From Quantification to Visualization: A Taxonomy of Uncertainty Visualization Approaches

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Abstract. Quantifying uncertainty is an increasingly important topic across many domains. The uncertainties present in data come with many diverse representations having originated from a wide variety of domains. Communicating these uncertainties is a task often left to visualization without clear connection between the quantification and visualization. In this paper, we first identify frequently occurring types of uncertainty. Second, we connect those uncertainty representations to ones commonly used in visualization. We then look at various approaches to visualizing this uncertainty by partitioning the work based on the dimensionality of the data and the dimensionality of the uncertainty. We also discuss noteworthy exceptions to our taxonomy along with future research directions for the uncertainty visualization community.

Keywords: uncertainty visualization

1 Introduction

In the past few years, quantifying uncertainty has become an increasingly important research area, especially in regard to computational science and engineering applications. Just as we need to quantify simulation accuracy and uncertainty, we must also convey uncertainty information, often through visualization. As the number of techniques for visualizing uncertainty grows, the broadening scope of uses and applications can make classifying uncertainty visualization techniques difficult. Uncertainty is often defined, quantified, and expressed using models specific to individual application domains. In visualization however, we are limited in the number of visual channels (3D position, color, texture, opacity, etc.) available for representing the data. Thus, when moving from quantified uncertainty to visualized uncertainty, we often simplify the uncertainty to make it fit into the available visual representations. In this paper, we identifying traditional types of uncertainty quantification and reduce them representations that are familiar to uncertainty visualization researchers. We then give an overview of different uncertainty visualization approaches targeted at these uncertainty representations. We then further differentiate them based on the dimensionality of the data and the dimensionality of the uncertainty. We also discuss a few noteworthy exceptions to our taxonomy. Our main goal is to position previous work on uncertainty visualization within the scope of uncertainty quantification in order to better connect the two.

2 Quantifying Uncertainty

To begin a discussion of uncertainty quantification, we must first define uncertainty into two overall, broad types: epistemic and aleatoric. *Epistemic* uncertainty describes uncertainties due to lack of knowledge and limited data which could, in principle, be known, but in practice are not. Such uncertainties are introduced through deficient measurements, poor models, or missing data. Quantification and characterization of epistemic uncertainty aims to better understand the underlying processes of the system and use methods such as fuzzy logic. *Aleatoric* uncertainty is defined as uncertainties that arise from, for example, running an experiment and getting slightly different results each time. This type of uncertainty is the random uncertainty inherent to the problem and cannot be reduced or removed by things such as model improvements or increases in measurement accuracy. Aleatory uncertainty can be characterized statistically and is often represented as a probability distribution function (PDF). The visualization of uncertainty focuses enhancing data understanding by unlocking and communicating the known aleatoric uncertainties present within data.

According to the NIST report on evaluating and expressing uncertainty [88], aleatoric uncertainty can be classified into two groups: type A and type B. While the distinction between the two classes may not always be apparent, they can be described as type A uncertainties arising from a "random" effect, whereas type B uncertainties arise from a "systematic" effect, where the former can give rise to a possible random error in the current measurement process and the latter gives rise to a possible systematic error in the current measurement process. The main difference between these two is in the evaluation of the uncertainties. Type A evaluation may be based on any valid statistical measure. However, the evaluation of type B is based on scientific judgment that will use all relevant information available, which often can include statistical reasoning.

While these classifications are important to note, and often have great impact on the quantification of uncertainty, their impact lessens when moving from quantification to visualization. The most straightforward understanding of uncertainty is often the easiest to expose visually, and thus uncertainty within the field of visualization is often thought of as type A - that is entirely statistically defined. Thus, unless otherwise noted, all of the papers in this taxonomy deal with statistically quantifiable uncertainty.

3 From Quantification to Visualization

The growing need to understand the effects of errors, randomness, and other unknowns within systems has lead to the recent upswing in research on uncertainty quantification. This growing body of work is creating an array of definitions of uncertainty differing in not only the mathematical measures defining uncertainty, but also in the way the uncertainty is expressed and used. These differences are often most apparent when crossing boundaries between scientific fields, but can also arise within the same field through various sources including data acquisition, transformation, sampling, interpolation, quantization, and visualization [70]. While understanding the measurement and propagation of uncertainty throughout a workflow pipeline is very challenging for quantifying the overall uncertainty of a system, this complexity can be prohibitive for visualization.

Using visualization as a tool for understanding leverages the high bandwidth of the human visual system, allowing for the fast understanding of large amounts of data. However our visual channels can be overwhelmed when increasing the amount and dimensionality of the data. For computational science applications, visualizing time-dependent, three-dimensional scalar, vector, or tensor field data is often the goal. However, depending on the complexity of the underlying geometry, such visual representations can suffer from problems such as occlusion, which may require user interaction to relieve. Even two-dimensional displays can suffer from visual clutter and overload leading to ineffective visualizations. Thus, regardless of additional uncertainty information, the visualization of data alone can be difficult to visually display in an effective way.

Adding uncertainty information is not only challenging in the design of the visual abstraction, it is also difficult to fully express the complexity of the uncertainty itself. While typically expressed as a PDF, very few visualization approaches can directly display this function, and those that can are restricted to 1D or limited 2D. Thus, to visualize the uncertainty, some type of assumptions are typically imposed on the data in order to reduce it to a manageable size or dimension. This is most often done by aggregating the uncertainty into a single value, such as standard deviation or defining an interval along which the value could possibly lie. This reduces the uncertainty to one or two values, which considerably eases its visual expression. However, this can often misrepresent characteristics of the actual data as mean and standard deviation often imply a normal distribution whereas an interval can be interpreted as uniform.

For visualization, these types of assumptions are often accepted since there are not yet readily available visual abstracts to address non normal distributions nor visual representations of high dimensional PDFs. It is very important to understand that these problems exist and that beyond the uncertainties associated with the data, there also exist uncertainties in the visualization - both in the technical mechanisms used to create the visual presentation, but also in the perception of the visualization itself. A handful of approaches have looked at exposing these assumptions by presenting information on the underlying PDF, however this greatly increases the complexity of the visualization, and most work to date uses a simplified view of uncertainty.

4 Taxonomy for Visualization Approaches

	Uncertainty Dimensionality		
Data Dim.	**Scalar**	**Vector**	**Tensor**
1D	[62, 77, 85][82]		
2D	[7, 13, 14, 22, 27, 30, 31, 34, 43, 45, 44, 49, 53, 51, 56, 60, 63, 69, 72, 78, 79, 77, 76, 83, 91, 95][16, 17, 28, 82]	[8, 9, 33, 53, 56, 64, 67, 65, 92, 97]	
3D	[12, 20, 19, 18, 42, 46, 47, 50, 54, 55, 59, 58, 71–73, 75, 80, 81, 86, 87, 93, 96] [15, 82, 61]	[5, 50, 53, 52, 68, 92]	[11, 35, 37, 41]
ND	[2, 23, 26, 32, 90]		

Table 1: Our taxonomy of uncertainty visualization approaches. Cells in light yellow represent categories with no known work. Citations in green refers to work with an emphasis on evaluation.

A wide array of taxonomies and typologies exist to help understand the field of uncertainty visualization. One of the first taxonomies to address uncertainty visualization separates methods by data type (scalar, multivariate, vector, and tensor) and visualization form (discrete and continuous) and proposes appropriate visual representations for each combination [70]. Skeels et al. [84] create a classification for information visualization which organizes the type of uncertainty by what it is trying to describe as well as commonalities between types and discusses exemplary visualizations for each type. Uncertainty has been a major theme in the area of geographic and information systems (GIS) and typologies have been created to focus on geospatial information visualization in the context of intelligence analysis [57, 89]. In contrast to these previous works, we differentiate our taxonomy by focusing on presenting the to-date uncertainty visualization approaches in as simple of a form as possible. We categorize approaches by two qualities: data dimension and data uncertainty dimension, and discuss the various visualization approaches based on these two categories.

4.1 Data Dimension

The data dimension is the most obvious of the categorization attributes. This is the dimension that the data lives in and may, or may not be the dimension that the visualization exists, or that the uncertainty is quantified. From a mathematical standpoint, this typically refers to the range of the function. For example, we have a computational science simulation that uses a model characterized by input parameters. The range refers to the output space of the simulation, which in many instances is spatial. This is the typical viewpoint for 1D-3D spatial data dimensions, however when moving to ND the interest may move to understanding the relationship between the parameter space, or domain, or of the function and the output. These types of questions are often answered by parameter-space studies and are treated in this work as ND.

1D A one dimensional data dimension can be thought of as a single variable at a single point such that the uncertainty describes the variation or possible values of that single data value. This type of data is rarely found alone, we typically see it expressed as bar charts where each bar expresses a single independent variable, however the collection of bars may have some relationship - for example populations of countries. Here, a bar chart will have a bar for each country, however there is no intrinsic relationship between country population values. Thus, the data can be represented by a single 1D PDF and any higher-dimensional aspect of the data is implied, rather than intrinsic to the data.

2D In contrast to 1D data, 2D can have a number of possibilities when it comes to the interpretation of the data. The data may be a 2D PDF, meaning a PDF defined over two variables, in which case the data is truly multivariate and can often be simplified to two distinct 1D PDFs. Alternatively, we can think of the data as having a 2D spatial domain, in which every location across the domain has a 1D PDF. This can be interpreted as a collection of 1D PDFs in space, or alternatively as a series of realizations across the 2D space where a single surface is made up of a sample from each of the PDFs. The term "ensemble" often comes up in this context and refers to the collection of output realizations, but may also include the particular parameter set associated with each ensemble member.

3D Similarly to 2D, 3D data in general refers to a variable defined across a spatial volume where a single PDF exists at each position within the volume. In contrast to much of the work in 2D, 3D often deals with spatial positioning and boundaries rather than variable value across the space.

ND Non-spatial, multivariate, and time-varying data is the final category of data dimension. The most often seen example of ND data is the addition of time, which can be added to 1,2, or 3D. Alternatively, ND data can refer to high-dimensional data often seen in parameter space explorations. In this case, there exists a parameter-space in R^M which maps to the target space in R^N. The M parameters can be modeled in some way such that an understanding of the relationship between the parameters and target is gained. While this relationship is often quite complex, the resulting data set is simply a set of realizations of the target space, and thus a collection of 1D PDFs across N and measures of uncertainty can be imposed on those 1D PDFs. While it can be the case that the dimension of the target space N is limited to 1, 2, or 3D, we are distinguishing work focusing on parameter-space uncertainties because the uncertainty in these works are more often focused on simulations that do not necessarily have a constrained dimension - they assume the target dimension as N and thus the actual value of N is irrelevant. Finally, multivariate data considers many variables simultaneously. This type of data may often be viewed as a collection of 1D data sets, unless there exists an inherent relationship between the variables. For our categorization, we reserve the ND classification to work which deals specifically with high-dimensional studies, for multivariate data which cannot be reduced to a collection of 1D variables, or for time-varying work which is separated from the lower-dimensional work by more than just animation.

4.2 Uncertainty Dimension

The uncertainty dimension refers to the dimensionality across which the uncertainty is quantified. This can often be a different dimension than the data. For example, many data sets attach a single value, i.e. a scalar, to points in 1D, 2D, or beyond. The uncertainty represented by these scalar values is still a 1D PDF. The data uncertainty dimension includes the categories of scalar, vector, and tensor representations.

Scalar The term *scalar* typically refers to a single data value. For uncertainty, we can think of the term *scalar uncertainty* as the uncertainty associated with a scalar variable. For a scalar variable, we define the scalar uncertainty as a 1D PDF.

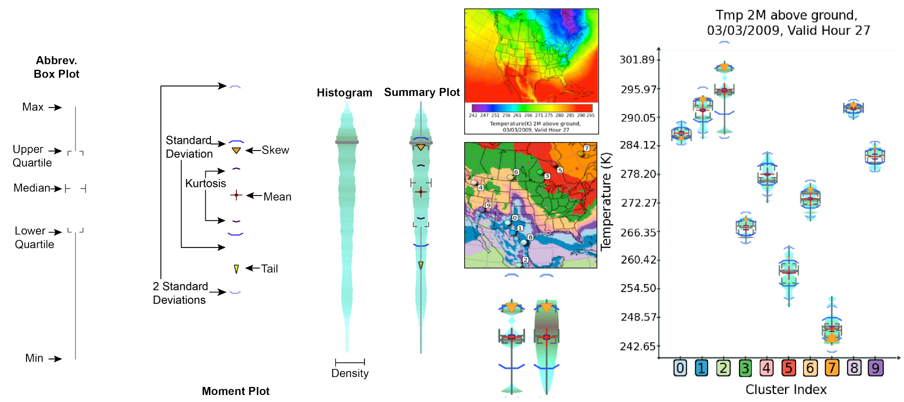
Vector A vector is usually thought of as consisting of two quantities, such as a magnitude and direction, defined over a grid and often changing with time. The uncertainty typically investigated using vectors looks at the quantities not as precise values, but rather random variables, which can be characterized as PDFs. These PDFs are influenced locally by noise, measurement and simulation errors, uncertain parameters, boundary and initial conditions, and inherent randomness due to turbulence.

Tensor Tensors are data types that define linear relationships between values for any dimensionality. While scalars and vectors are both technically low-order tensor data, we differentiate our discussion of tensors to be higher-order tensors only. These approaches do not visualize the tensors directly, but instead visualize some derived representation. For example, in [36], the authors visualize uncertainty in white matter tract reconstruction based upon ensembles of orientation distribution functions from diffusion tensor images.

5 Scalar Field Uncertainty

5.1 1D Data

As mentioned in Section 4.1, we typically see uncertainty in 1D scalar field data expressed as error bars or boxplots [74] in charts and graphs. These mechanisms typically show the expected value along with a range of possibilities. While this often may be enough to express the “unknownness” of the value, both of these techniques can be misleading by implying a normal or Gaussian distribution. Most work in 1D scalar fields have been on trying to express the actual distribution of the variable, in order to get away from the assumption of a specific distribution and more accurately express the uncertainty. An example of visualizing non-Gaussian distributions comes from work on dealing with bounded uncertainty. This type of uncertainty is defined as an interval in which the actual data value lies. To express this visually, rather than having a line for the expected value and the range with error bars, the entire interval is depicted as fuzzy [62]. Thus, there is no line for expected value and the user can clearly see the location of where the data may lie. This “ambiguation” can be used for graphs and charts with an absolute scale, such as bar charts, and can also be applied to absolute scale charts such as pie charts. The expression of bounded uncertainty can be thought of as displaying a uniform distribution within the



(a) (b)

Fig. 1: Construction (a) and application (b) of the Summary Plot used by Potter et al. [77]. The Summary Plot highlights the variation of a distribution from normal by combining three glyph-based plots of statistical characteristics of the data. Similar to error bars, the application of the Summary Plot must be constrained to individual 1D points to avoid overwhelming visual clutter. Here (b), a clustering technique is used to select regions of interest for further exploration using the Summary Plot.

range, as each value within the range is equally likely, and values outside of the range are not possible. A similar idea is to express more characteristics of the data in order to fully express the distribution. Potter et al. [77] use this idea by expressing higher-order statistics of the distribution. As seen in Figure 1, the *summary plot* shows not only the traditional box plot (abbreviated to reduce visual clutter) but also a histogram which shows an approximation of the probability distribution function, and a glyph-based moment "signature" which shows the mean, standard deviations, skew, kurtosis, and tail. This hybrid plot allows for a better understanding of the distribution underlying the uncertainty and quickly shows the non-normal behaviour of the data. These characteristics of 1D uncertainty are also present in tabular data where the cell value may be interpreted as average, estimated, possible, or likely. These terms express different understandings of the value, and may or may not be statistically grounded. To show the difference between these meanings, different line types are used to plot the value. For example, a dashed line is used for *estimated* and *possible* is expressed by widening the line to cover all valid values [85]. A few visualization methods have been developed to explore the characteristics of the PDFs underlying the uncertainty in 2D scalar fields. Most of these techniques employ some sort of dimensionality reduction or abstraction because, even as a low resolution grid, having a PDF at every point leads to too much visual complexity. Clustering is a common technique for grouping similar things. Bordoli et al. [7] use clustering techniques to group similar PDFs across the 2D spatial domain or to group 2D realizations. In a similar manner, Kao et al. [43, 44] use pixel-wise or feature-wise summaries to reduce the data to groups. Difference measures have been developed to compare a collection of PDFs against each other [76] to show the differences or similarities between them (shown in Figure 2b), and a defined set of operators has been used to reduce the distributions down to scalars [56]. It should be noted that the interpretation of the data as a set of 1D PDFs is an approximation and that linear interpolation between the points across the surface may not always be the most accurate or correct representation. Gerharz et al. [27] advocating looking at full joint PDFs and compare statistical methods for both marginal and joint PDFs defined across the spatio-temporal domain. All of the above techniques allow for the application of traditional 2D visualization techniques such as color mapping, however this leaves the third dimension free to be leveraged for the exploration of the PDFs. A density estimate volume can be computed [45] that creates a comparison volume across all PDFs allowing for the interrogation with cutting planes, local surface graphs, PDF isosurfaces, and glyphs. Thinking of the data as a set of realization surfaces allows for the creation of a volume which can be visualized using volume rendering and streamlines [78]; however this type of interpretation of the data imposes some sort of ordering on the realization surfaces which is not actually existent in the data. While the above techniques attempt to maintain the presence of the PDF in the visualization, it is often easier to reduce the understanding of uncertainty down to mean and standard deviation, a range of uncertainty, or a single scalar value depicting the magnitude of uncertainty. While this type of interpretation may impose assumptions on characteristics of the uncertainty quantification, it greatly reduces the difficulty in visualization.

5.2 2D Data

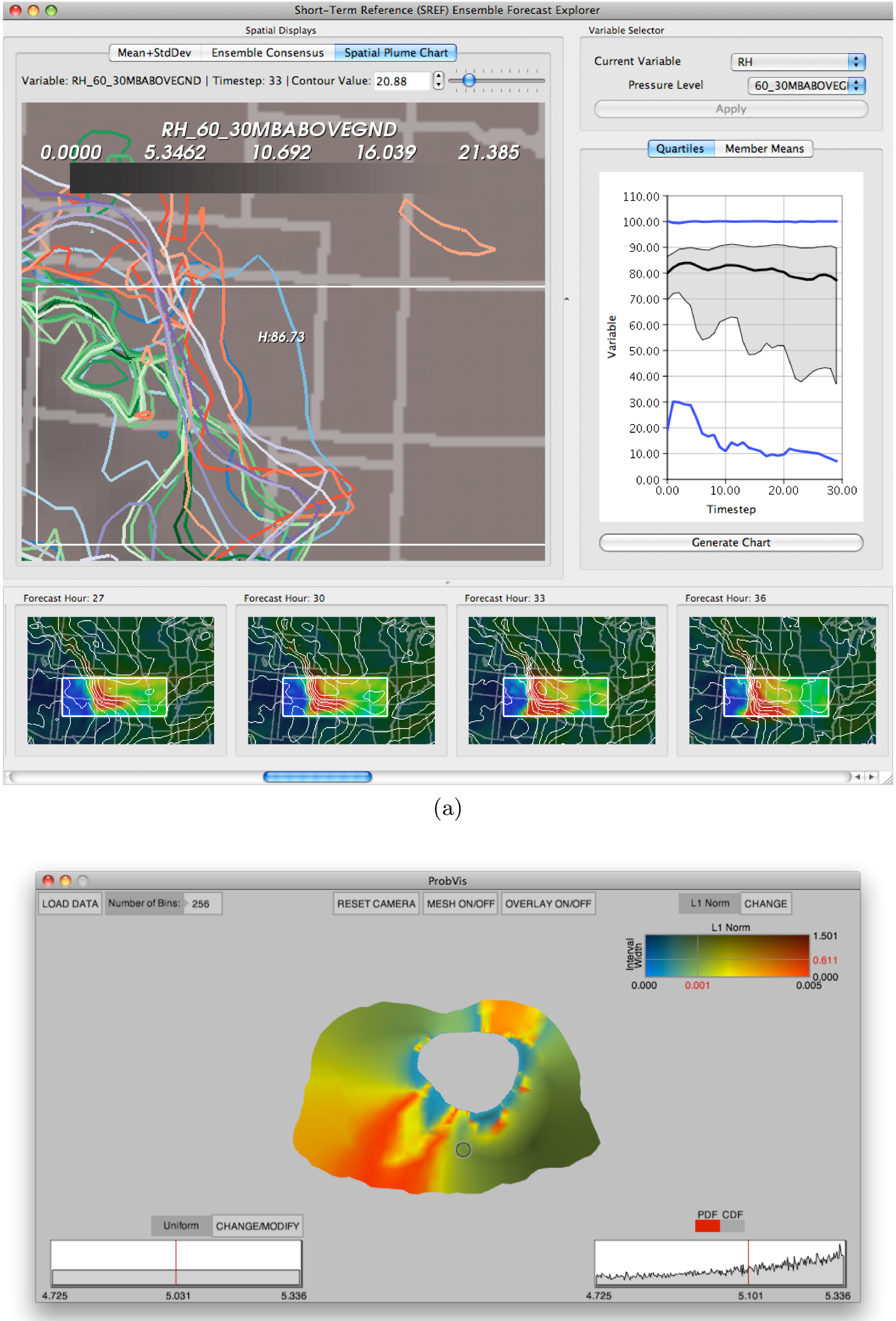
Methods that use this type of approach often employ color maps [13, 51, 60, 79, 83, 91] such as those in Figure 2, texture irregularities, opacity [34, 69], surface displacement [31], animation [30, 22] and glyphs [13, 51, 60, 83, 91, 95] to show uncertainty. Modifying contour color, thickness, and opacity [63, 72, 83] can show regions of uncertainty across the spatial domain. These types of displays can be augmented with uncertainty annotations [13] which modulate properties of information external to the data display, such as longitude and latitude lines, in order to show uncertainty in a way that does not interfere with the data display. Multiwindow methods can help expose underlying information of the PDF that these types of approaches hide [79, 83], as shown in Figure 2a, and can also provide for application specific types of visualizations. Finally, in contrast to 2D spatial domains, uncertainty can exist in 2D lattice and tree structures. This type of uncertainty arises as data structures for many statistical processing systems where the structure usually represents the "best guess" and alternate branches or leaves are shown with reduced opacity, or variations in positioning, color, or size [14, 49].

5.3 3D Data

Moving into 3D data, the number of visualization channels available has significantly diminished which limits the amount of information that can be readily displayed on the screen. The direct display of each PDF contributing the data set possible in 1D and less so in 2D, is now greatly diminished. Rather than expressing these full PDFs in this context, it is necessary to reduce the uncertainty information into an aggregated form, such a summarizing through a small set of numbers, or as an interval. The emphasis of 3D techniques is more often on displaying the location and relative size, rather than the exact quantification of the uncertainty.

The most commonly found techniques for showing uncertainty in 3D include color mapping, opacity, texture, and glyphs [15, 50, 61, 82], with Figure 3 showing some examples. This is used in volume rendering [19] where the transfer function is used to encode uncertainty with color and opacity, or as a post-processes composite with texture. This work was later extended to include depth cuing and improved transfer function selection [18]. Rather than simply encoding a single value of uncertainty, the transfer function can be used to encode different measures of uncertainty, such as risk or fuzzy classifications of tissues [47, 81]. This idea can also be applied to the fuzzy classification of isosurfaces [54].

In contrast to mapping a quantity of uncertainty onto a 3D visualization, it is noteworthy to point out uncertainties created by the visualization itself. Probabilistic marching cubes and uncertain isocontours [72, 73] are techniques which investigate the uncertainties in calculating underlying 2D and 3D visual



(b)

Fig. 2: Two examples of the visualization of 2D scalar data. (a) EnsembleVis [79] uses multiple windows to show various characteristics of the uncertainty and provides linking and brushing through a gui. (b) PDFVis [76] uses a color map to compare all PDFs across the 2D spatial domain.

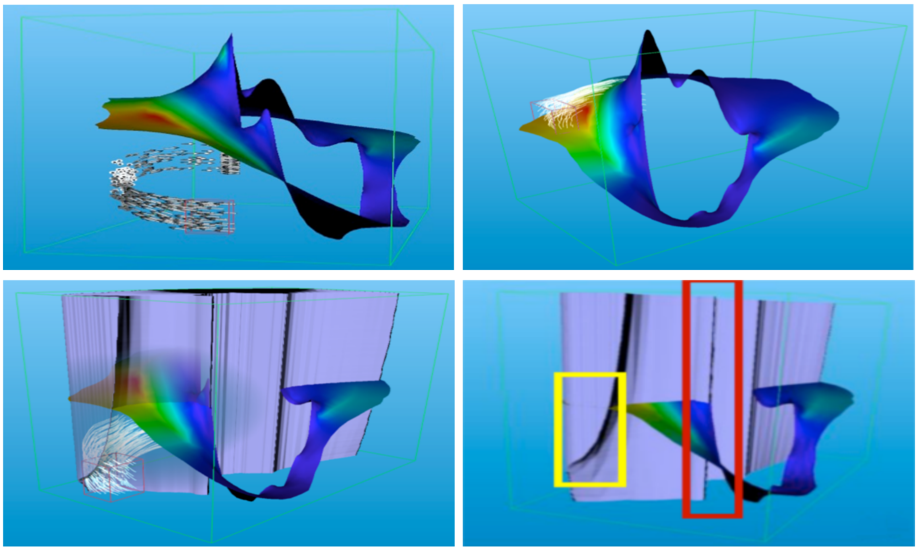


Fig. 3: 3D scalar field uncertainty visualization using glyphs, color maps, isosurfacing, and volume rendering [78].

representations. Djurcilov et al. [20] construct visualization geometries (point-clouds, contours, isosurfaces & volume rendering) with missing data, and Pauly et al. [71] investigate surfaces generated from 3D data acquired from scanners. Overlay, pseudo-coloring, transparency, and glyphs can be used to compare differences in isosurface generation algorithms and volumetric interpolation techniques [42, 80], and color mapping using flowline curvature [46] can be used to gain knowledge on the quality of an isosurface for representing the underlying data. Animation is often used to highlight these discrepancies by vibrating through possible isosurface positions [12], or looping through visualization parameter settings [55].

The last body of work on uncertainty in 3D deals with data reliability data and is most often found in fields such as archeology and virtual architectural reconstruction [58, 86]. In these works, the uncertainty is defined as the confidence an expert in the field has in the construction of a 3D model. Scientific judgment is used to fuse what is know about particular archaeological site, such as existing structures, and historical background of the regions and peoples. The construction of the 3D models reflects uncertainty or points of contention based on the way the model is rendered. Opacity [87], sketch-based texture [75], animated line drawings [59], and temporal animations [96] can be used to express this type of uncertainty. Because highly-realistic imagery tend to be interpreted as truth [21], the unifying theme of these works is to add an illustrative quality to the rendering technique to lower the rendering quality to directly reflect the reliability of the data [93].

5.4 ND Data

As mentioned in Section 4, ND data deals with high-dimensional data typically defined as time-varying, multivariate, or parameter space explorations. Most work on time-varying data simply extends the 2D or 3D using techniques such as animation, and thus these works have been discussed in the previous sections. Here, we will focus our discussion on multivariate and parameter space data.

Multivariate data involves many related variables. Simple visualization of multivariate data is, in itself, a challenge and much work towards visualizing this type of data has been done [24]. A common approach for this type of data is parallel coordinates, which creates a coordinate system and plots the location of points across all axes. Adding uncertainty to parallel coordinates can be done through blurring, opacity, and color [23, 25]. While parallel coordinates do indeed display many dimensions within the same window, they are often hard to understand. As an alternative to parallel coordinates, multiple visualization windows can be used to expose uncertainties in relationships between spatial, temporal, and other dimensionalities [32]. This type of approach, however, reduces the multivariate aspect of the data to a lower dimensional representation more appropriate to visualization.

Parameter-space explorations expose the uncertainties within systems by analysing the relationships between input parameters and outcomes and are often used to better understand and improve simulations. While a full discussion of work in parameter-space analysis is outside the scope of this paper, the connection to uncertainty visualization is of interest. Here, we discuss a few notable works that relate parameter-space analysis to visualization, and we refer the reader to the papers for a treatment of the underlying mechanics.

The first exemplary work uses a combination of parallel coordinates and scatterplots to show the parameter-space sensitivity [2]. For each dimension, a PDF defines the uncertainty, which is then expressed as a histogram on top of each axes in the parallel coordinate display, or the user can select two dimensions to be displayed as a scatterplot with overlaid boxplots. An alternative to parallel coordinates, Gerber et al. [26] propose using the Morse-Smale complex to summarize the high-dimensional parameter space with a 2D representation that preserves salient features, and provides an interactive framework for a qualitative understanding of the effect of simulation parameters on simulation outcomes. The final example is World Lines which [90], using the demonstrative application of a flooding scenario, visualizes the multiple output scenarios individually, allowing the user to interactively explore the various world outcomes of the simulation.

6 Vector Fields

Vector fields are typically found as 2D and 3D with a time component. While these are different domains, both have equal treatment in the visualization space; the majority of techniques are either applied to both 2D and 3D or have been extended from 2D to handle 3D. 1D vector fields equate to a 1D scalar fields which are discusses in Section 5.1. Both 2D and 3D fields often have a time component and thus, for sake of our discussion here, we classify data with a time-component as 2D or 3D and assume any ND vector field work deals with parameter exploration. This eliminates the discussion of 1D vector fields, and postpones the consideration of ND vector fields to a later date, as no work in this area has been done at this time.

The visualization of uncertain vector fields can be classified into four types and we will assume the description of each type applies to both 2D and 3D, unless otherwise noted.

A common visualization technique for both 2D and 3D vector fields are glyphs [68]. Glyphs typically encode the two variables of the vector within their construction, such as an arrow pointing towards the direction with length scaled by magnitude. Expansions of glyphs to uncertainty information include using area, direction, length, and additional geometry to indicate uncertainty [92], line segment or barbell glyphs [52], or ellipsoidal glyphs depicting regions of possible vector positions [50]. Finally, time can be included in the glyph itself [33] or through animation of the glyphs [97].

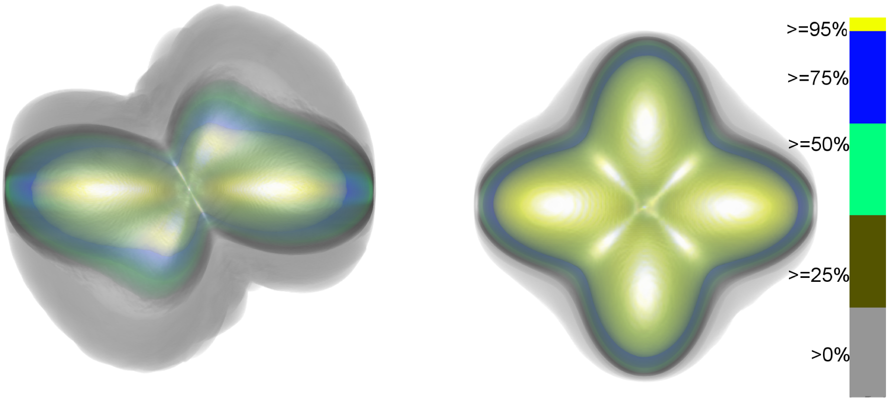
Stream and particle lines show the path of flow from a particular seed point through time. In 2D these can be represented as lines [56] and as ribbons or tubes in 3D [52], both of which can use color, opacity, width, and animation to show uncertainties such as interpolation error in meteorological trajectory [5, 50] or differences in integration methods for particle tracing [52]. Texture-based streakline methods are more often used for 2D vector fields and use attributes such as noise, color and fog to modulate streaklines with uncertainty as they move across the domain [8, 9, 64] .

Most recently, Otto et. al. suggested that instead of creating new uncertainty representations for vector fields, we should instead recycle the approaches used in scalar field visualization. Their approach calculates multiple scalar fields with probabilities that represent topological features such as sinks, sources, and basins. Then, scalar field uncertainty approaches can be applied for 2D [65] and 3D [66] or used to analyze uncertainty of motion in video data [67]. In another topological approach, [3, 4] create a new data structure called the “edge map” to represent 2D flow and bound error. Once they have quantified the associated error, they use streamwaves to visualize the fuzzy topological constructions.

A notable break from traditional visualization techniques, sonification is the use of sound to indicate areas of uncertainty and has been included in the UFLOW system [53] to visualize flow fields using glyphs and streamlines as well as a system for visualizing the uncertainty of surface interpolants.

7 Tensor Fields

Most of the work in 3D tensors fields has focused on brain fiber tracking in diffusion tensor imaging. The first set of methods display the data as a glyph representation [35, 41], which indicates the fiber directional or fiber crossing uncertainty at a given location. The other set of approaches track fibers under



(a) b-value 7000 s/mm2, SNR=10, (0.6,0.4) 60 degrees

(b) b-value 1000 s/mm2, SNR=20, (0.5,0.5) 90 degrees

Fig. 4: Visualization of the uncertainty in two diffusion shapes from diffusion tensor imaging using volume rendering applied to an ensemble of 3D orientation distribution functions. (a) Two fibers crossing at 60 degrees with relative weight of 0.6:0.4 and SNR of 10. (b) Two fibers crossing at 90 degrees with equal weight and SNR of 20 (with much less uncertainty).

uncertain conditions, giving either a color map for confidence [11] or an envelop of potential fiber routes [37]. Figure 4 shows a recent effort by [36] to use volume rendering with multi-dimensional transfer functions to visualize the uncertainty associated with high angular resolution diffusion imaging (HARDI). The authors use ensembles of orientation distribution functions (ODF) and apply volume rendering to 3D ODF glyphs, which they term SIP functions of diffusion shapes to capture their variability due to underlying uncertainty. Beyond these few approaches, very little work has been performed on visualizing uncertainty within tensor fields.

8 Evaluation

An often overlooked aspect in the field of visualization is evaluation. This is also the case in uncertainty visualization, which is doubly problematic in that the visualizations often represent highly complex phenomenon, and the assessment of effectiveness must take into account not only good visual design, but also appropriate understanding, transformation, and expression of the data.

A handful of papers have been dedicated to evaluating visualizations in the context of uncertainty. Most of these look at the method of visual encoding such error bars, glyph size, and colormapping in 1 and 2D [82], glyph type in 3D [61], or comparing adjacent, sequential, integrated, and static vs dynamic displays [28]. While each work identified a “better” technique for their unique study; surface and glyph color work better than size, multi-point glyphs perform better than ball, arrow, and cone glyphs, and adjacent displays with simple indications of data and uncertainty were preferred by the users, however none of the techniques performed well enough to be called the best display of uncertainty in all circumstances. A more human-centered approach evaluated the psychophysical sensitivity to contrast and identified particular noise ranges appropriate for uncertainty visualization [15]. Finally, indications of uncertainty in a visualization were found to influence confidence levels in the context of decision making [16, 17].

While each of the above works is significant in improving our understanding of the effectiveness of uncertainty visualizations, much more work must be done. The number of fields turning to visualization for understanding and decision making is growing, as well as the range of users, and this again reiterates both the great challenge in evaluation as well as the great need. While the work done to date does not necessarily point out specific techniques that will work in any situation, it does, as a collection, point to the necessity to understand the perceptual issues in visualization, as well as the needs tailored specifically to the problem at hand. Thus future work in evaluation should continue to study in what circumstances particular visual devices work, how overloading visual displays with information such as uncertainty effect understanding, and the ramifications of the human visual system. A formal treatment in this regard will allow future developers of uncertainty visualizations to position work within a tested set of constraints which, while not guaranteeing a successful visualization, may help foster good design.

9 Conclusion

Uncertainty visualization has been identified as a top visualization research problem [39, 40]. The increased need for uncertainty visualization is demonstrated not only by the various taxonomies and typologies referenced in Section 4, but by the discussions found in numerous positional papers and reports [38], which motivate the need from the viewpoints of several scientific domains, including GIS and decision making [1, 10, 29, 48, 69]. This has inspired significant growth of work in the area of uncertainty visualization and with this amassing number of emerging works there has also become a need for an organization of that work. As a compendium to previous surveys on uncertainty visualization [6, 57, 68], this paper organizes the state of the art in uncertainty visualization based on the dimensionality of the data and of the uncertainty. Our main contribution is the classification of the work into groups, as well as identifying common visualization techniques for each group and point out specific unique techniques.

9.1 Directions for Uncertainty Visualization

Below we outline areas of uncertainty visualization we have identified as still needing further study.

Scalar Fields The majority of work in uncertainty visualization has focused on scalar fields. These visualization methods almost always depend upon a single uncertainty value for their visualization. This limits the uncertainty information they can convey. New methods of visualizing the underlying PDF would allow visualizations to more accurately convey the possibilities for the shape of the underlying data without increasing visual clutter. We see new glyphs representations as one promising direction for solutions. Additionally, clustering of similar uncertainty might offer a possibility to reign in visual cluster.

Significant needs also remain for parameter space visualizations of uncertain data, as this type of data are becoming more widespread. The current approaches most often take standard parameter space visualizations, such as parallel coordinates, and apply standard scalar field uncertainty approaches. New abstractions and visual designs are needed to better convey the richness of this data.

Vector Fields One of the more intriguing directions of work is that suggested by Otto et al., recycling scalar field visualization techniques by finding topological features within the vector field. Topological analysis of vector fields is a robust field of research. The uncertainty visualizing community could certainly leverage topological techniques as a way to better communicate uncertain vector field data, such as that put forth by Bhatia et al. [3].

There has been limited work performed on joint-histograms and correlations between two variables. While we do not consider this as a vector field within our taxonomy, it is in some sense related. Most techniques assume an independent 1D PDF for each variable, no matter the number of variables, with higher dimensional PDFs only available combinatorially. In fact, multiple variables can correlated in such a way that the structure of their uncertainty is not separable into multiple 1D PDFs. Instead they need higher dimensional PDFs and new visualization methods for those additional dimensions. Potter et al. [77] suggest some simple techniques for addressing these problems using 2D plots but more work is still needed.

Tensor Fields Little work has been done for visualizing uncertainty associated with tensor field. The small body of work in visualizing uncertainty within tensors has mostly focused its efforts towards visualizing uncertainty of derived values, such as white matter fibers. Future work on this problem must focus on both the uncertainty of derived values and the uncertainty present in the tensor itself. However, visualizing a tensor directly, irregardless of uncertainty, is in itself a challenging problem, especially as the order of tensor increases. For high dimensional uncertainty tensor visualization, one avenue for future research is to combine traditional tensor field visualization techniques from information and statistical visualization techniques, such as [94], which combines 3D view of diffusion tensor fiber tracts with 2D and 3D embedded points along with multiple histograms that show derived quantities including fractional anisotropy, fiber length, and average curvature.

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