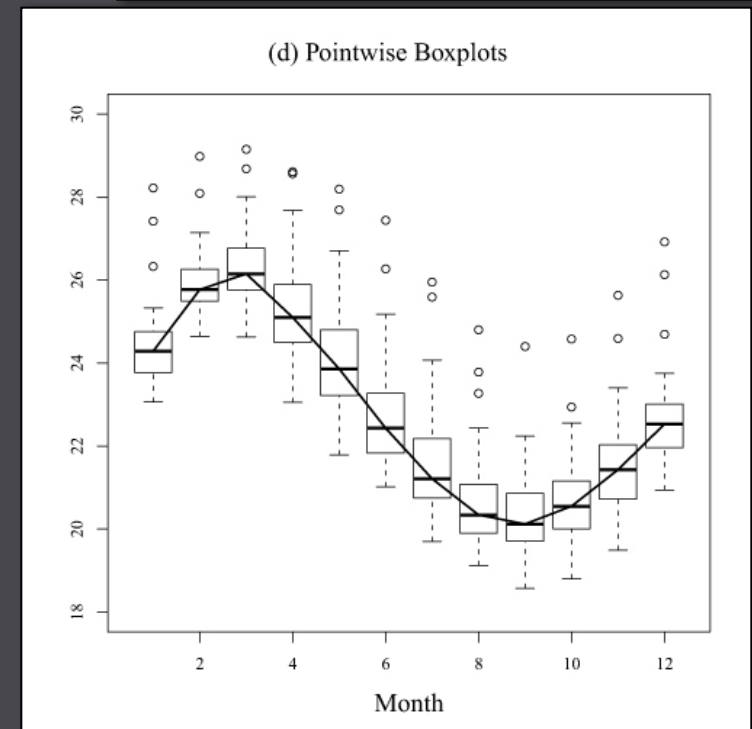
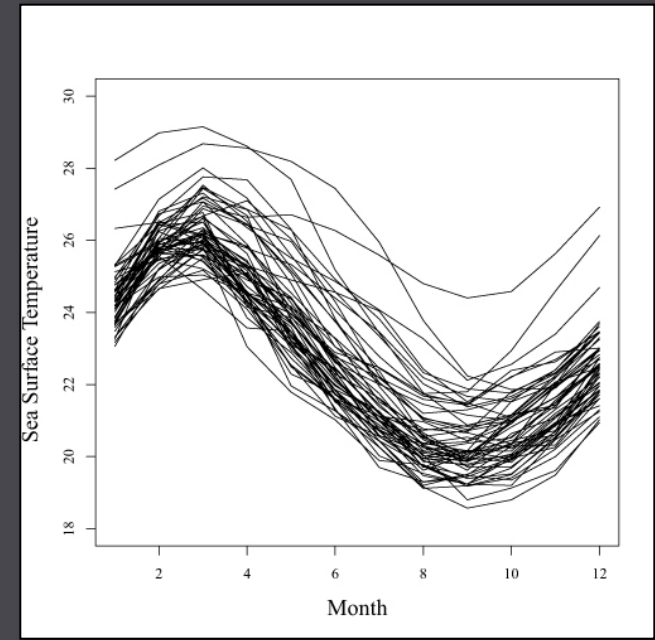
Boxplot statistics on 2D functions

defined on the function, rather than point-wise

Functional Boxplots.

Ying Sun, Marc G. Genton.

J. of Comp. and Graphical Statistics 20:2, 2011, 316-334.



Functional Box Plot

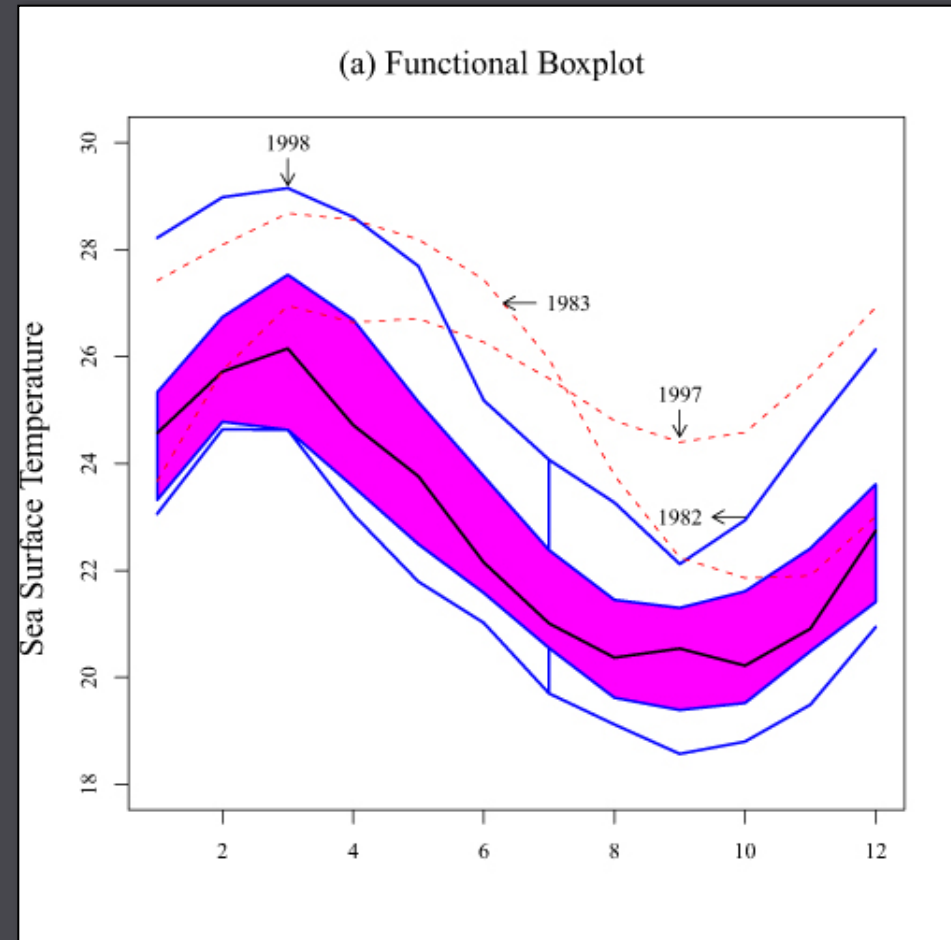
Band Depth

the amount of time a function lies within the set of functions

Functional Boxplots.

Ying Sun, Marc G. Genton.

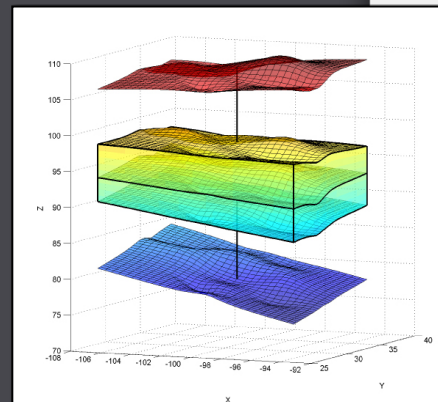
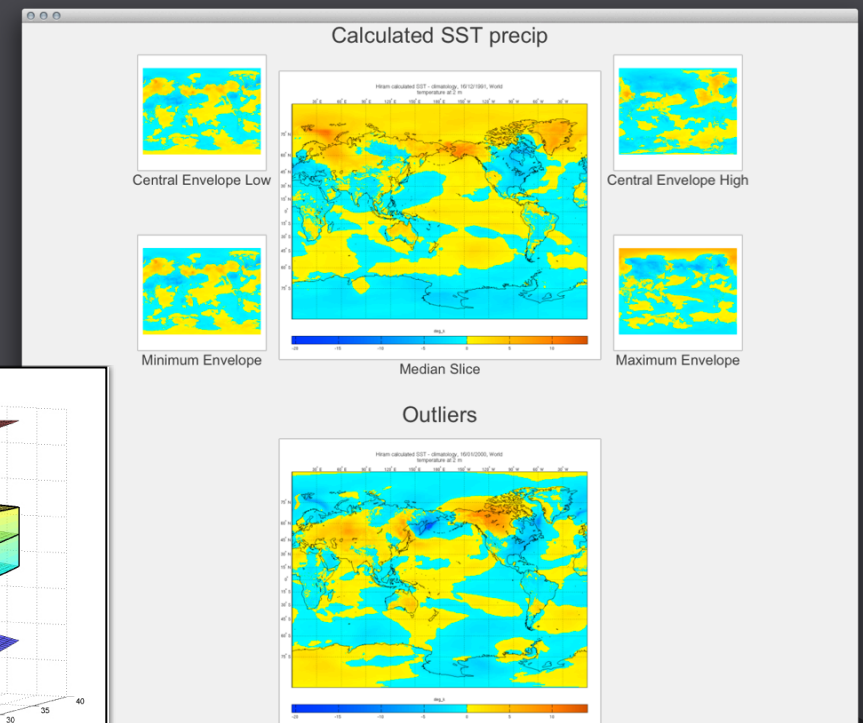
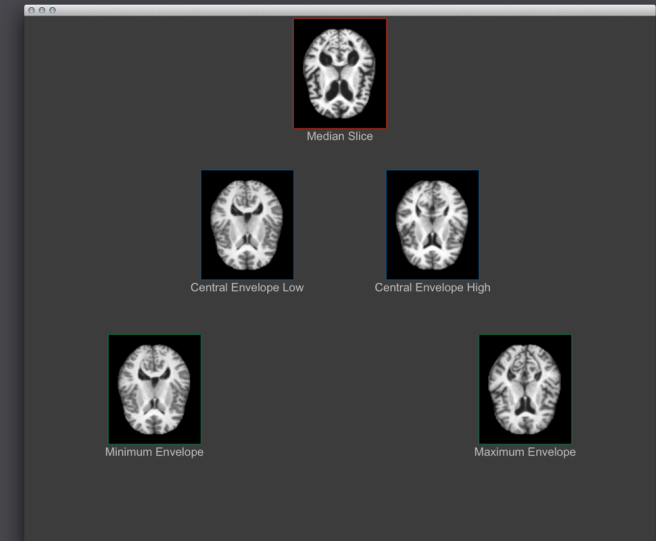
J. of Comp. and Graphical Statistics 20:2, 2011, 316-334.



Surface Box Plots

Extension to 3D

- *images rather than curves*
- *volume-based band-depth*



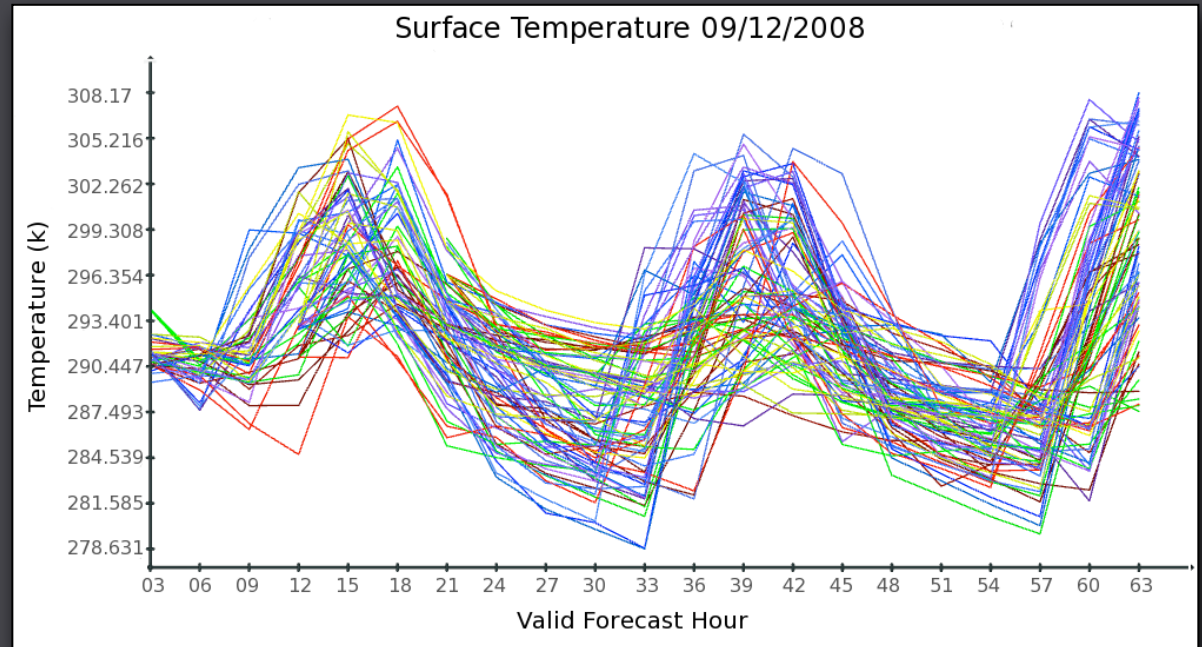
Visualization

incorporate uncertainty information into data display

Challenges

Increased complexity

- *Visual clutter*
- *Data concealment*
- *Confusion*



Information-Seeking Mantra

“Overview first, then zoom and filter,
and finally, details on demand.”

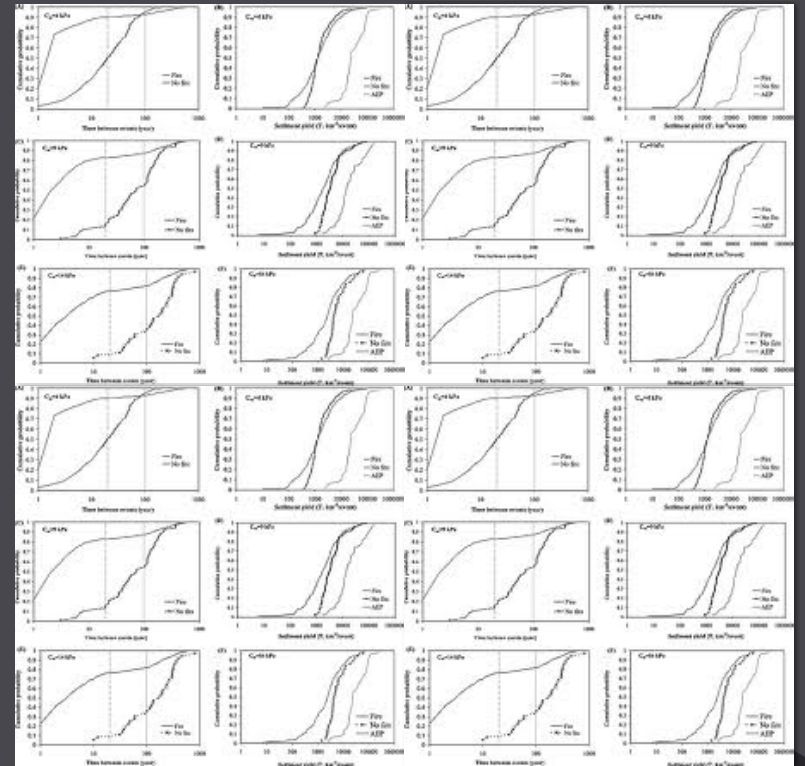
-Ben Shneiderman



Overview vs Summary

Overview

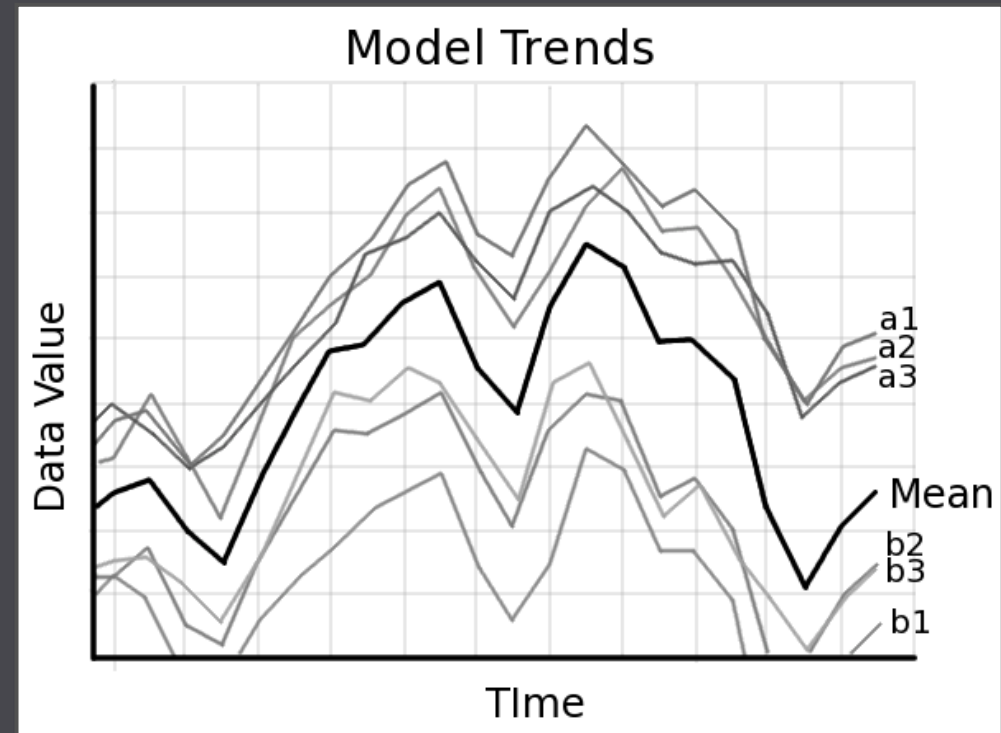
- *Show all data at once through many charts*
- *Manual search for patterns*
- *Finite screen resolution*



Overview vs Summary

Summary

- *Aggregate data along some dimension*
- *Automation summaries*
- *Have an idea of the questions*



Aggregation for Visualization

REDUCE DATA USING SUMMARIES

- *mimic human visual system*
- *done implicitly*

phenomena modeling
floating pt quantization
limited # of pixels

Aggregation for Visualization

BUT How?

- *In what dimension do we summarize?*
- *Is mean/standard deviation appropriate?*
- *Do we need multiple summarizations?*

Ex: Weather Forecast Data

Weather Forecasting

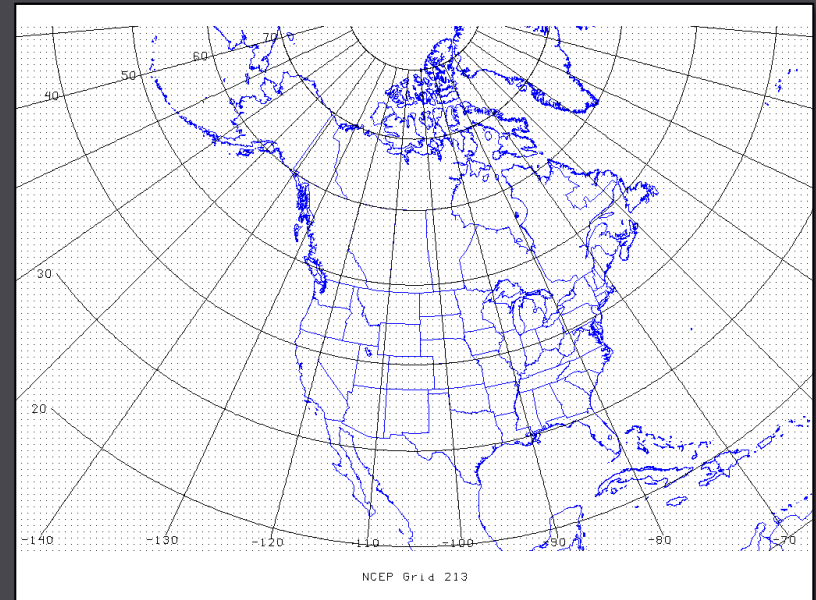
Short-Range Ensemble Forecasts (SREF)

NOAA / NCEP

- *Domain across North America*
- *Forecast weather variables out to ~3.5 days*

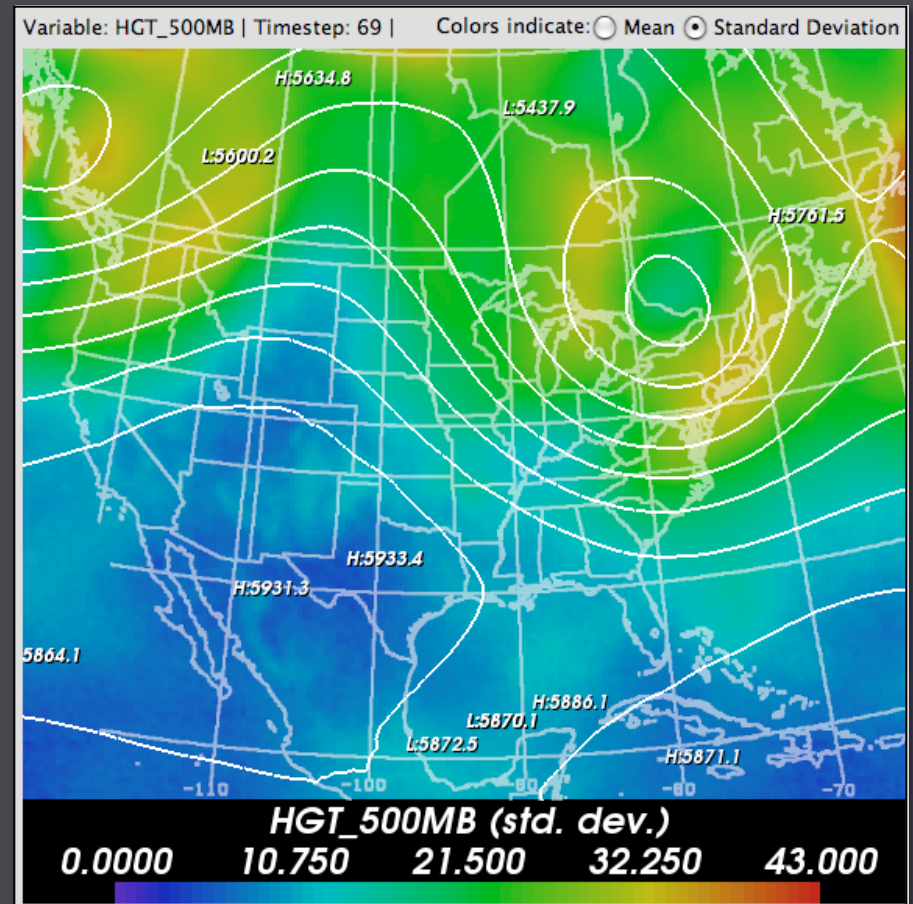
GOAL

- *Public notification, warnings, aviation*



Data

- *Study the variations of output between models*
- *Vary input parameters, initial conditions*



Ex: Forecast Data

QUESTIONS

- *what is the the weather going to be tomorrow?*
- *what information is available from this data?*

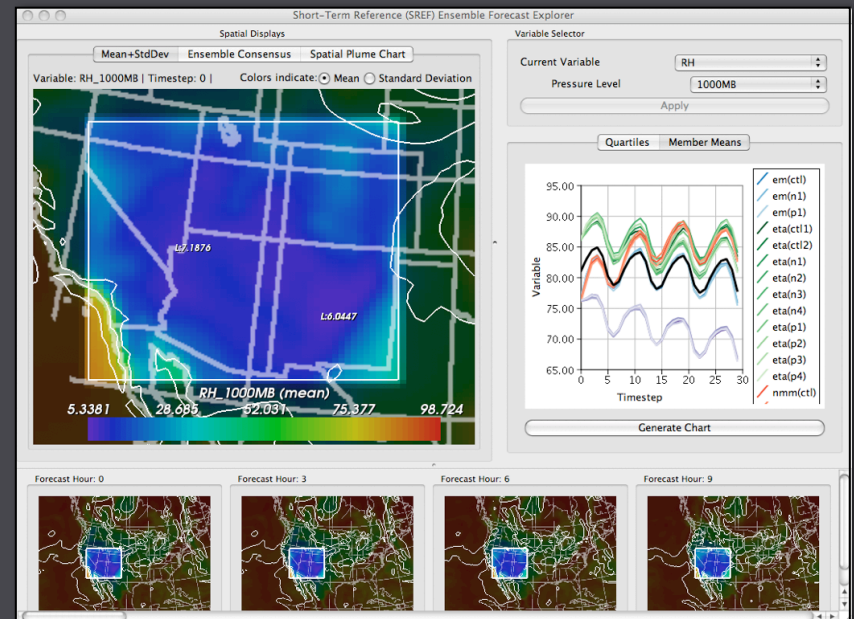
Ensemble-Vis

User-driven, component-based framework

- *Combine multiple summaries*
- *explore range of possible forecasts*
- *reveal probabilities of outcomes*
- *interrogate the ensemble*

Ensemble-Vis: A Framework for the Statistical Visualization of Ensemble Data.

K. Potter, A. Wilson, P.T. Bremer, D. Williams, C. Doutriaux, V. Pascucci, and C. R. Johnson
In IEEE Workshop on Knowledge Discovery from Climate Data: Prediction, Extremes,
and Impacts, pp. 233-240, 2009.



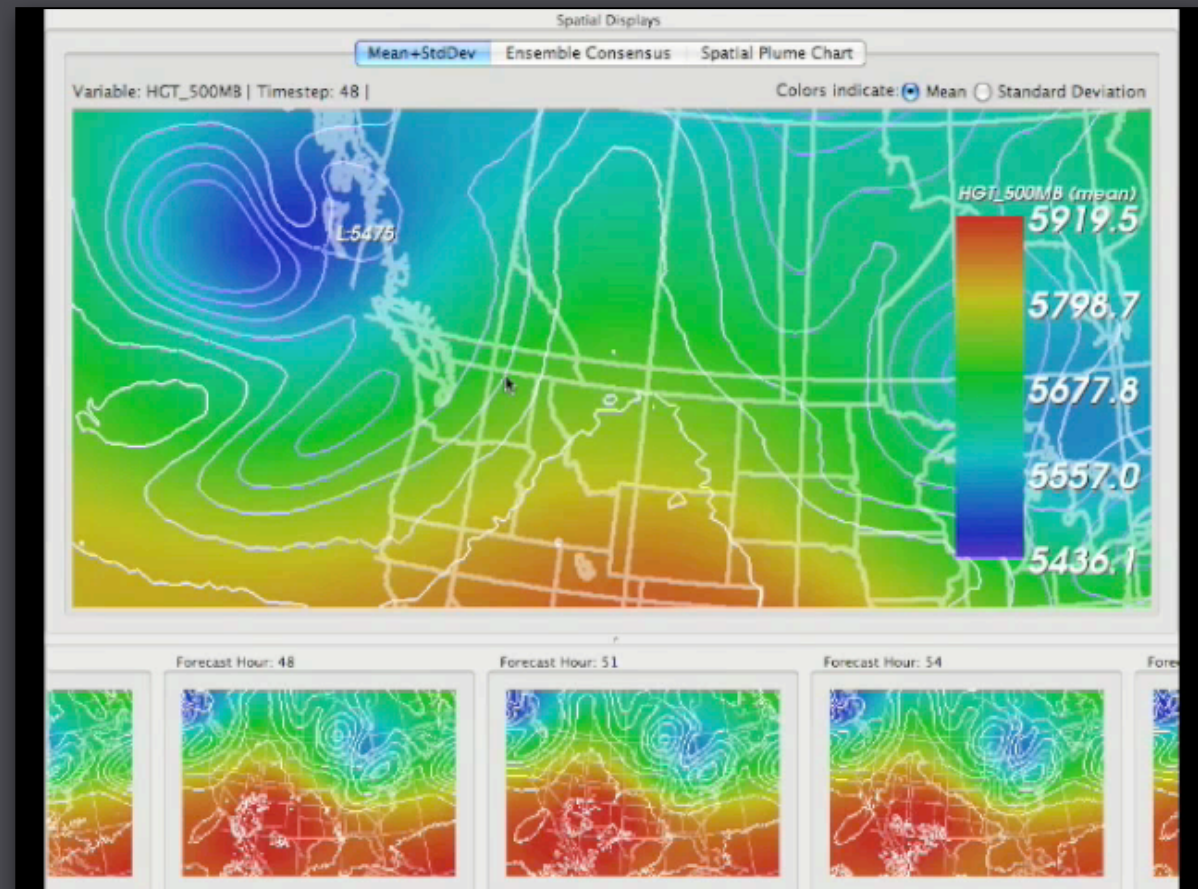
Global Summaries

QUESTIONS

- *What is the average model temperature at a given time step?*
- *Where do the models vary?*

Global Summaries

- *Aggregate over models at each grid point*
- *Colormap & contour*
- *Single timestep*



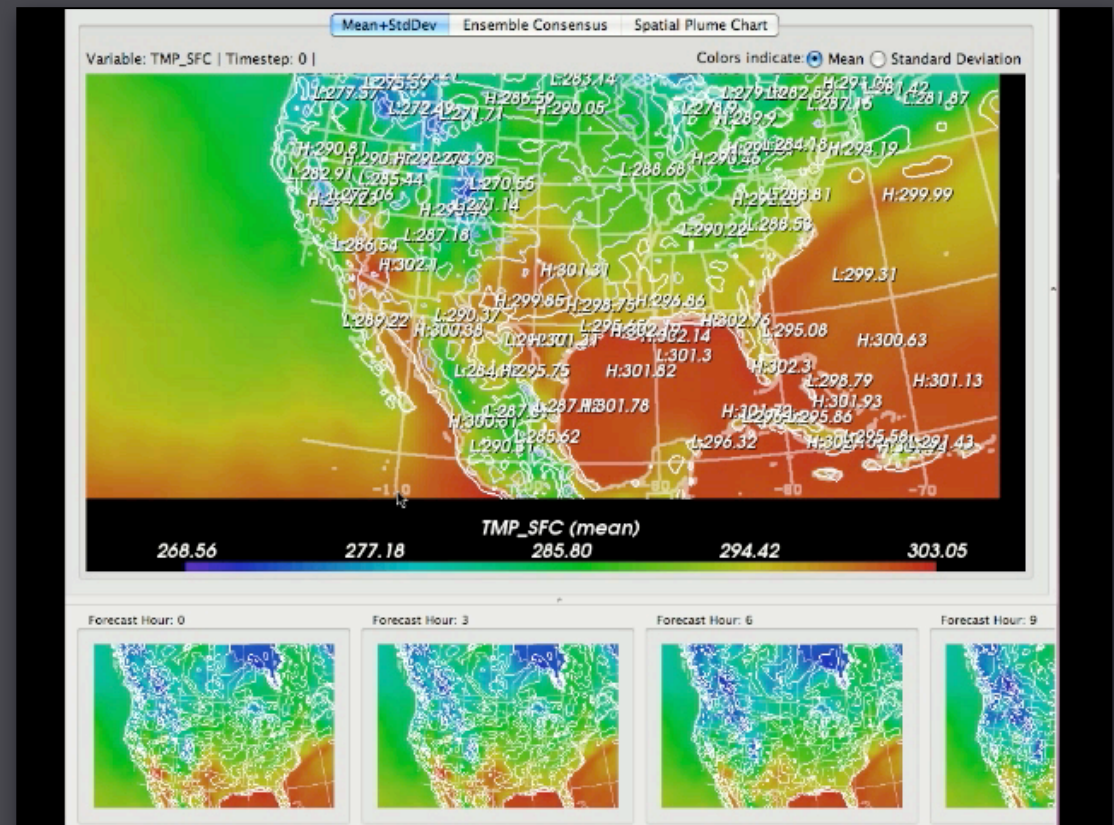
Time Summaries

QUESTIONS

- *How does the temperature change across time?*
- *What time step am I most interested in?*

Time Summaries

- *Small multiples*
- *User interaction to scroll through time*
- *Select step of interest, reflect to overview*



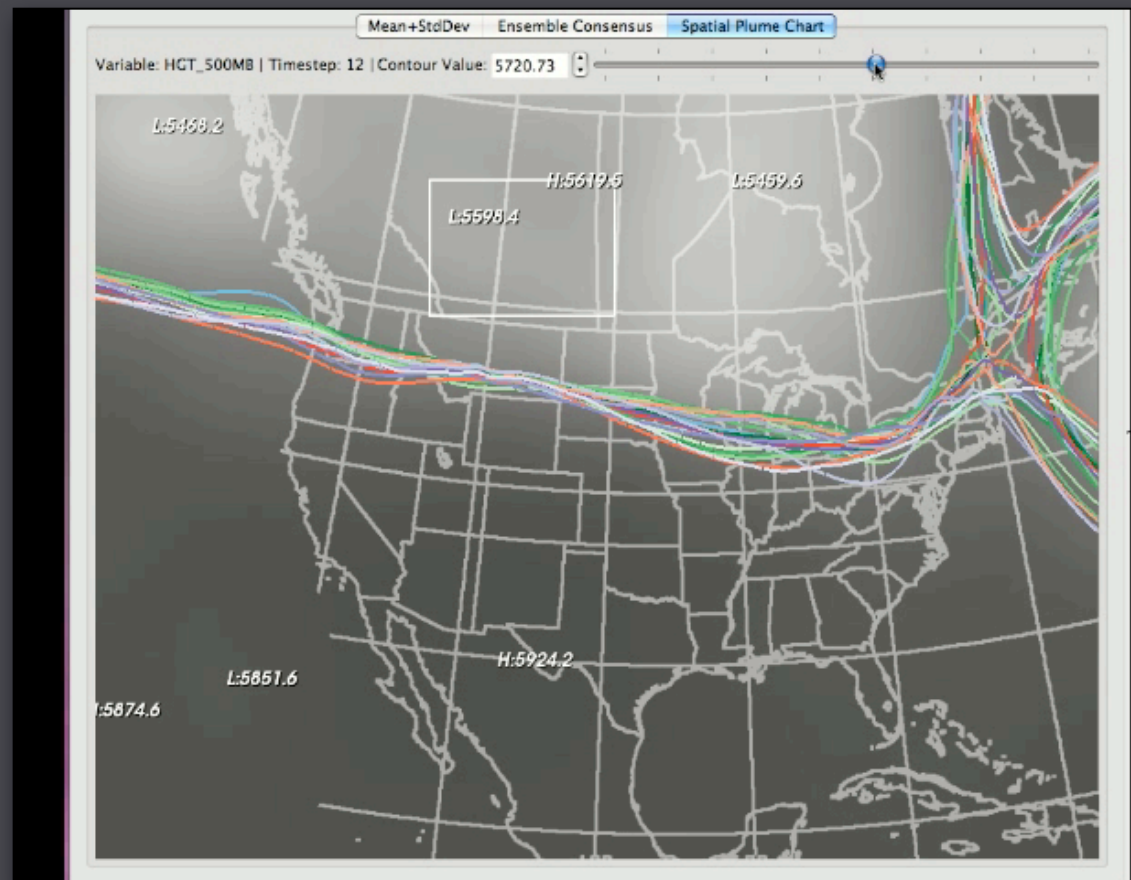
Contour Summaries

QUESTIONS

- *Where does a particular value exist for each model?*
- *Where does that value move across time?*

Contour Summaries

- *Isocontour of value across spatial domain for each model*
- *Model bundle shows variation, outliers, divergence*



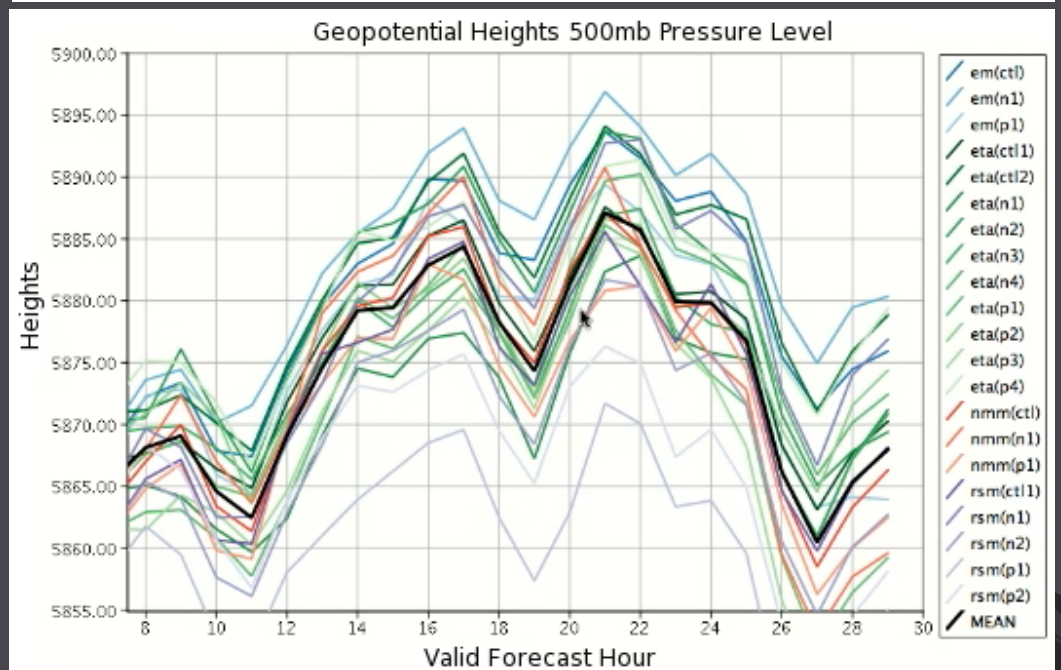
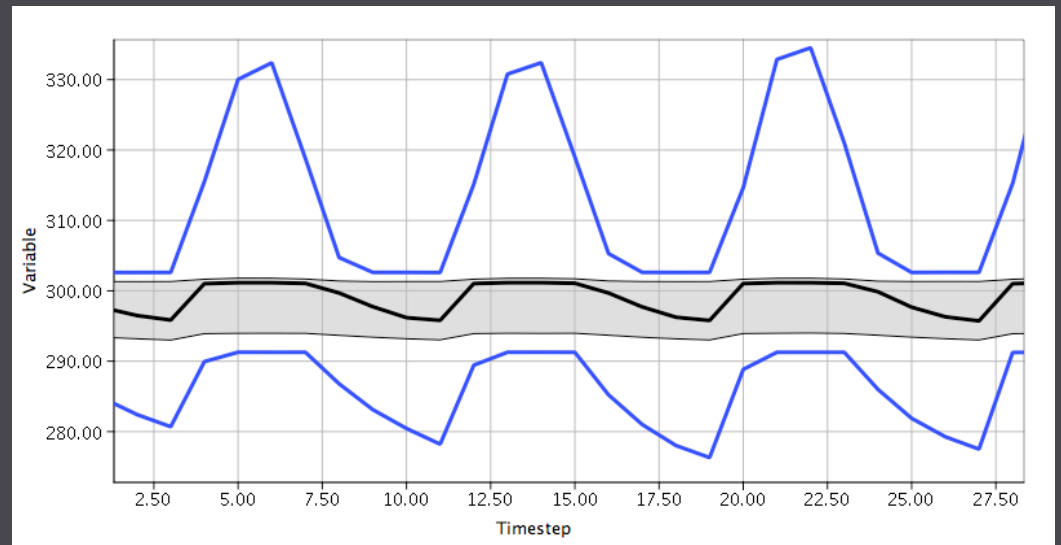
Spatial Summaries

QUESTIONS

- *What is the trend of the models over a region?*
- *What is the average trend?*

Spatial Summaries

- *summarize over user selected region*
- *average per model*
- *overall average/ boxplot statistics*



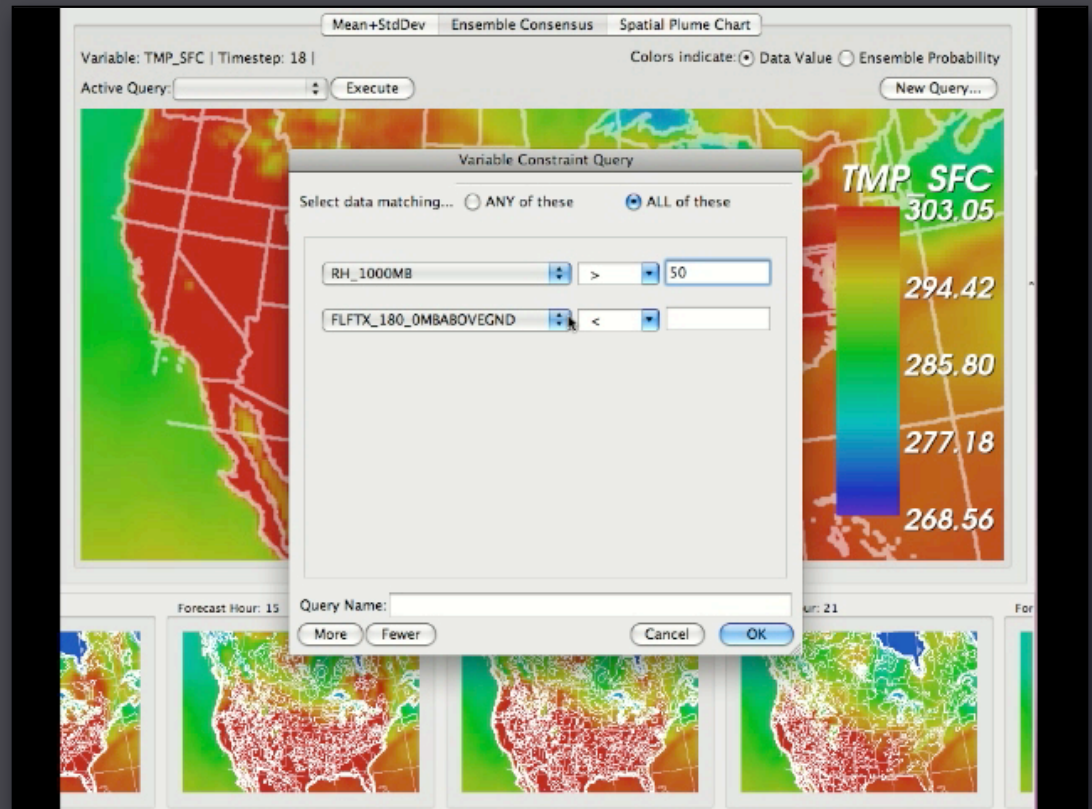
Query Summaries

QUESTIONS

- *Where does the data express particular characteristics?*
- *What fraction of the data expresses it?*

Query Summaries

- *SQL type queries to filter the data*
- *contours of ensemble fraction that predicts the condition*



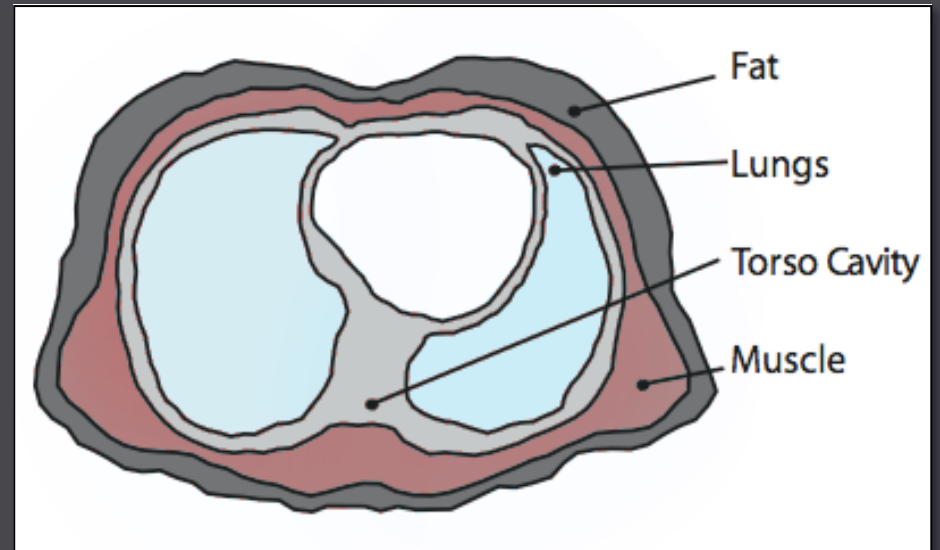
Are we asking the right questions?

Ex: Torso Potentials

Electrical Conductivity of the Heart

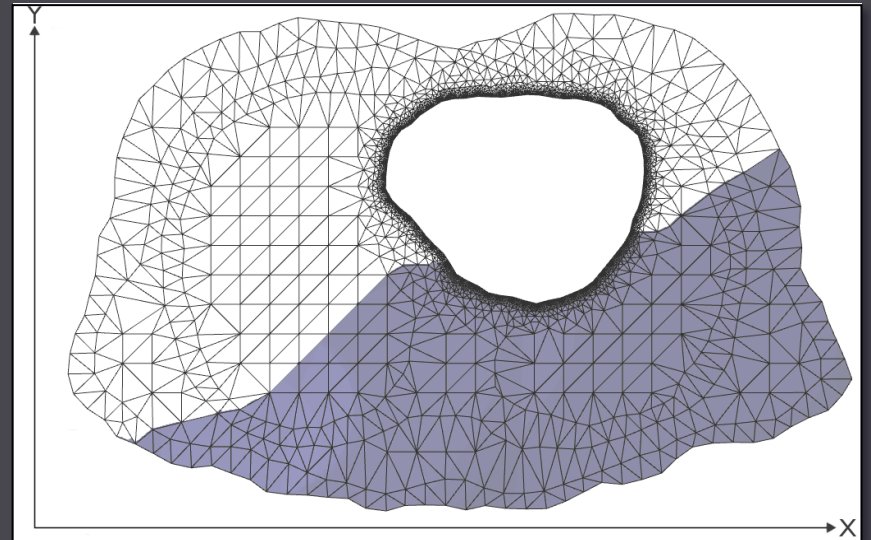
Goal

- *Simulate how signals from the heart propagate across the torso*
- *Distinguish normal changes (breathing, movement) from abnormal heart function*



Data

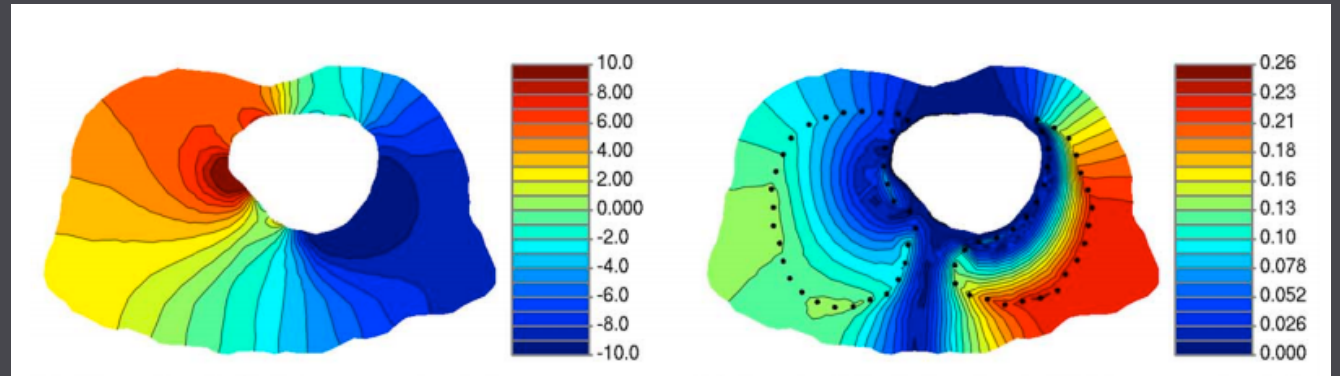
- *Study the impact of variation on input conductivity*
- *Vary lung conductivity uniformly $\pm 50\%$ from the reference*
- *10,000 realizations of potentials*



Ex: Torso Data

QUESTIONS

- *what is the average potential across the domain?*
- *what is the variation of potential across the domain?*



Mean

Standard Deviation

Global Summary

QUESTIONS

- *what is the average potential across the domain?*
- *what is the variation of potential across the domain?*

But is it meaningful?

- *are mean & standard deviation appropriate statistics?*
- *can we look at the data without individually inspecting each grid point?*

Torso Data

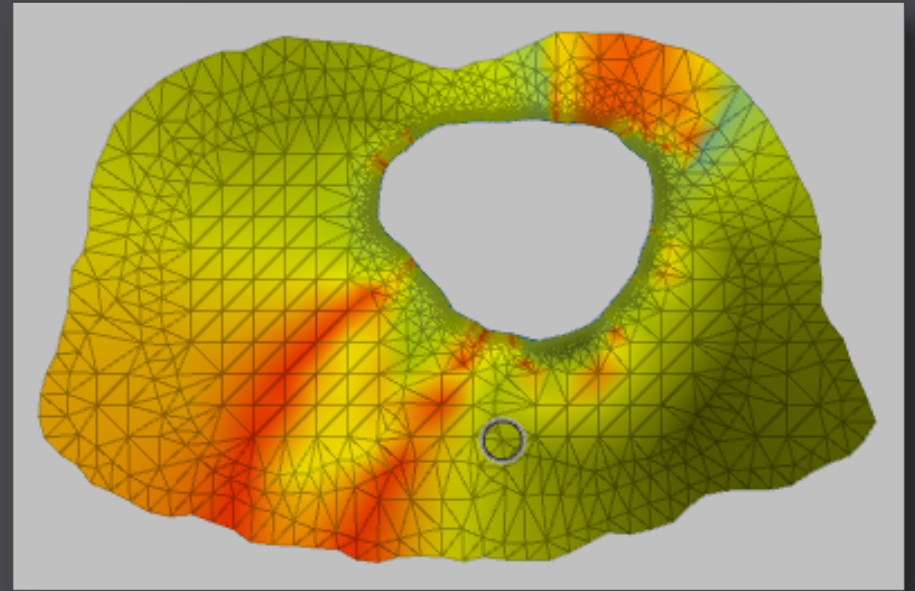
NEW QUESTIONS

- *what do the PDFs look like across the domain?*
- *where are the PDFs similar or different?*

ProbVis

VISUALIZATION & EXPLORATION FOR DISTRIBUTIONS

- *show differences between PDFs*
- *summarize all data in a single view*



Method

COMPARISON BETWEEN TWO PDFS

- *compare all data points to a single PDF*
- *get a single metric for each data point*
- *compare between points*

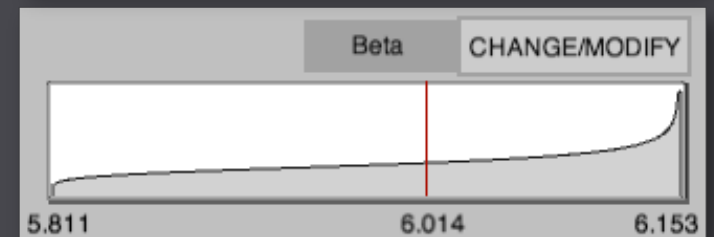
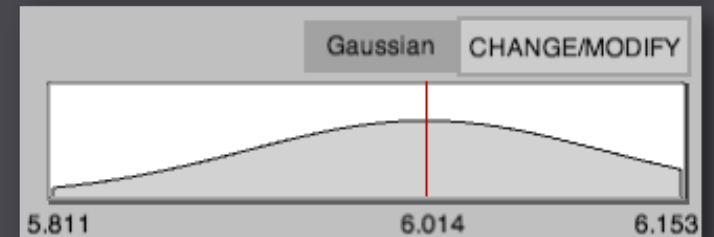
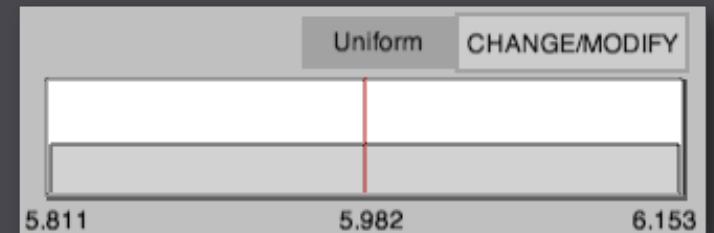
Method

LIBRARY OF CANONICAL DISTRIBUTIONS

uniform

gaussian

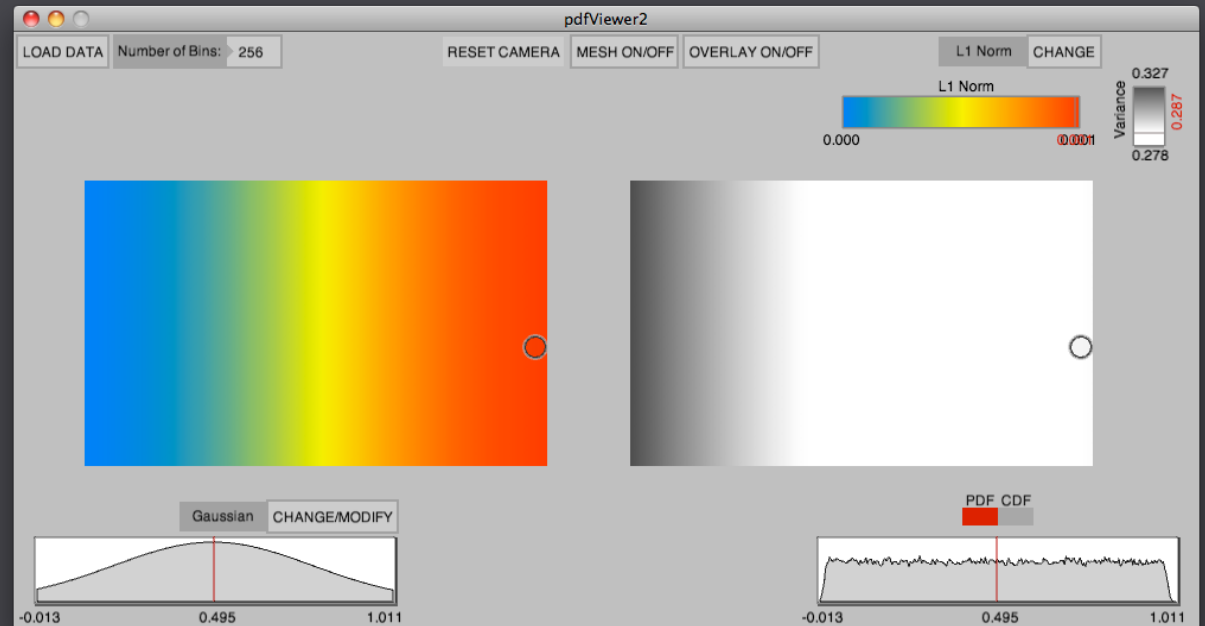
beta

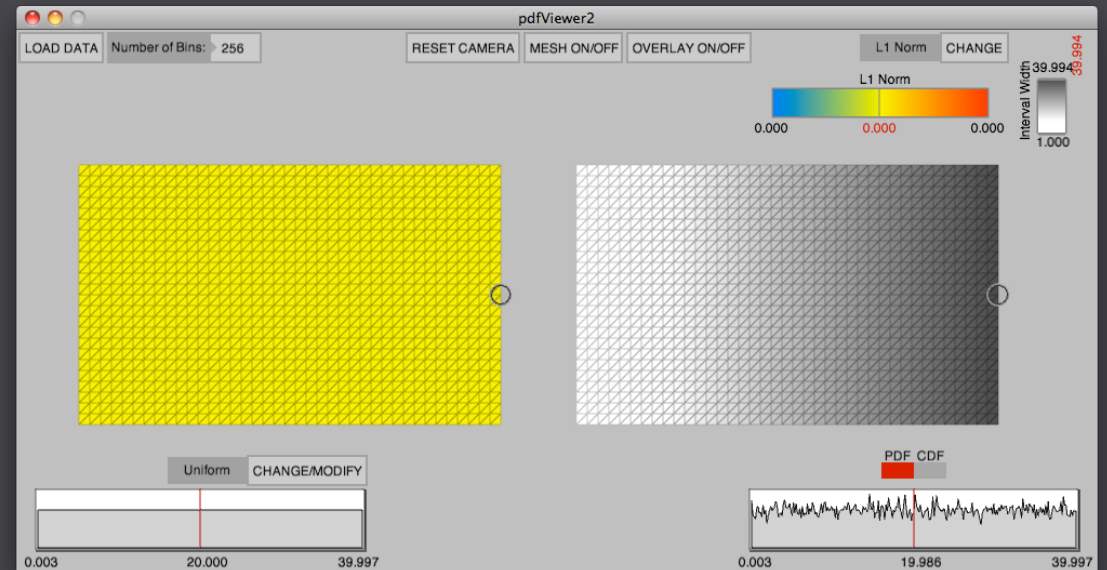


Method

DEFINE A DIFFERENCE MEASURE

shape: L1 Norm or Hellinger Distance





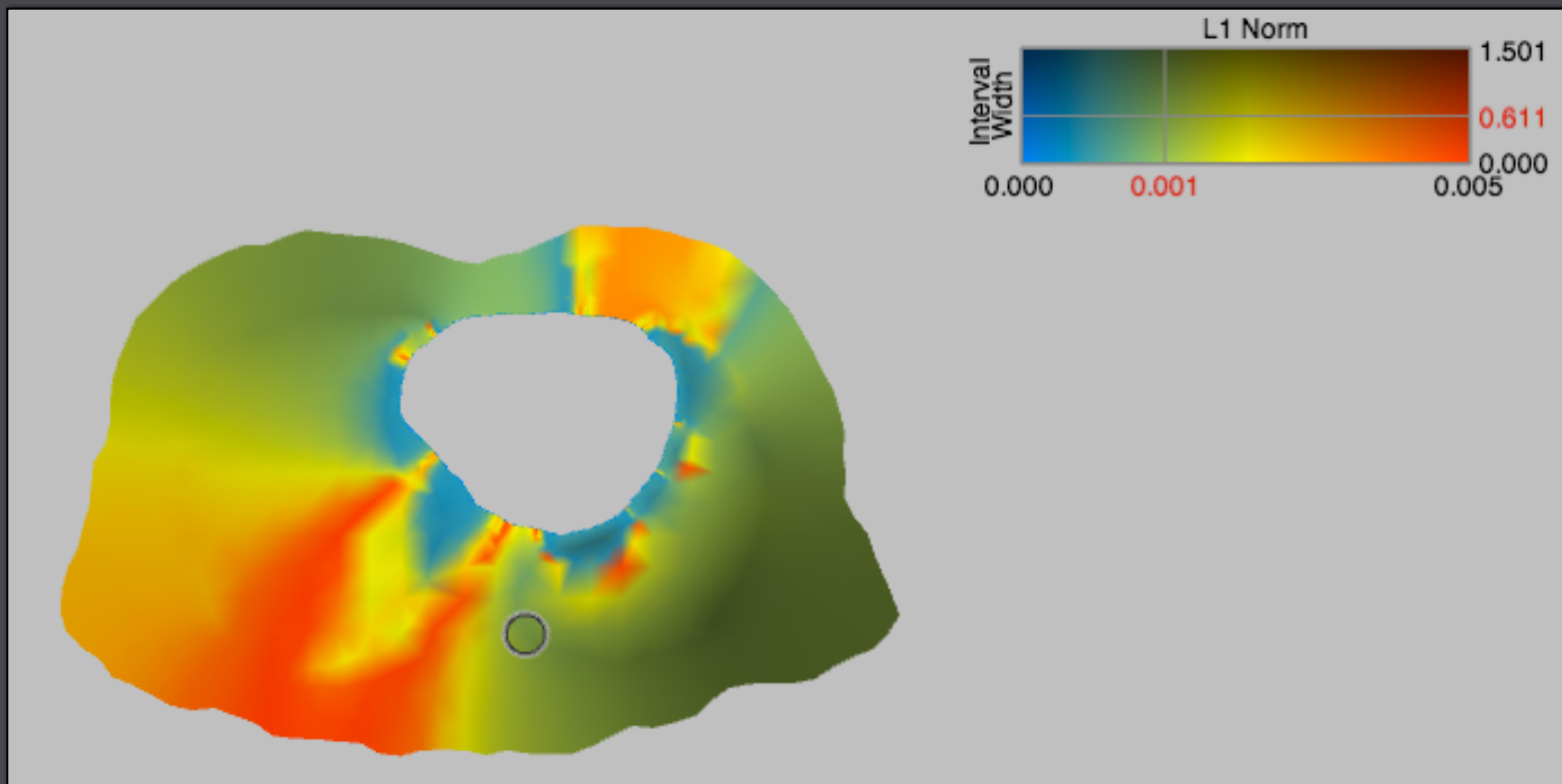
Method

DEFINE A DIFFERENCE MEASURE

interval: (range of sample values)

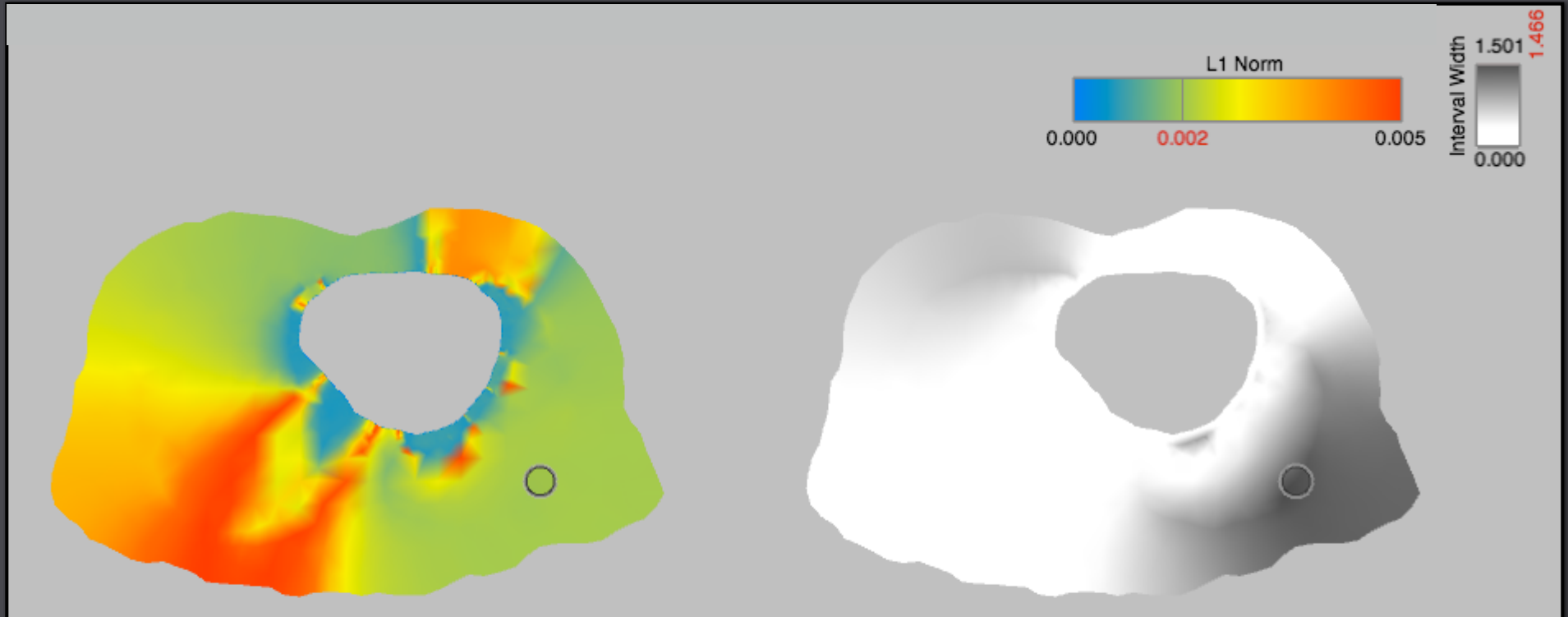
Visualization

Colormap distance measure

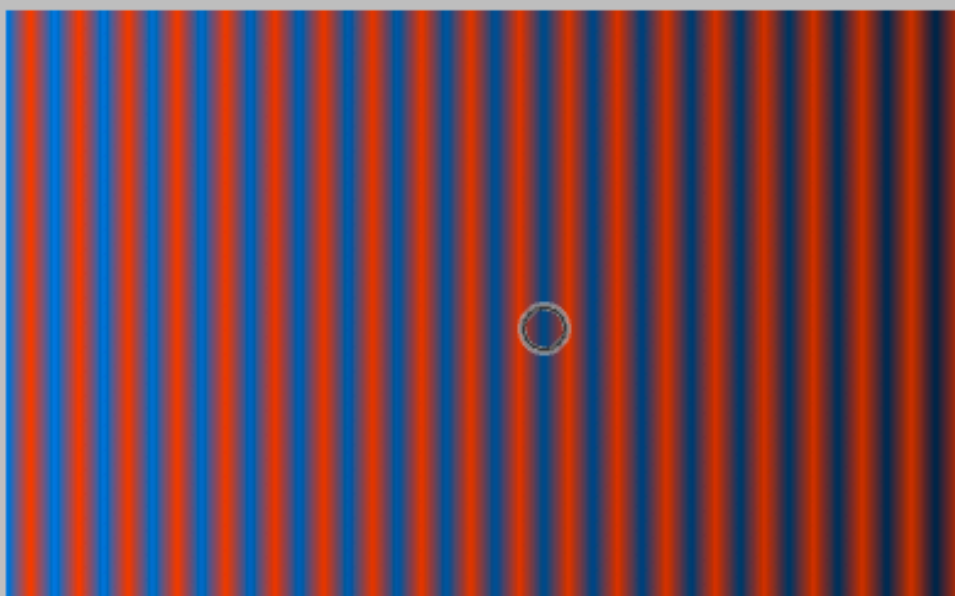
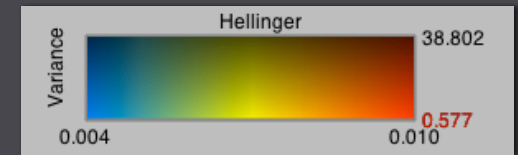


Visualization

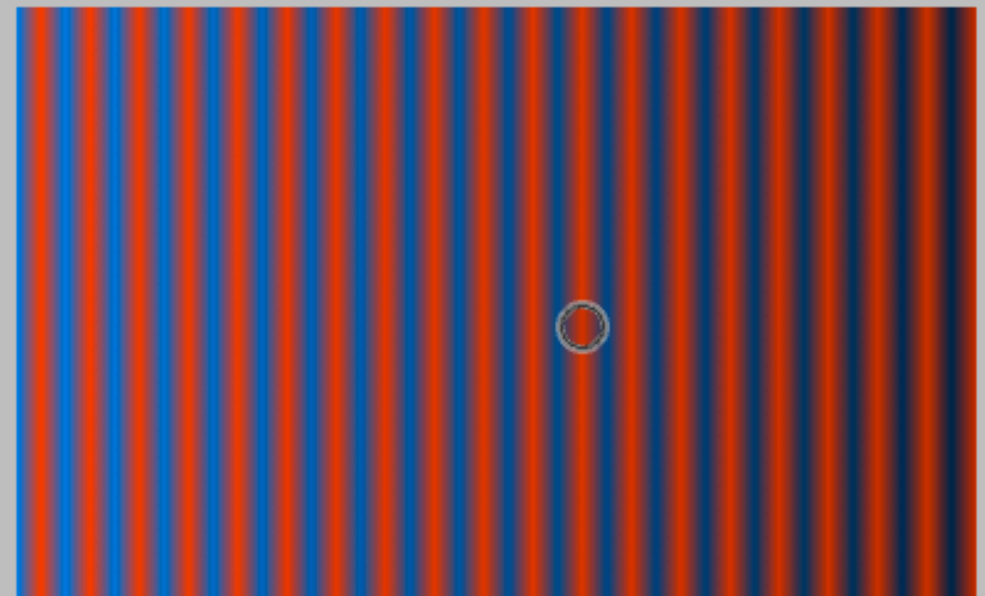
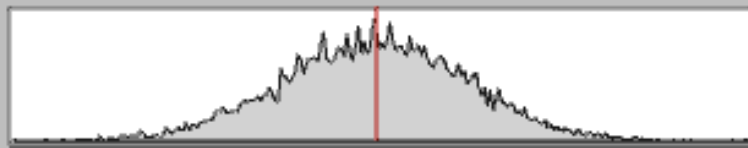
Colormap distance measure



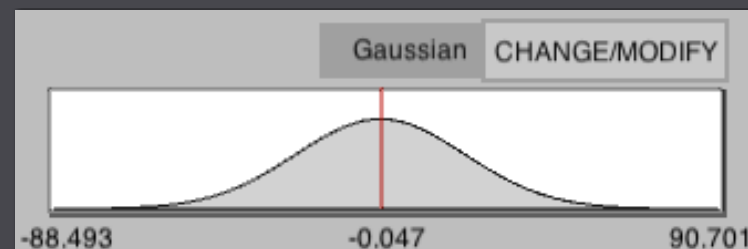
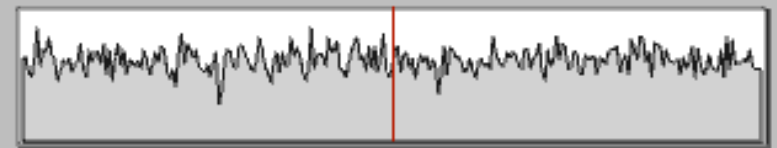
Alternating Example



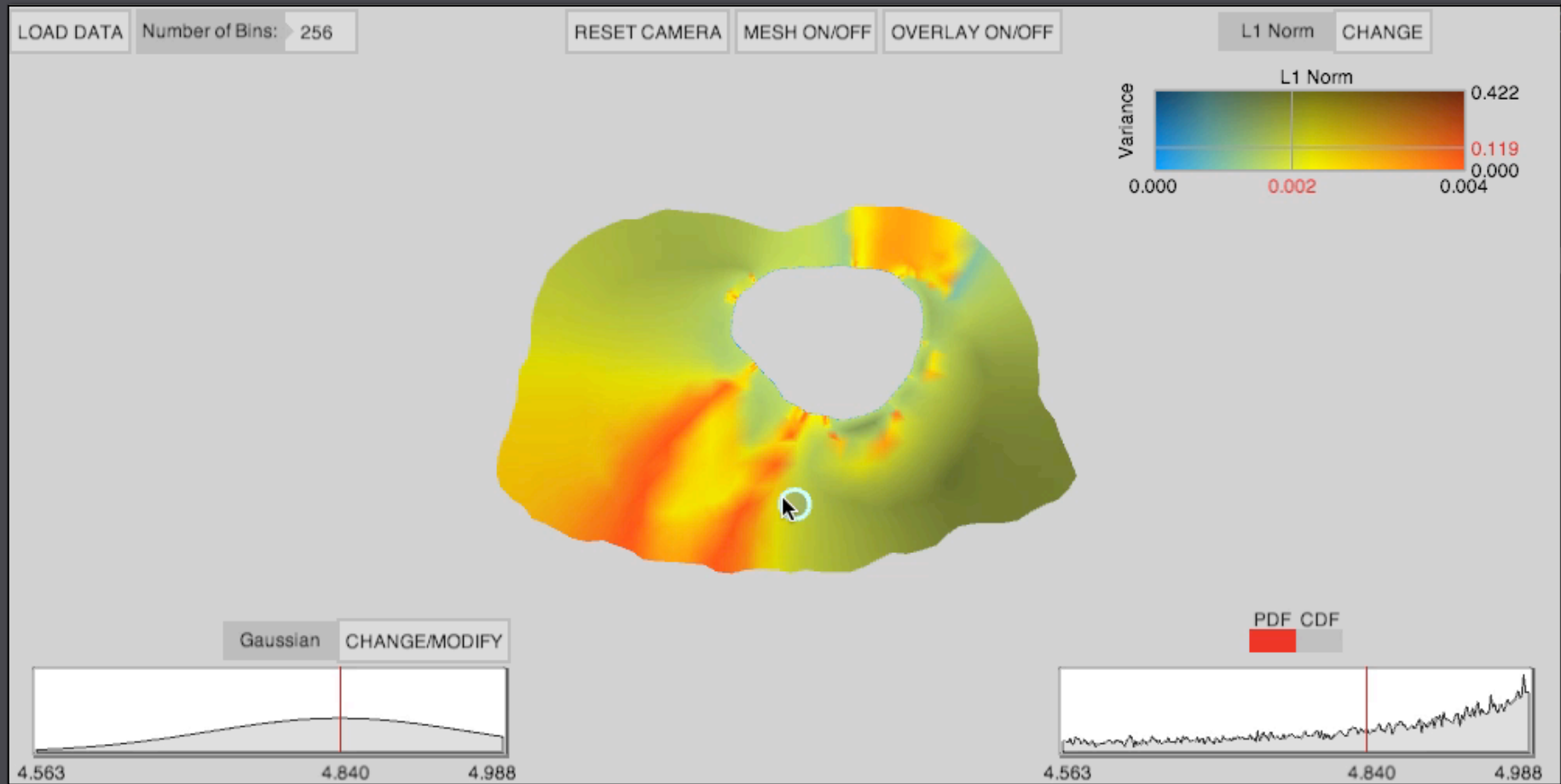
PDF CDF



PDF CDF



Interactivity



What about Categorical Data?

Ex: Brain Segmentation

DISCRETE DISTRIBUTIONS

- *11 possible tissues*
- *11 distributions, one for each tissue type, describing every voxel*
- *statistics such a mean are not defined*

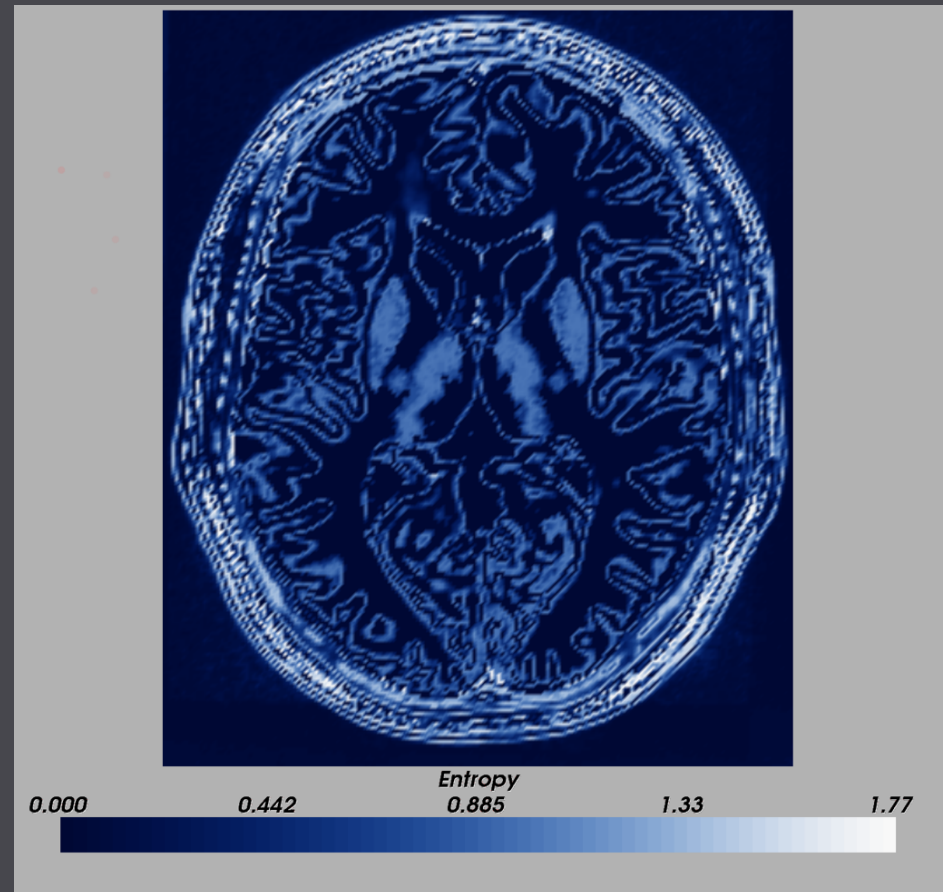
BrainWeb

www.bic.mni.mcgill.ca/brainweb.

Measures other than mean

Entropy (information theory)

- *describes the randomness of a voxel*
- *i.e. 0 entropy a tissue's type is defined*
- *higher entropy, more uncertainty*

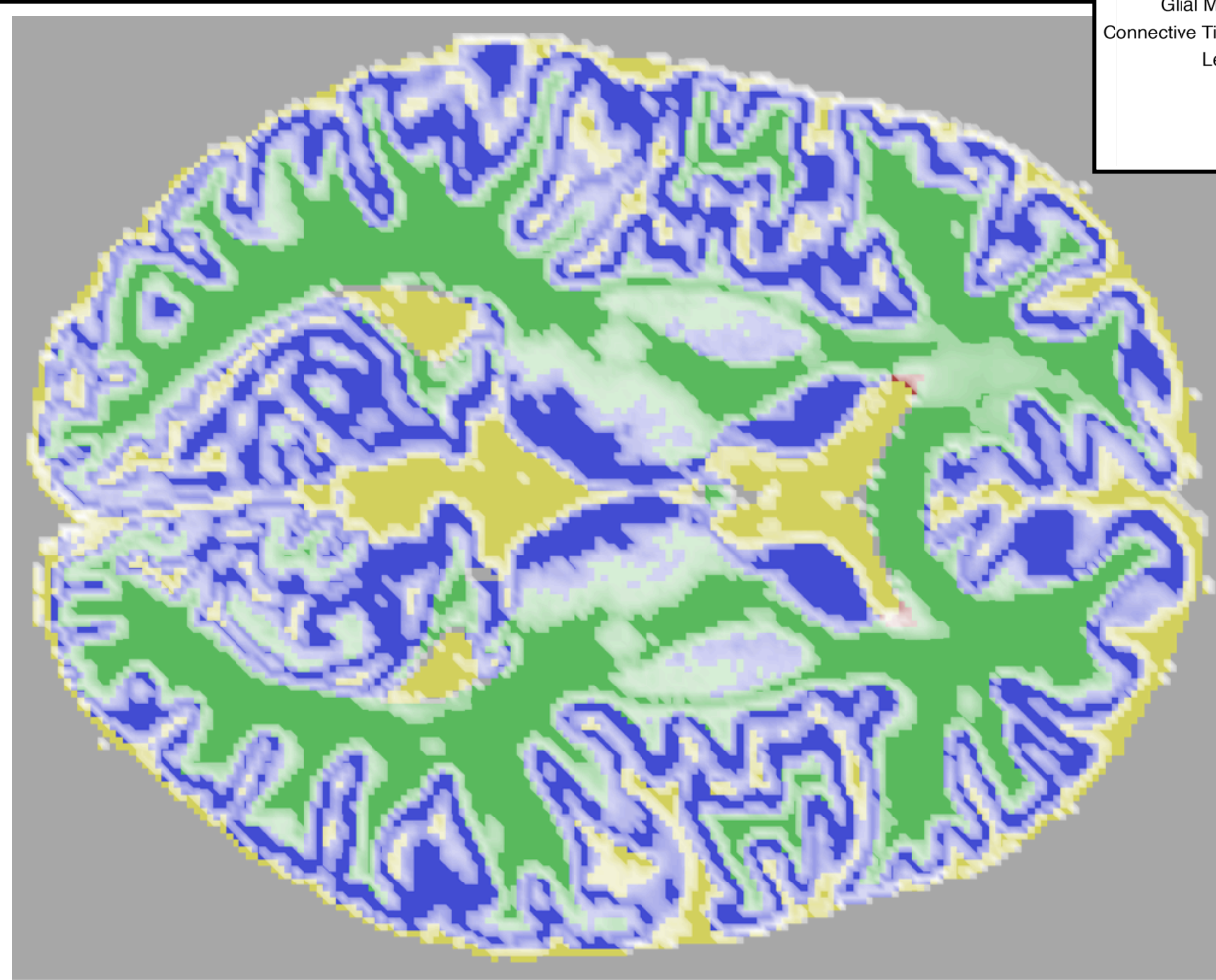
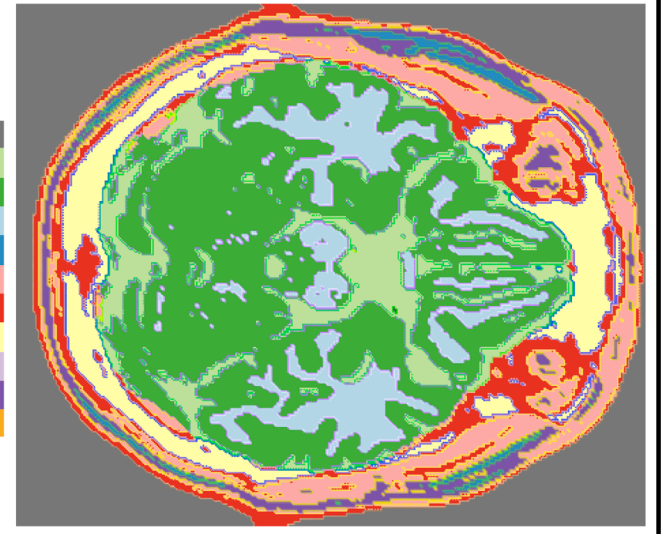


Visualization of Uncertainty without a Mean

K. Potter, S. Gerber, and E. W. Anderson. CG&A Vis Viewpoints, 2013, to appear.

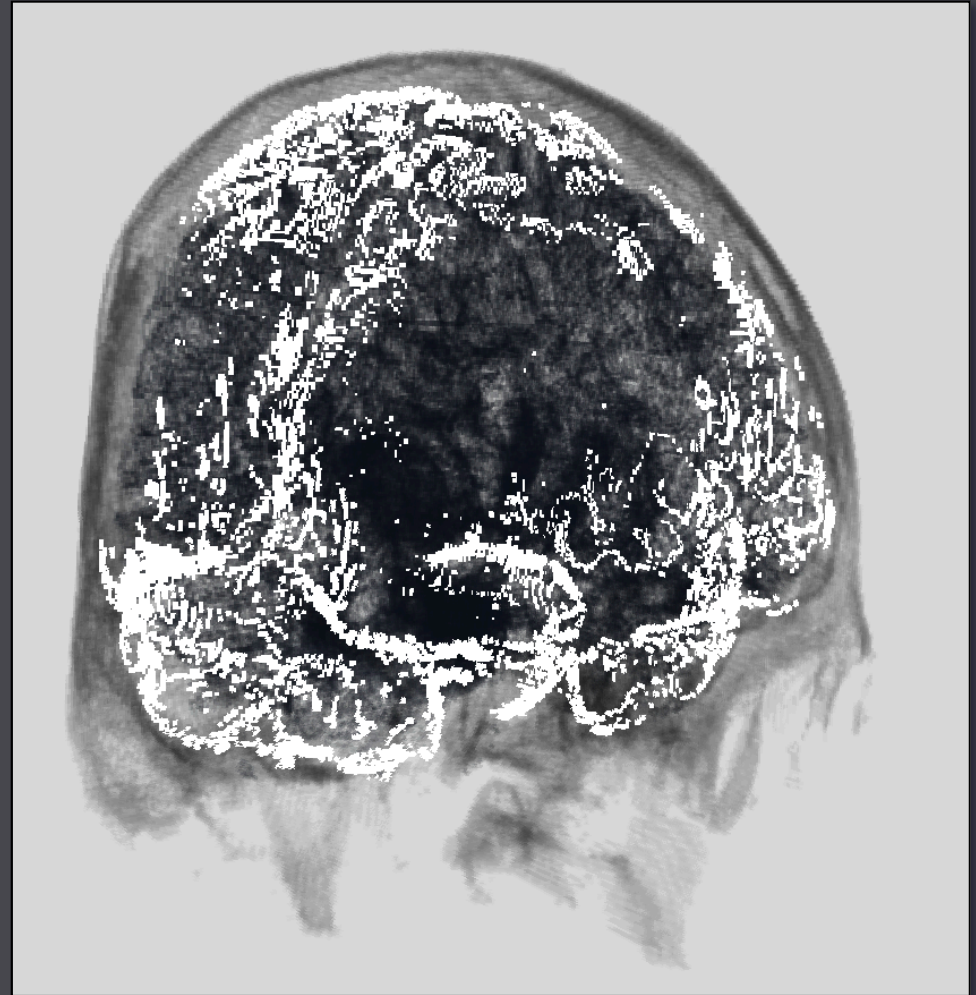
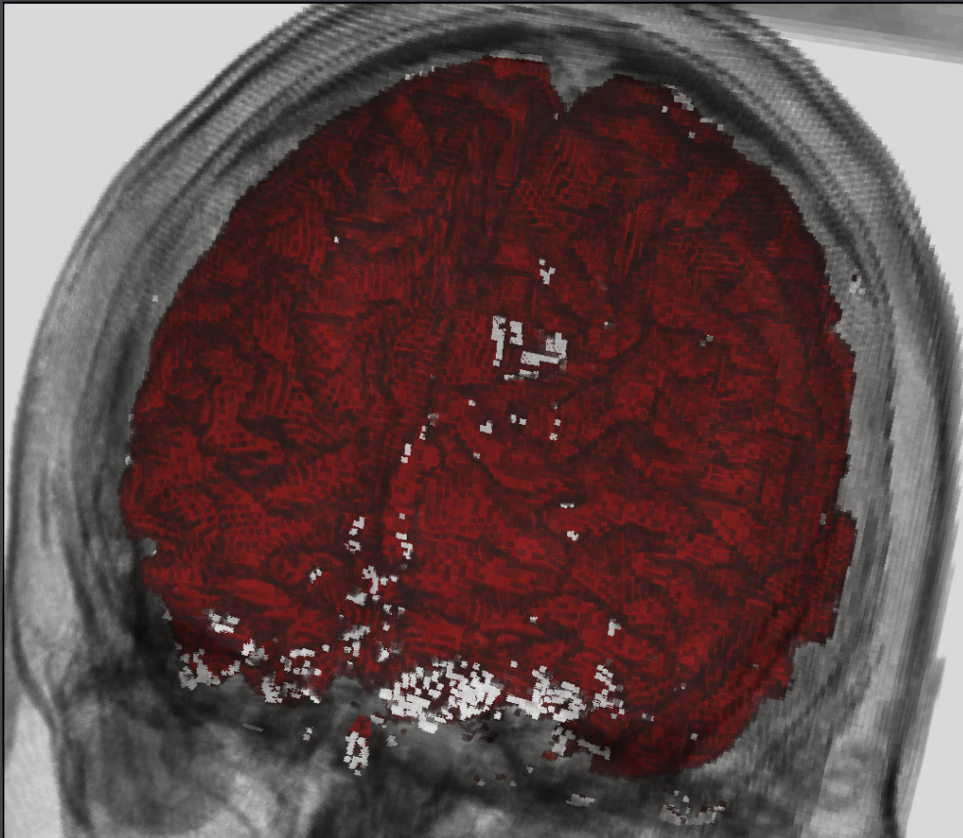
2D Display

- Background
- CSF
- Grey Matter
- White Matter
- Fat
- Muscle & Skin
- Skin
- Skull
- Glial Matter
- Connective Tissue
- Lesion



- Grey Matter
- White Matter
- CSF
- Lesion

3D Display



In Conclusion

Aggregation takes place everywhere

- *required for visualization*
- *controllable for certain questions*

Future Directions

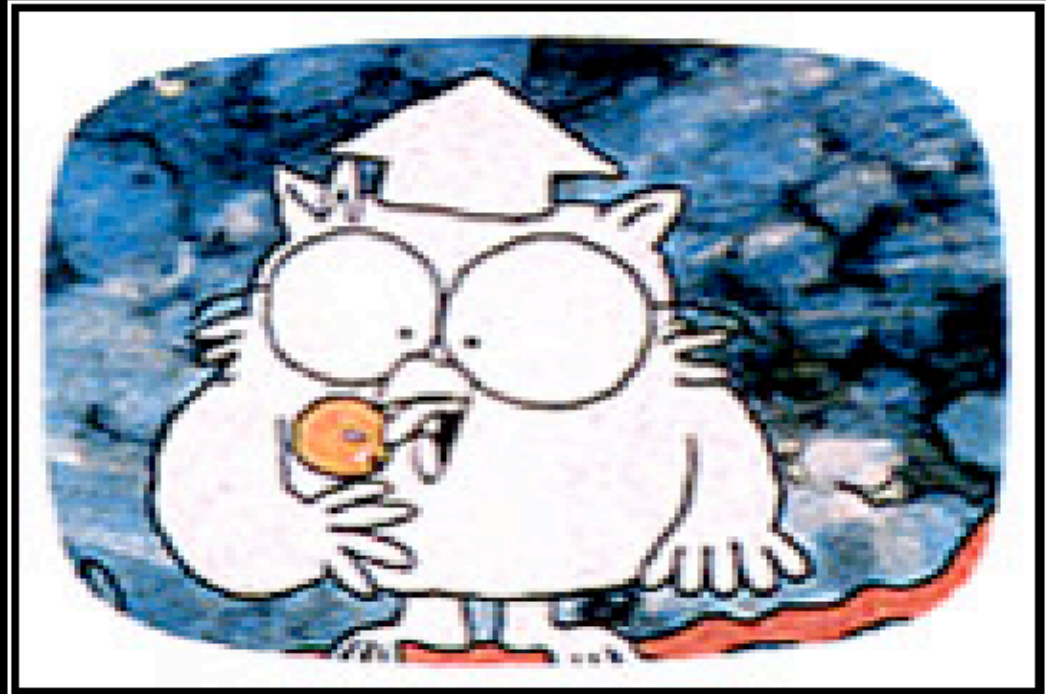
Let's keep working!

- *increase our vocabulary of uncertainty measures*
- *further the visual metaphors used for uncertainty*
- *interaction will be required for most applications*

Thanks!

QUESTIONS

- KAUST No. KUS-CI-016-04
- DOE NETL DE-EE0004449



Uncertainty

Face it kid, Not even Mr. Owl knows how many licks it takes.

<http://tinyurl.com/8hwtzr>