

# Surface segmentation for improved isotropic remeshing or What has John been doing for the last four months?

John Edwards

The University of Texas at Austin

HKU group meeting, May 30, 2012

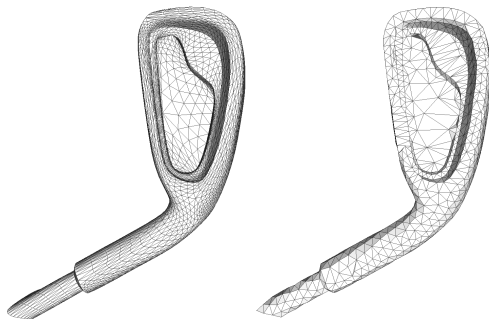
# John's adventures



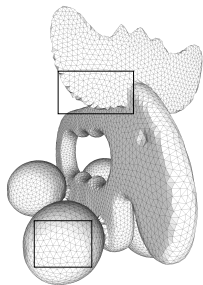
- Motivation and problem statement
- Centroidal Voronoi Tessellation
- Segmentation
- Stitching
- Results

# Surface remeshing

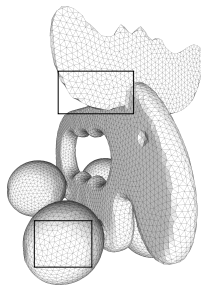
- Surface remeshing is the process of transforming one surface mesh into another
- Reasons for remeshing
  - Decimation
  - Triangle quality improvement
- Many techniques sample the input surface  $S$  and then triangulate the points



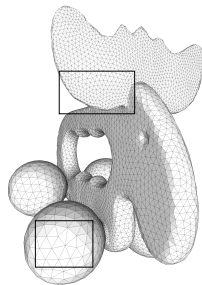
# Motivation



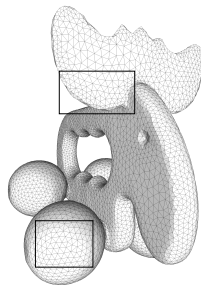
[6]



uniform



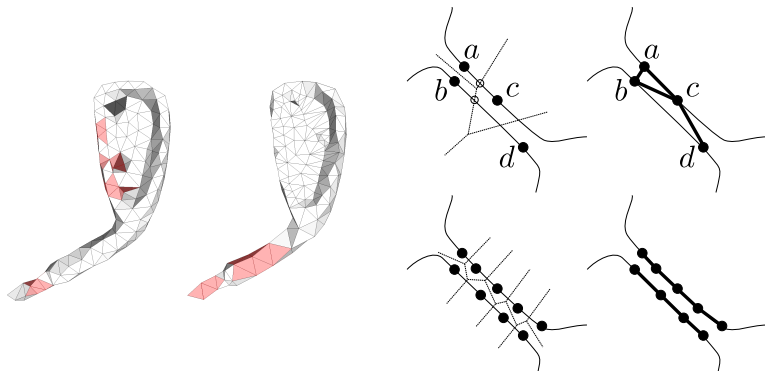
*lfs*



$\kappa$ CVT

# Sampling density

- Some techniques resample in parameter space [1, 2, 7]
  - Global parametrization can cause distortion
  - Local parametrization: optimization is not global, stitching is required
- Direct sampling techniques require minimum sampling density
  - Reconstruction is done using the Restricted Voronoi Diagram
  - Required sample density is based on local feature size ( $lfs$ ) [3]



- Centroidal Voronoi Tessellation (CVT) is a method of tessellating a space
- It can be formulated as a critical point of the CVT energy function [4]

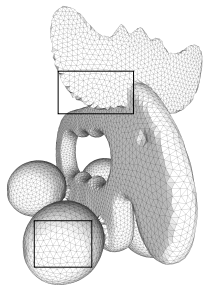
$$F(X) = \sum_{i=1}^n \int_{\Omega_i} \rho(x) \|x - x_i\|^2 d\sigma \quad (1)$$

$X = \{x_i\}$  is the set of sample points,  $\Omega_i$  is the Voronoi cell of  $x_i$  with respect to the other sample points,  $\rho$  is a density function

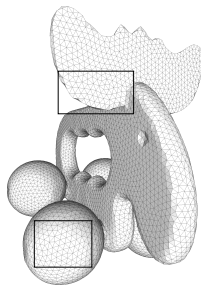
- In the context of surface remeshing:
  - We use the Restricted Voronoi Diagram for the final mesh [5]
  - Typical density functions are  $\rho = 1$  and  $\rho = 1/lfs^2$  [8]

# Introducing $\kappa$ CVT

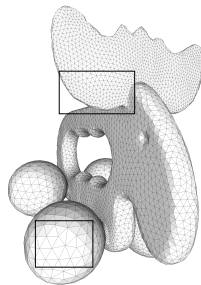
- [6]:  $\rho = \sqrt{\kappa}$
- uniform:  $\rho = 1$
- $lfs$ :  $\rho = 1/lfs^2$
- $\kappa$ CVT:  $\rho = \sqrt{\kappa}$



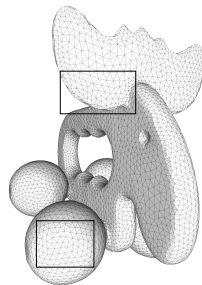
[6]



uniform



$lfs$



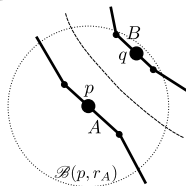
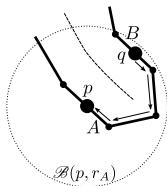
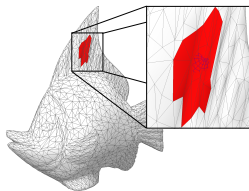
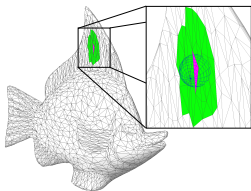
$\kappa$ CVT

- The \$1,000,000 question: how can we use  $\kappa$  as the density function while still meeting the sampling theorem?



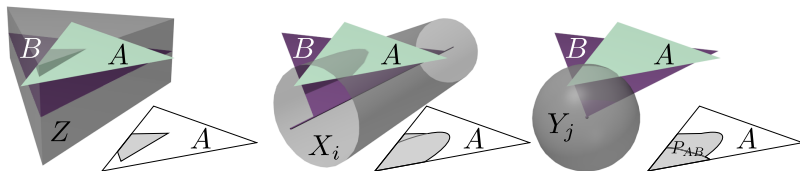
# Segmentation

- Segment surface  $S$  such that flattish areas have high  $lfs$
- Let  $A$  be a triangle in  $M_i$ . Heuristic: partition  $S$  into subsurfaces  $M = \{M_i\}$  such that the ball  $\mathcal{B}(p, r_A)$  centered at any point  $p \in A$  will yield a single connected component when intersected with  $M_i$ .
  - $r_A$  is an approximation of  $lfs(A)$ .



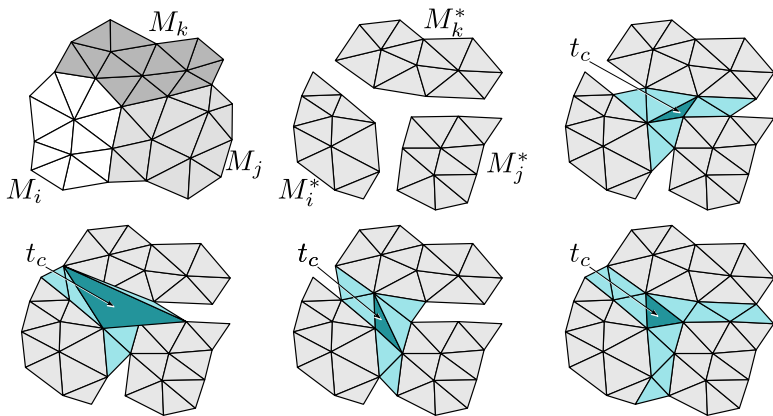
# Building compatibility table and segmentation

- Find which triangles are compatible with triangle  $A$
- Given triangle  $B$ , find the set of all points on  $A$  that are within  $r_A$  of  $B$
- Segmentation is done using the compatibility table and a flood-fill algorithm

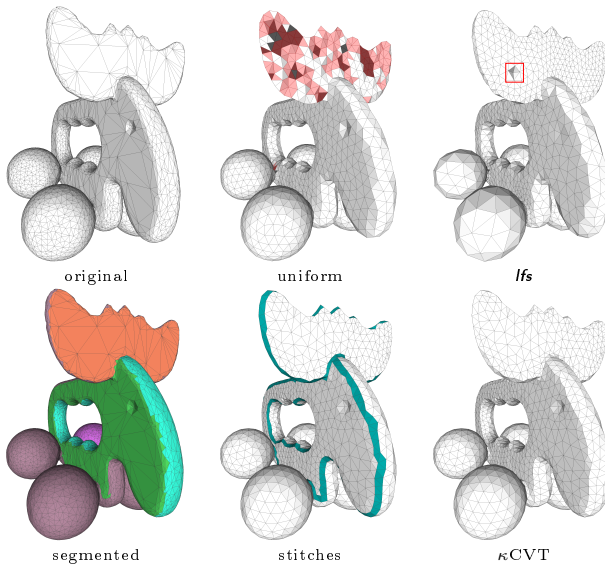


# Subsurface remeshing and stitching

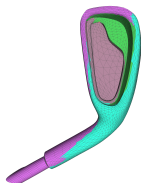
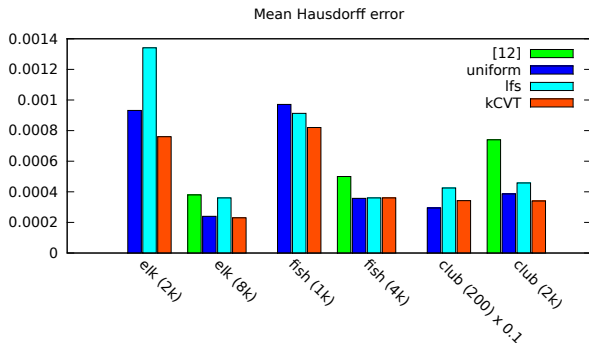
- Remesh each subsurface  $M_i$  individually using CVT with  $\rho = \sqrt{\kappa}$  (hence the name of our method)
- Stitch remeshed subsurfaces  $\{M_i^*\}$  back together using a search algorithm and cost function



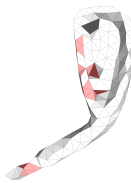
# Method overview and results



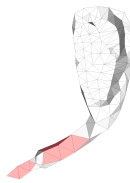
# Results



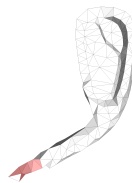
segmented



uniform

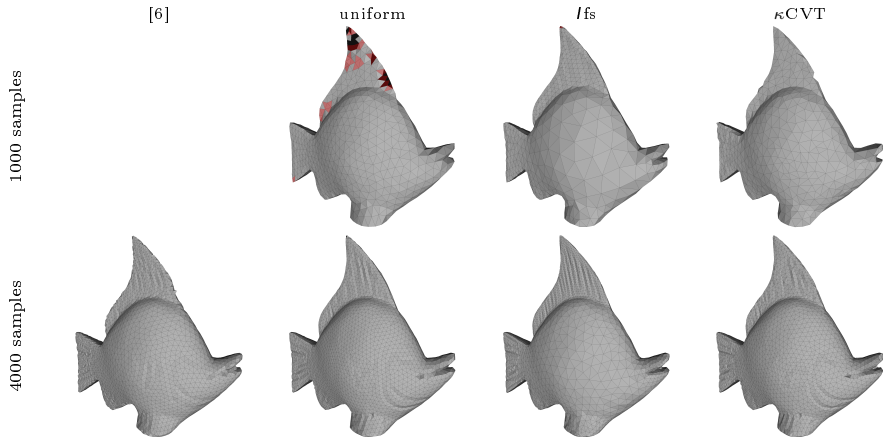


*lfs*



$\kappa$ CVT

# Results



- $\kappa$ CVT performed better than the other two CVT methods in terms of geometric error in every test but one, and showed as much as 20% improvement over the next-best method
- Topological errors were reduced to 0 in every case but one. In that case topological errors were reduced by 30% of next-best method
- Average triangle quality was similar to that of *lfs* method
- So what's the catch?
  - Min triangle quality was reduced, due to stitching
  - Improvement is specific to models with flattish areas that have low local feature size

# 谢谢



# References

- [1] P. Alliez, EC de Verdire, O. Devillers, and M. Isenburg.  
**Isotropic surface remeshing.**  
In *Shape Modeling International, 2003*, pages 49–58. IEEE, 2003.
- [2] P. Alliez, M. Meyer, and M. Desbrun.  
**Interactive geometry remeshing.**  
*ACM Transactions on Graphics*, 21(3):347–354, 2002.
- [3] N. Amenta and M. Bern.  
**Surface reconstruction by voronoi filtering.**  
*Discrete & Computational Geometry*, 22(4):481–504, 1999.
- [4] Q. Du, V. Faber, and M. Gunzburger.  
**Centroidal voronoi tessellations: Applications and algorithms.**  
*SIAM review*, pages 637–676, 1999.
- [5] Q. Du, M.D. Gunzburger, and L. Ju.  
**Constrained centroidal voronoi tessellations for surfaces.**  
*SIAM Journal on Scientific Computing*, 24(5):1488–1506, 2003.
- [6] S. Fuhrmann, J. Ackermann, T. Kalbe, and M. Goesele.  
**Direct resampling for isotropic surface remeshing.**  
In *Vision, Modeling, and Visualization*, pages 9–16, 2010.
- [7] V. Surazhsky, P. Alliez, C. Gotsman, et al.  
**Isotropic remeshing of surfaces: a local parameterization approach.**  
2003.
- [8] D.M. Yan, B. Lévy, Y. Liu, F. Sun, and W. Wang.  
**Isotropic remeshing with fast and exact computation of restricted voronoi diagram.**  
In *Computer graphics forum*, volume 28, pages 1445–1454. Wiley Online Library, 2009.

## References (cont.)

# Sampling density theorem

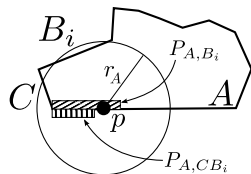
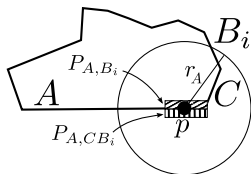
## Theorem

*$r$ -sampling theorem [3] If no point  $p$  on surface  $S$  is farther than  $r \cdot lfs(p)$  from a seed point  $x \in X$  where  $r$  is a constant then the Restricted Delaunay Triangulation induced by  $X$  is homeomorphic to  $S$ .*

# Computation of $r_A$

- We define  $r_p = 2 \cdot \alpha \cdot lfs(p)$  and  $r_A = \arg \min_{v_i \in V_A} r_{v_i}$ .
- All of our experiments use  $\alpha = 1.1$ .

# Flood fill segmentation



# Stitching cost function

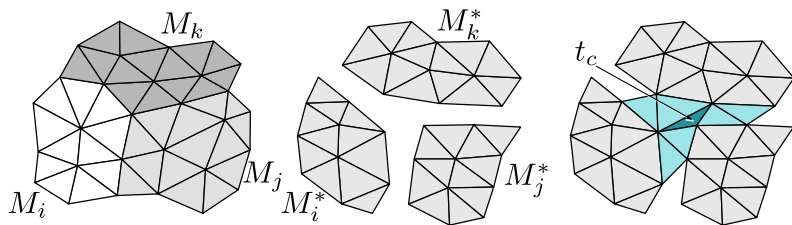
$t_c$  is a candidate “connector”

$$\text{cost}(t_c) = \sum_{t \in T_c} \text{area}(t) \cdot Q(t)^{-\gamma}. \quad (2)$$

$\gamma$  is a user-defined parameter (we used  $\gamma = 0.5$ ) and  $Q(t)$  is the triangle quality measure

$$Q_t = \frac{6}{\sqrt{3}} \frac{r_t}{h_t} \quad (3)$$

where  $r_t$  and  $h_t$  are the inradius and longest edge length of  $t$ , respectively.



# Results - table

model	# seeds	method	errors	$H_{\text{mean}} \times 10^3$	$H_{\text{RMS}} \times 10^3$	$Q_{\text{min}}$	$Q_{\text{ave}}$	$\theta_{\text{min}}$	$\theta_{\text{min,av}}$
Elk	2000	uniform	400	0.94	1.48	0.448	0.884	22.4	50.8
		$I_{\text{fs}}$	15	1.31	1.76	0.347	0.858	19.3	48.7
		$\kappa\text{CVT}$	0	0.76	1.00	0.220	0.849	11.7	48.1
Elk	8000	[6]	0	0.38	0.63	0.058	0.902	2.6	52.2
		uniform	0	0.24	0.37	0.509	0.916	24.4	53.2
		$I_{\text{fs}}$	0	0.36	0.49	0.451	0.893	22.6	51.4
		$\kappa\text{CVT}$	0	0.23	0.34	0.259	0.885	15.2	50.9
Fish	1000	uniform	95	0.97	0.16	0.525	0.872	28.6	49.7
		$I_{\text{fs}}$	14	0.91	0.12	0.420	0.830	18.3	46.4
		$\kappa\text{CVT}$	0	0.82	0.12	0.236	0.809	13.1	45.0
Fish	4000	[6]	0	0.50	0.85	0.070	0.898	2.7	51.8
		uniform	11	0.36	0.53	0.580	0.898	26.3	51.7
		$I_{\text{fs}}$	0	0.36	0.51	0.407	0.864	19.4	49.1
		$\kappa\text{CVT}$	0	0.36	0.58	0.160	0.863	6.5	49.0
Club	200	uniform	51	2.94	4.08	0.570	0.842	30.1	47.4
		$I_{\text{fs}}$	31	4.25	6.36	0.362	0.770	13.8	41.8
		$\kappa\text{CVT}$	19	3.42	4.92	0.173	0.728	9.2	39.5
Club	2000	[6]	-	0.74	1.54	$\sim 0$	0.832	$\sim 0$	47.5
		uniform	0	0.39	0.70	0.555	0.893	32.9	51.5
		$I_{\text{fs}}$	0	0.46	0.85	0.314	0.834	12.5	46.8
		$\kappa\text{CVT}$	0	0.34	0.62	0.082	0.855	4.5	48.5