Simulation of Sand using Material Point Method (MPM)

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- **Dr. Jin Ma and Rohit Raghav**
  - For providing, the basic 2D and 3D versions of combined SAMRAI and MPM computational framework

- **SAMRAI** – Richard Hornung and Andrew Wissink

- **Dr. John A. Nairn** -- For providing the 2D MPM code

High speed/high strain-rate mechanics of particulate media

To understand the physics of high-speed particulate flow:

- **Mechanical behavior and characterization of sand under high-speed compression, shear, and penetration into sand using combined experimental and numerical approaches.**

- **To develop essential scaling laws using multi-scale models for modeling of particulate media.**

Background

Shear band characterization of triaxial sand specimens using computer tomography

Batiste *et al.* (2004)

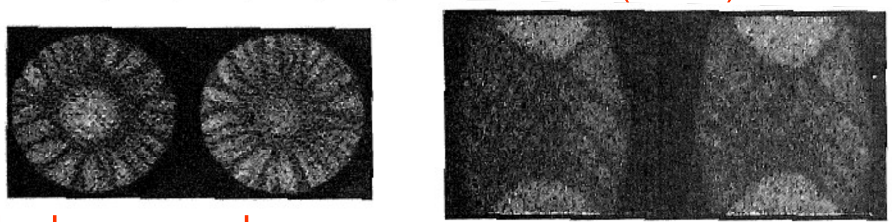


FIG. 4—CT scans for KSC experiments.

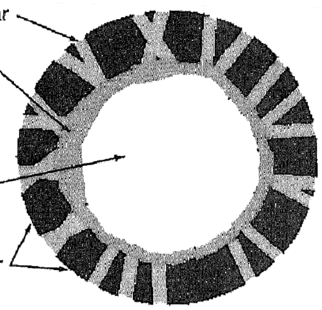
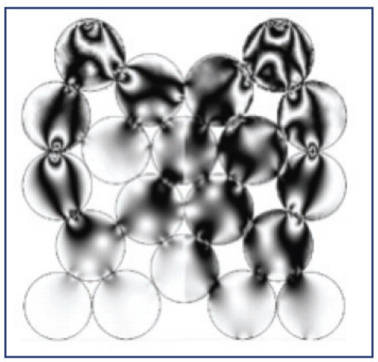


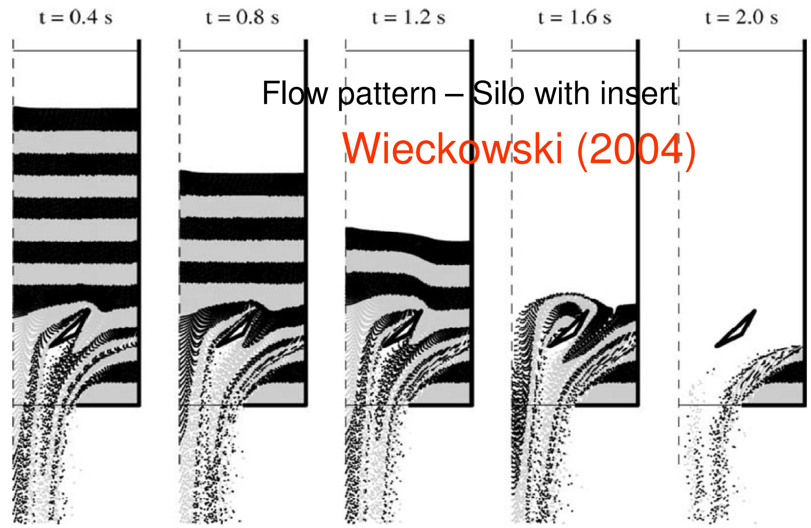
FIG. 5—CT slice of specimen with different deformation patterns.

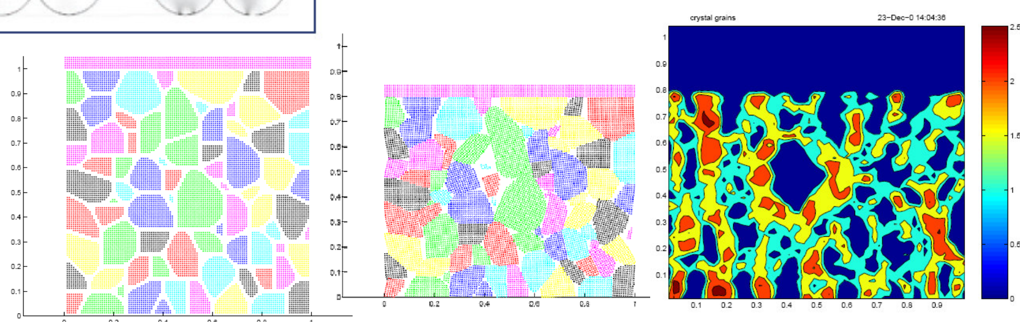
Bardenhagen and Brydon (2005)



Flow pattern – Silo with insert

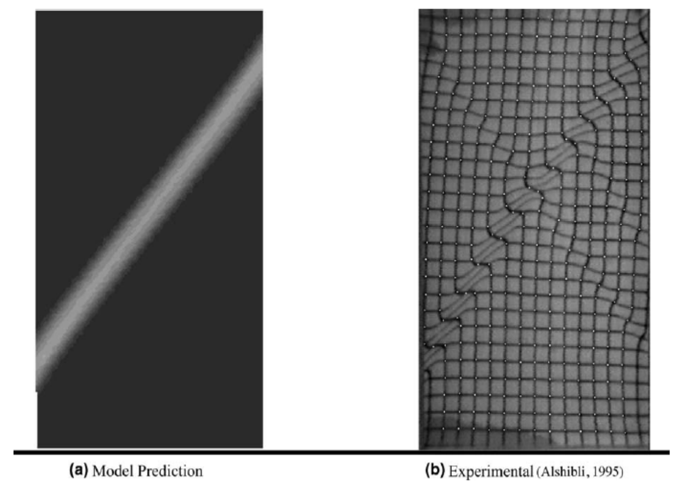
Wieckowski (2004)





MPM simulation of granular material showing and stress chains at 20 % deformation.

Sulsky (2003)



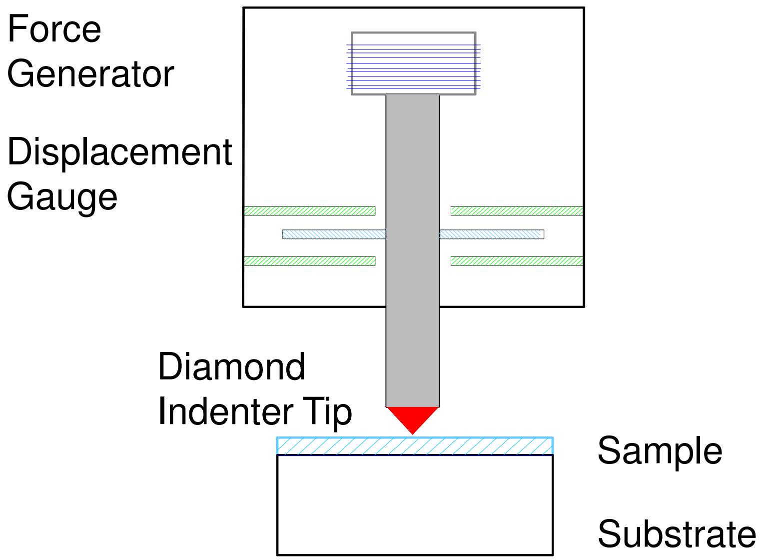
FEM simulation of shear banding in sand and comparison with experiments

Voyiadjis *et al.* (2005)

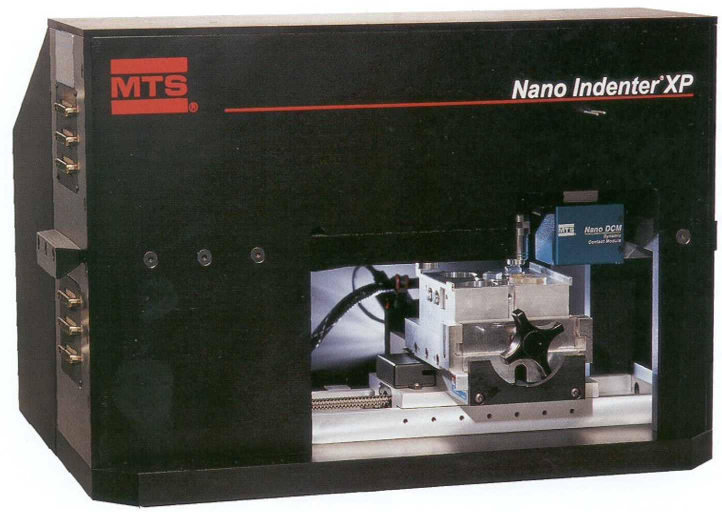
Approach

- Measurement of mechanical properties of sand using nanoindentation at the granular scale.
- Develop MPM as a utility for simulation of granular materials. Implement contact algorithm (Bardenhagen *et al.* 2001) into MPM and use the granular structure determined from Micro-Computed Tomography (μ-CT) to perform high-strain rate compression and shear simulations on sand

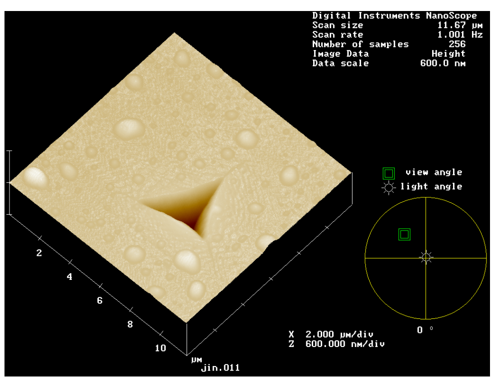
Nanoindentation



Schematic of mechanism of nanoindentation

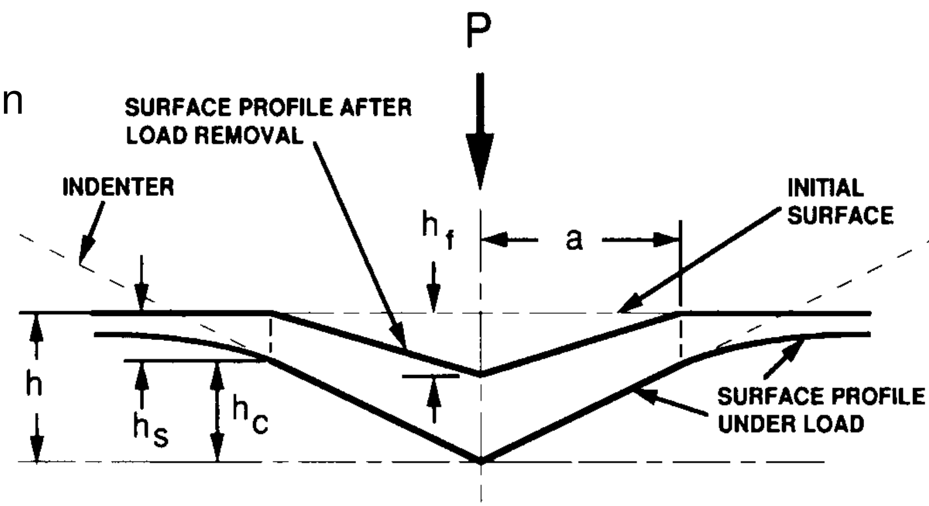


MTS Nano Indenter XP



An indent on silicon

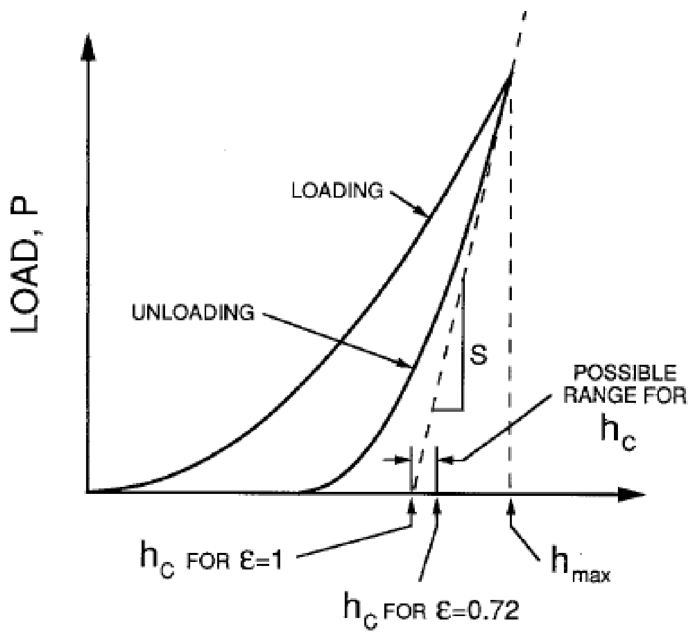
Oliver and Pharr, 1992



Schematic of indentation profile

A method for material without time-dependence

(Oliver & Pharr, 1992)



A typical nanoindentation P-h curve

$$P = C(h - h_f)^m$$

$$S = \frac{dP}{dh} = \frac{2}{\sqrt{\pi}}\sqrt{A}E_r$$

$$\frac{1}{E_r} = \frac{1-\nu^2}{E} + \frac{1-\nu_i^{\,2}}{E_i}$$

$$A = a_0 h_c^{\,2} + a_1 h_c + a_2 h_c^{\,1/2} + a_3 h_c^{\,1/4} + a_4 h_c^{\,1/8} \cdots$$

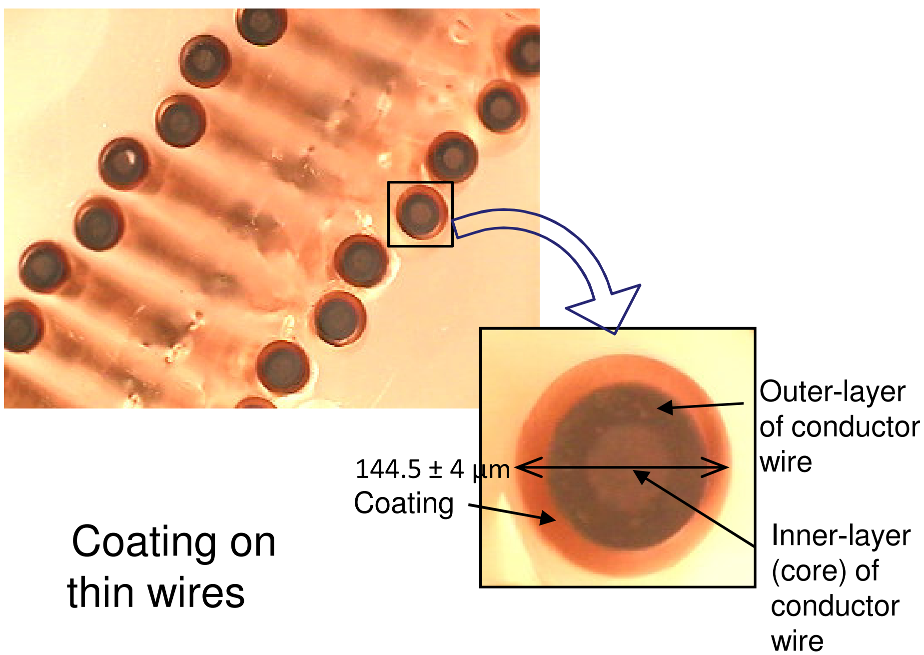
(Indenter tip calibration)

$$h_c = h - h_s = h - \varepsilon \frac{P}{S}$$

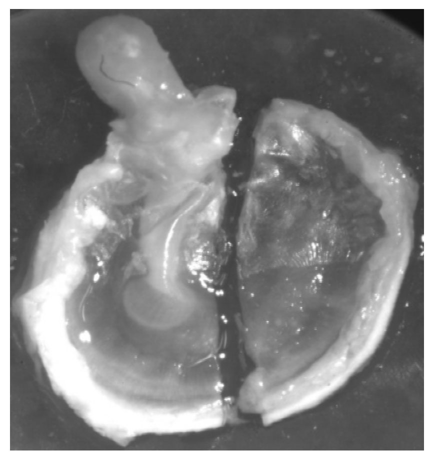
[From Sneddon's solution (1965)]

Some advantages of using nanoindentation

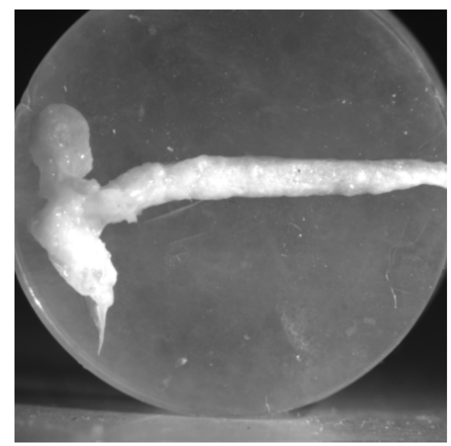
- A technique to measure mechanical properties of small amounts of materials — a challenge with conventional testing.
- An effective measurement technique to characterize material behavior at micron/submicron scale.
- Applicable to heterogeneous materials with mechanical properties varying spatially.



Coating on thin wires

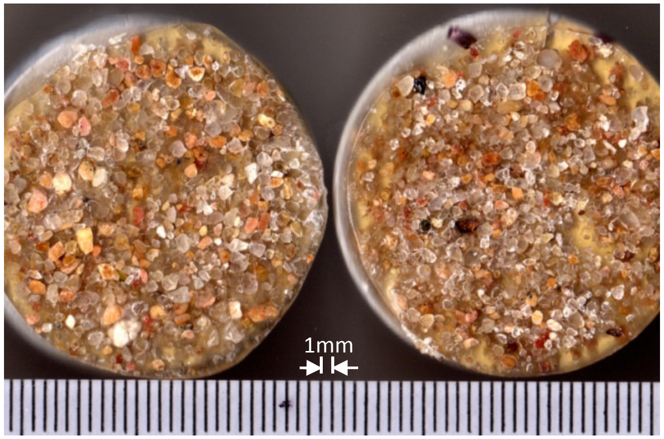


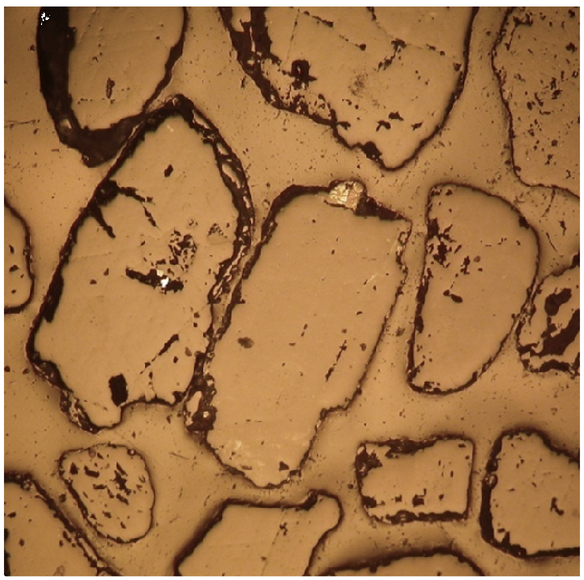
Tympanic Membrane



Middle ear ossicular chain

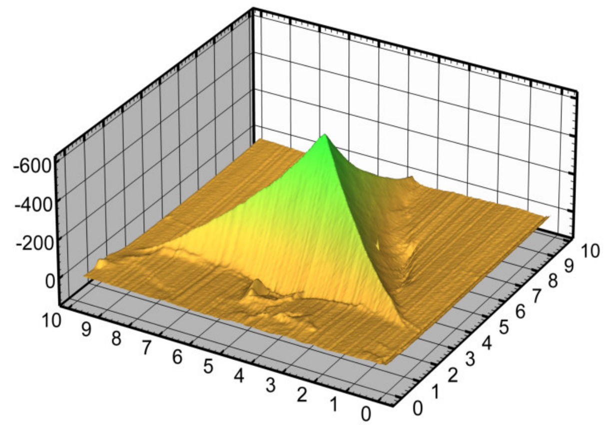
Nanoindentation on sand grains

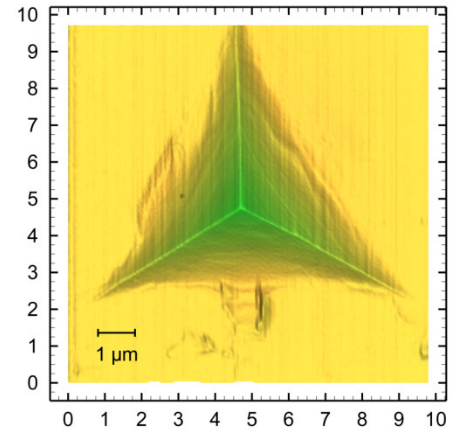


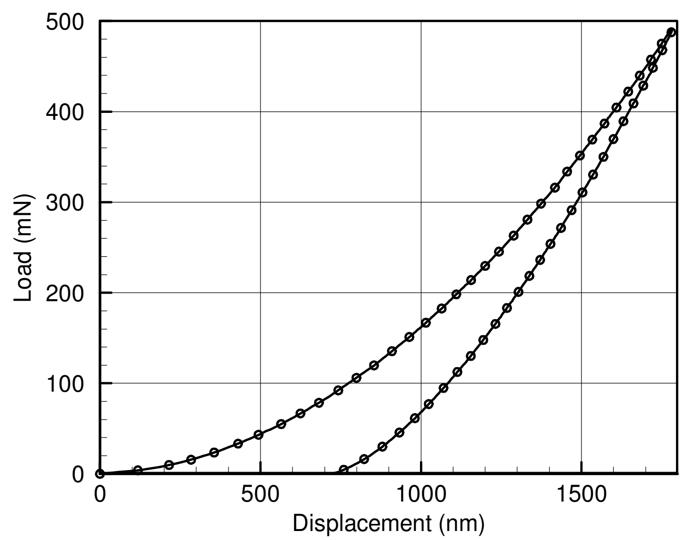


Embedded sand grain samples in an epoxy matrix; Size 0.5 – 1 mm

MTS Nanoindentation XP system with Nanovision



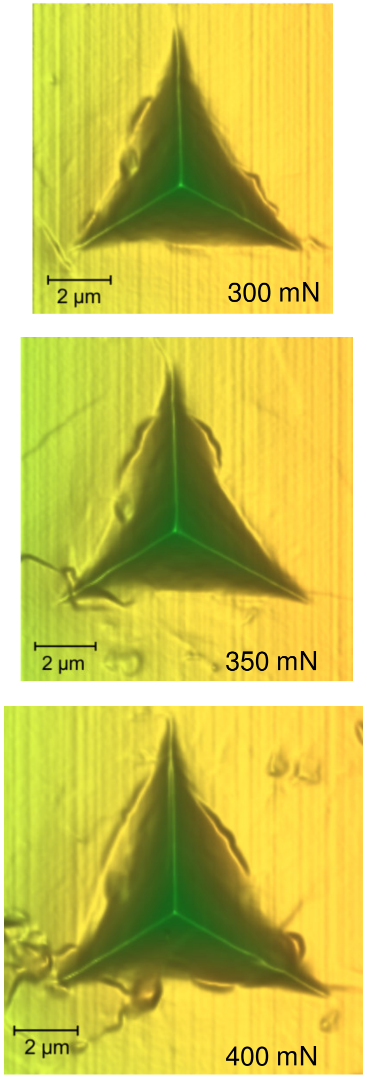




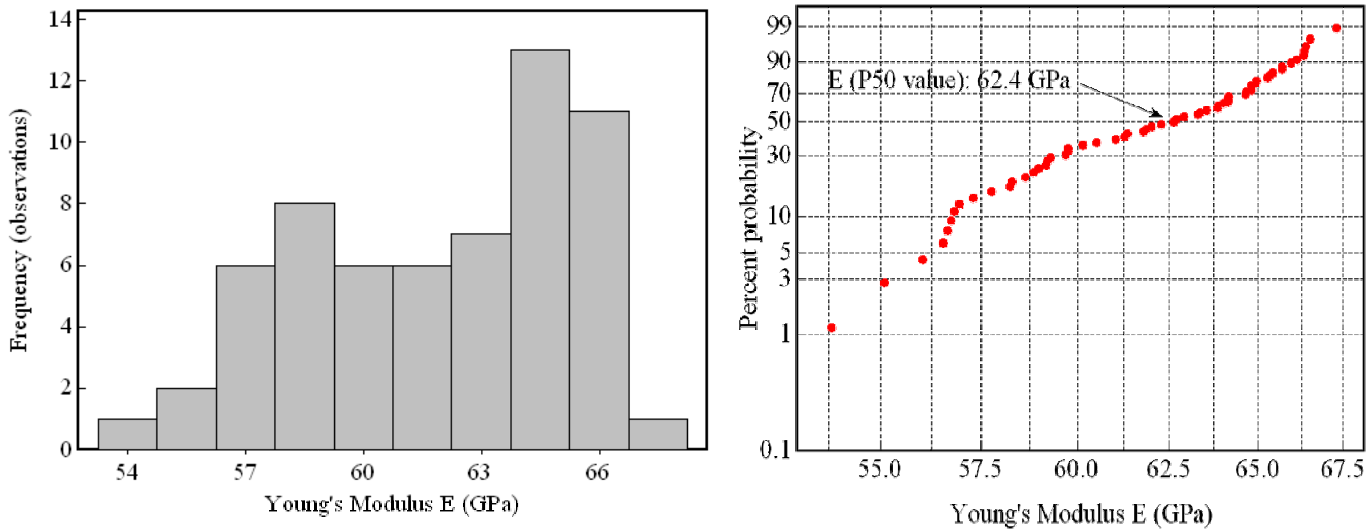
Typical results from nanoindentation test on sand grain using Berkovich tip

Modulus & hardness from a single sand grain

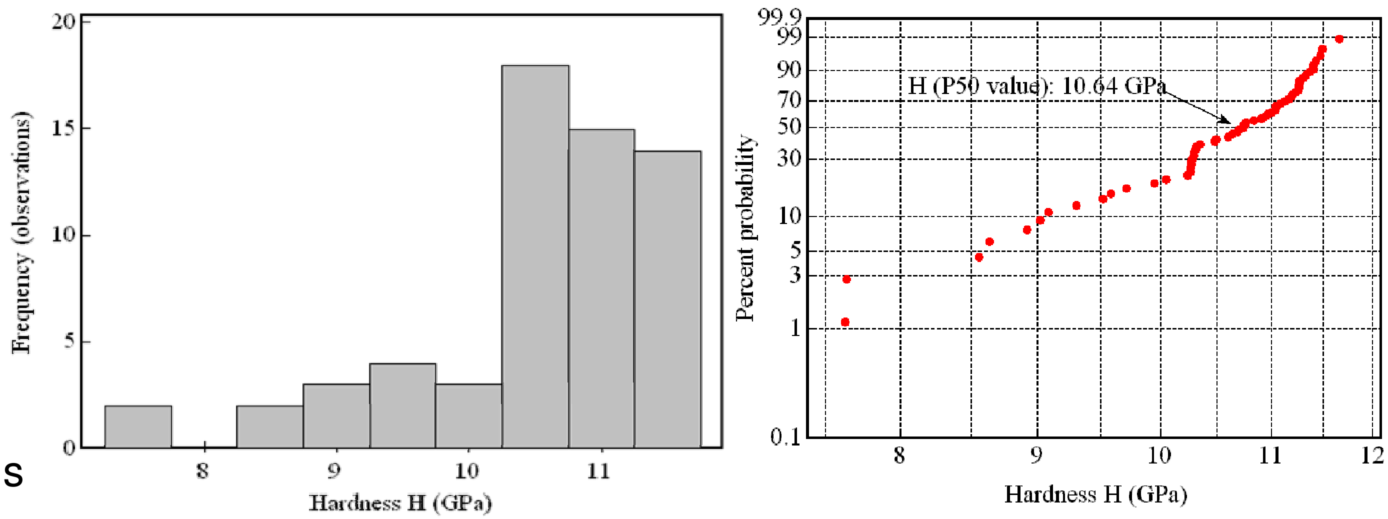
Oliver *et al.* (1986), Oliver and Pharr (1992)



Indentation impressions from Berkovich tip, at different loads

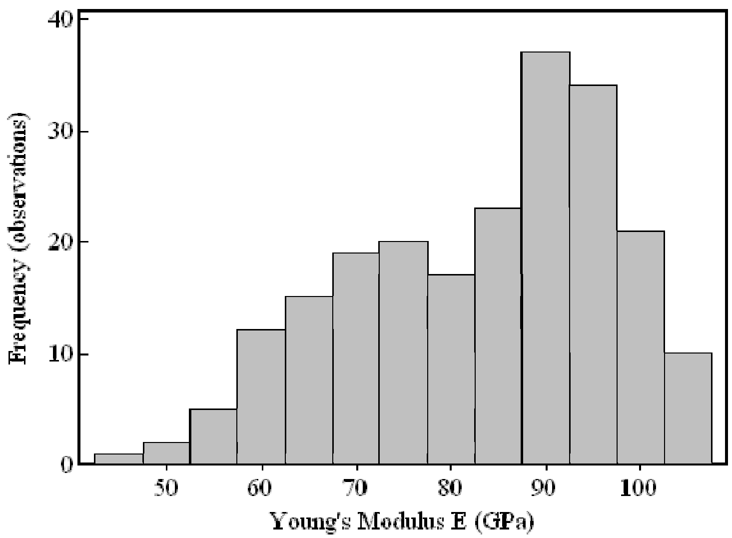


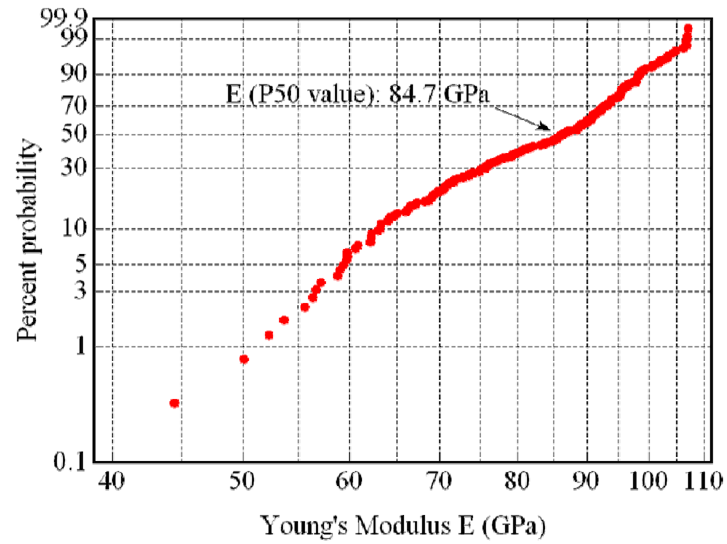
Variation of Young's modulus



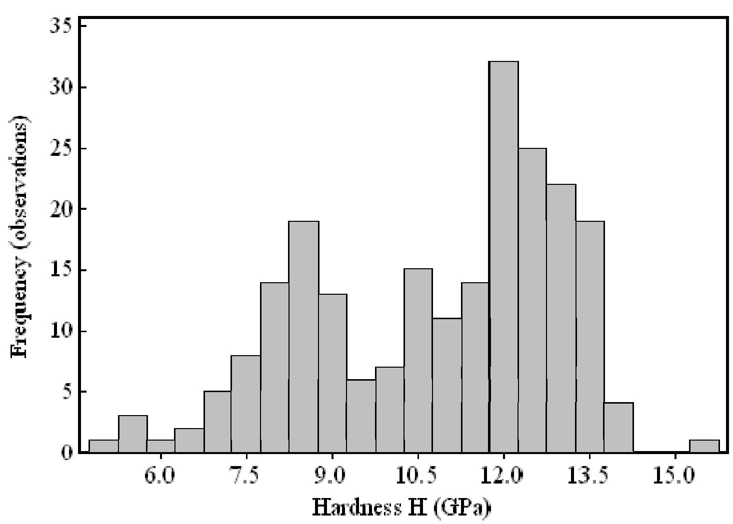
Variation of Hardness

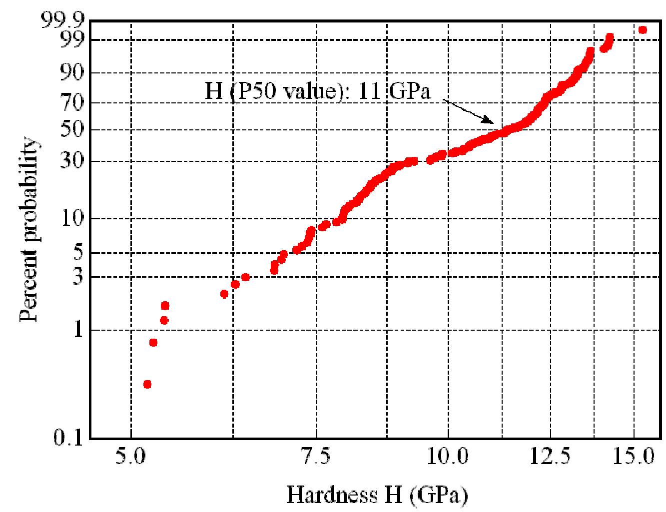
Modulus and hardness from multiple sand grains





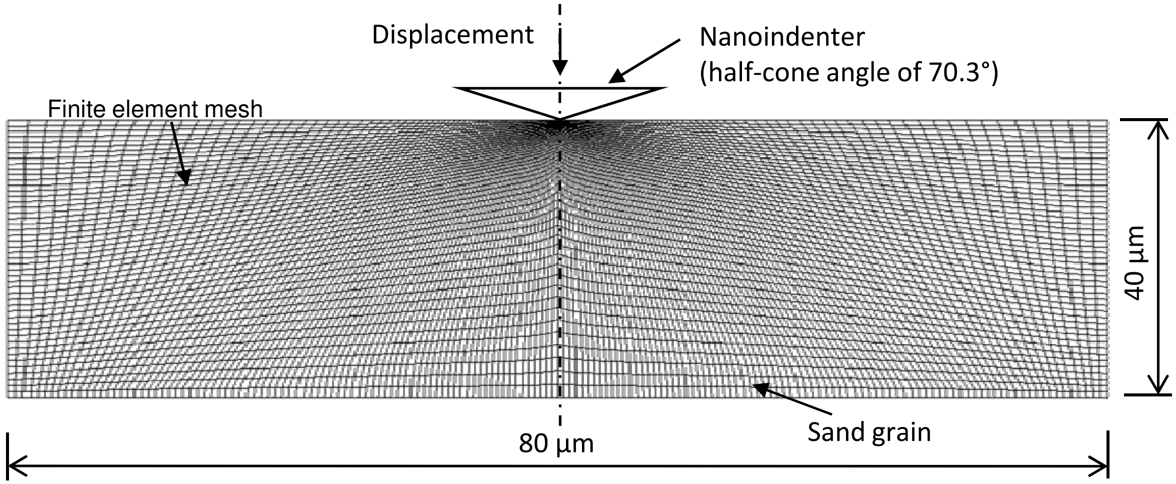
Variation of Young's modulus



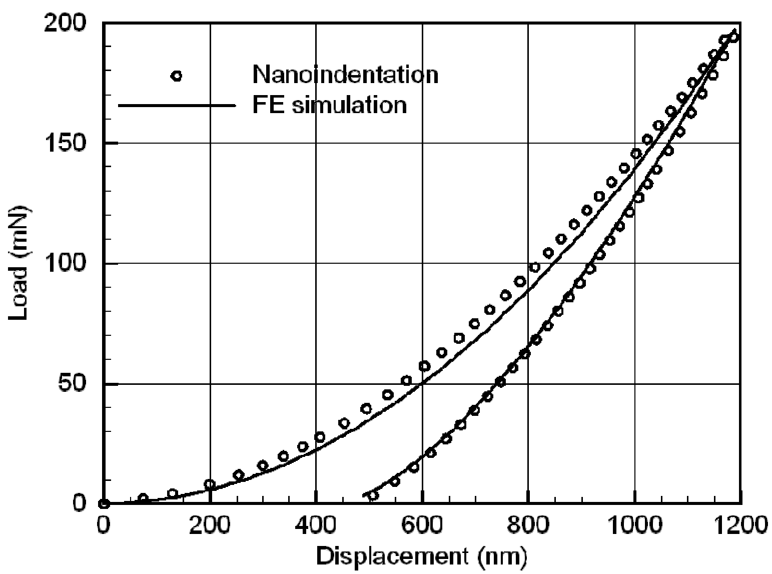


Variation of Hardness

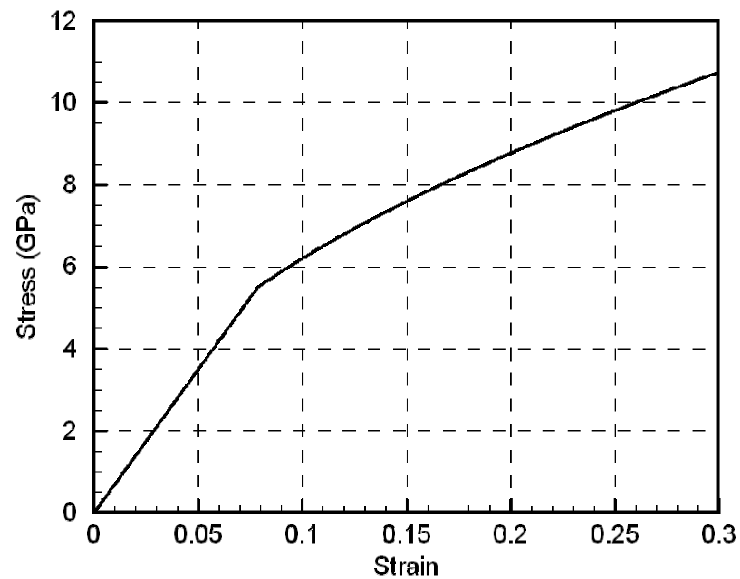
Predicting of Stress-strain using simulation



Axisymmetric FEM model of nanoindentation on sand grain



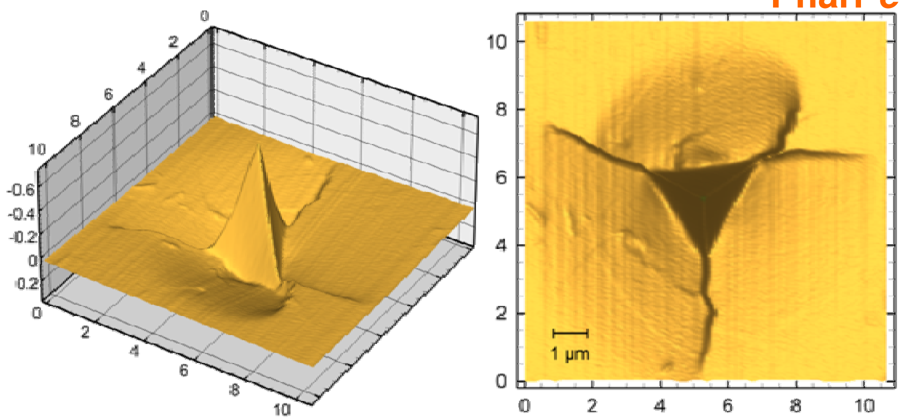
Fitting FEM results to experiments



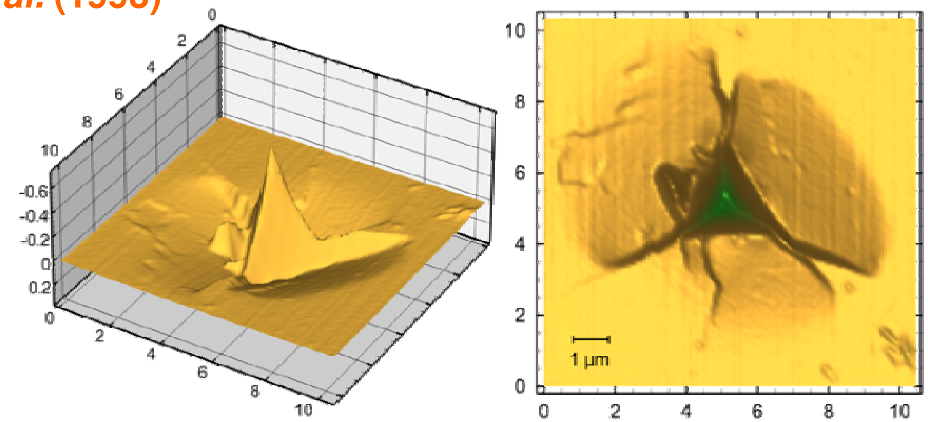
Predicted uniaxial stress-strain in compression

Fracture toughness from multiple sand grains

Pharr *et al.* (1993)

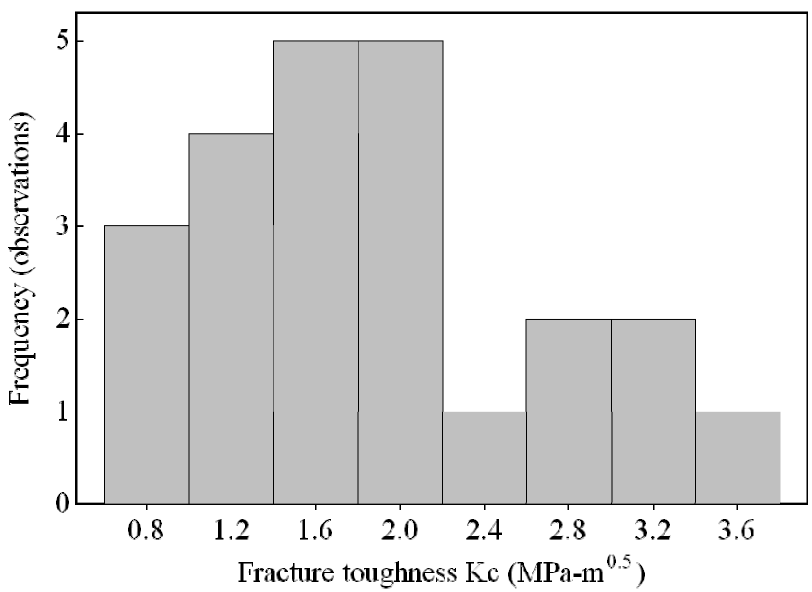


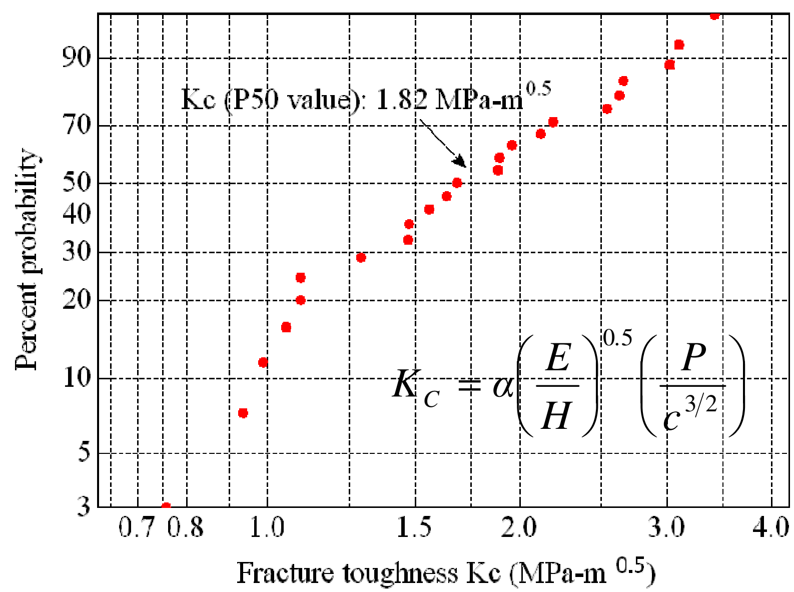
Max. Load: 80 mN



Max. Load: 70 mN

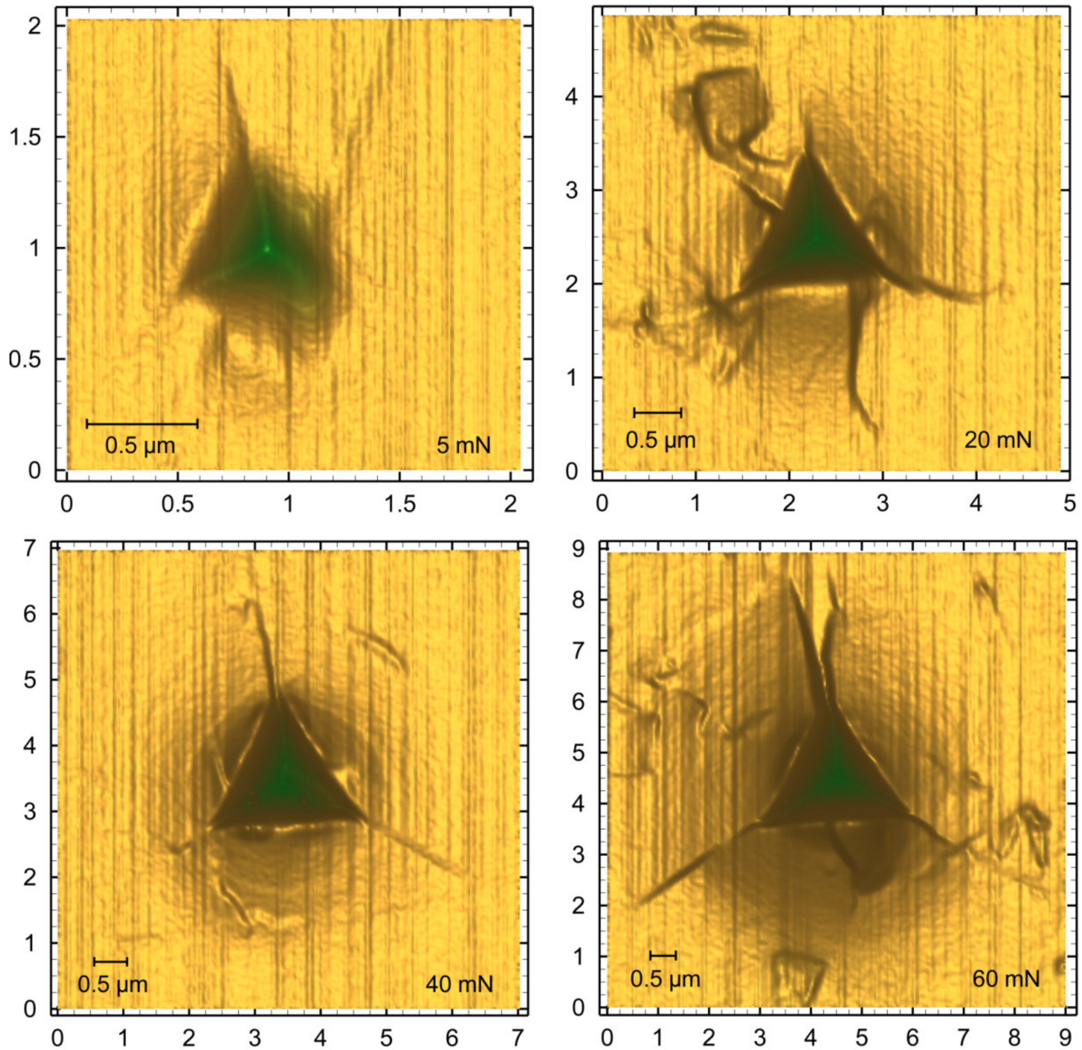
Indentation impressions from cube-corner tip, at different loads, for measuring crack length



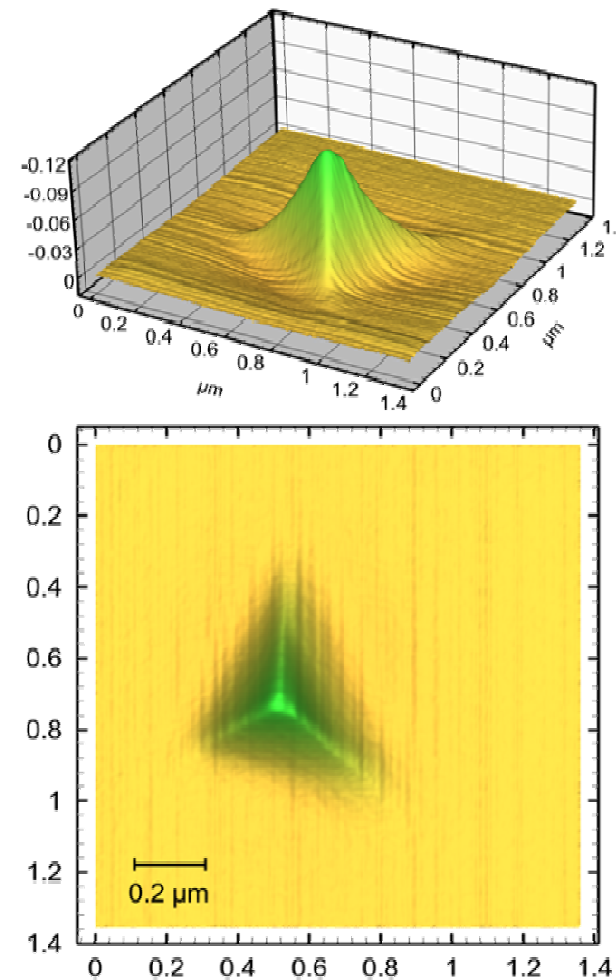


Variation of Fracture Toughness

Anisotropy & heterogeneity of sand grains



Indentation impressions from cube-corner tip, at increasing loads on a single sand grain

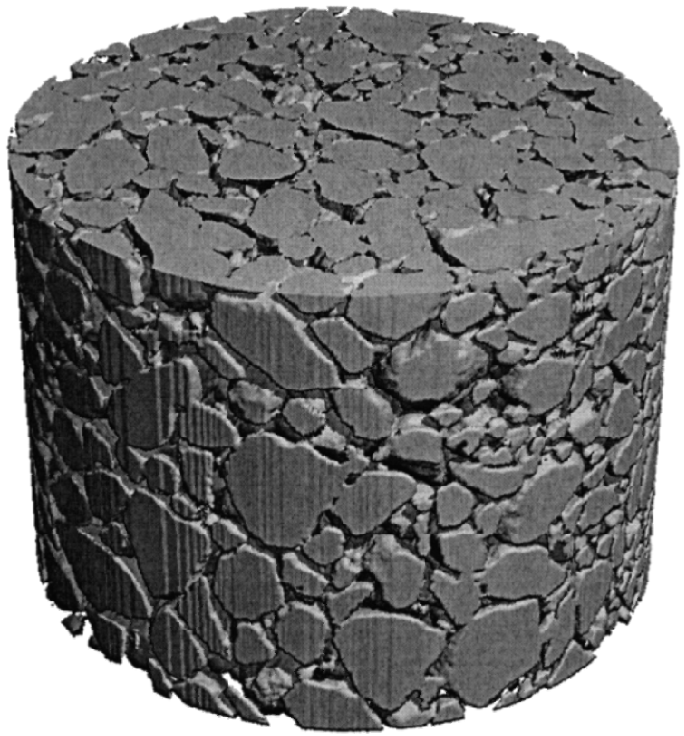


Inverse image of nanoindentation on a sand grain at 5 mN with no cracks formed

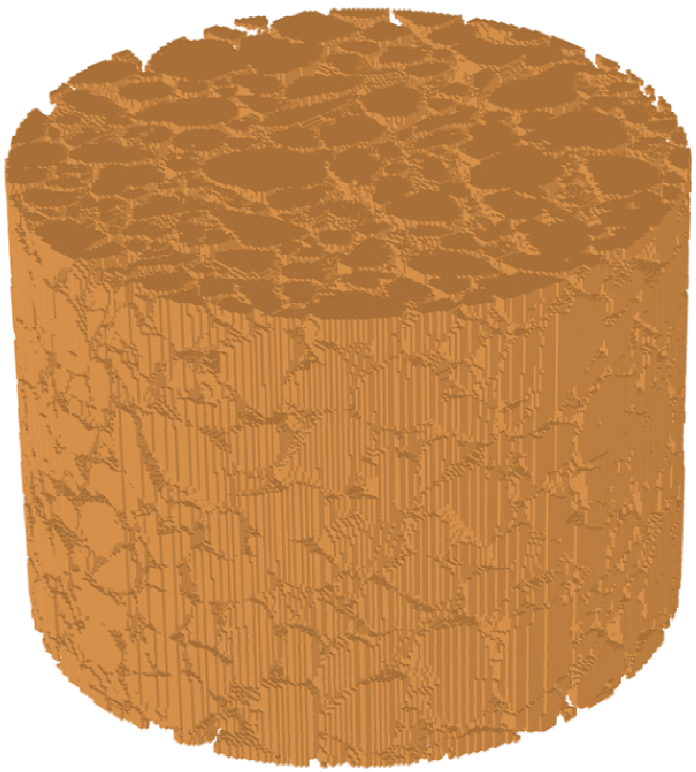
Simulation of sand in compression using MPM

Sulsky and Schreyer (2004), Daphalapurkar *et al.* (2007), Coker *et. al.* (2005)
Sulsky (2003), Bardenhagen and Brackbill (2000), Roessig *et al.* (2002),
Bardenhagen (1998), Bardenhagen *et al.* (2001), Voyiadjis *et al.* (2005)

Image from X-Ray μ-CT , after thresholding and 3D reconstruction.



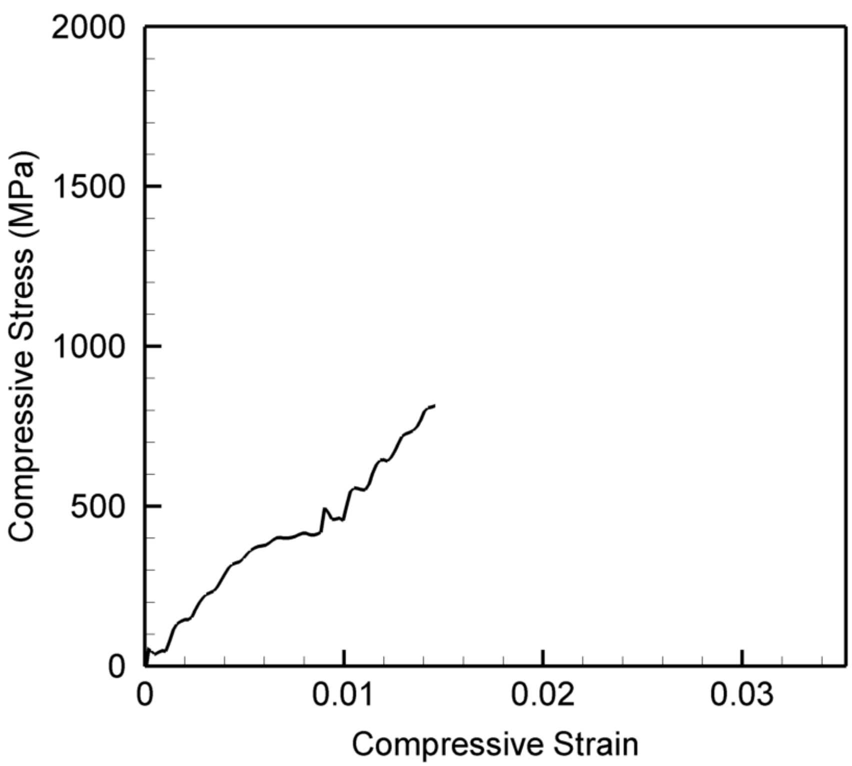
Representative 3D Model for high strain-rate compression, shear, and high-speed penetration into sand

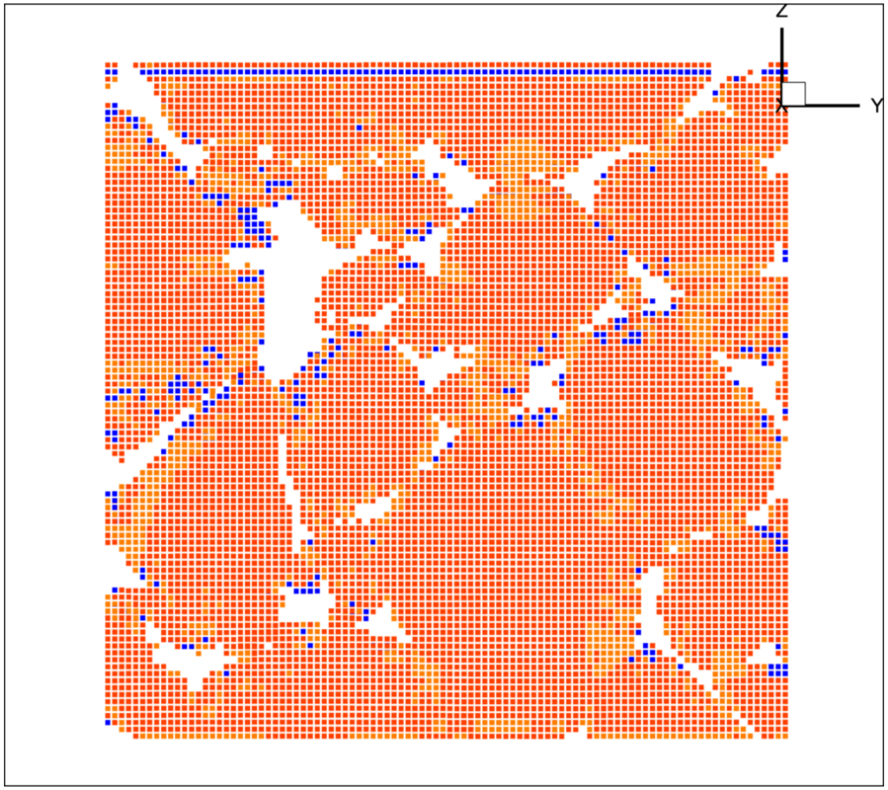


Discretized model in MPM

Simulation of sand in compression using MPM

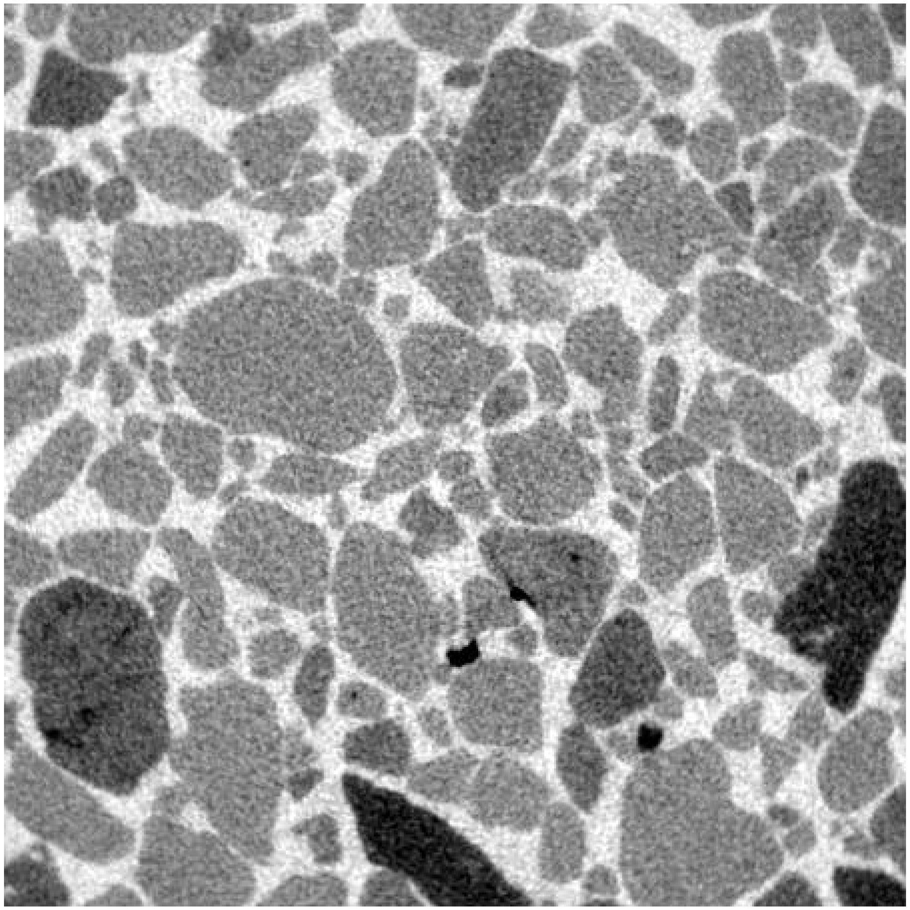
Stress-strain curve in compression





MPM simulation snapshot showing a section of sand in compression Colors indicate stress component σ_{zz} (blue indicates highest stress compressive)

Micro-computed tomography of sand



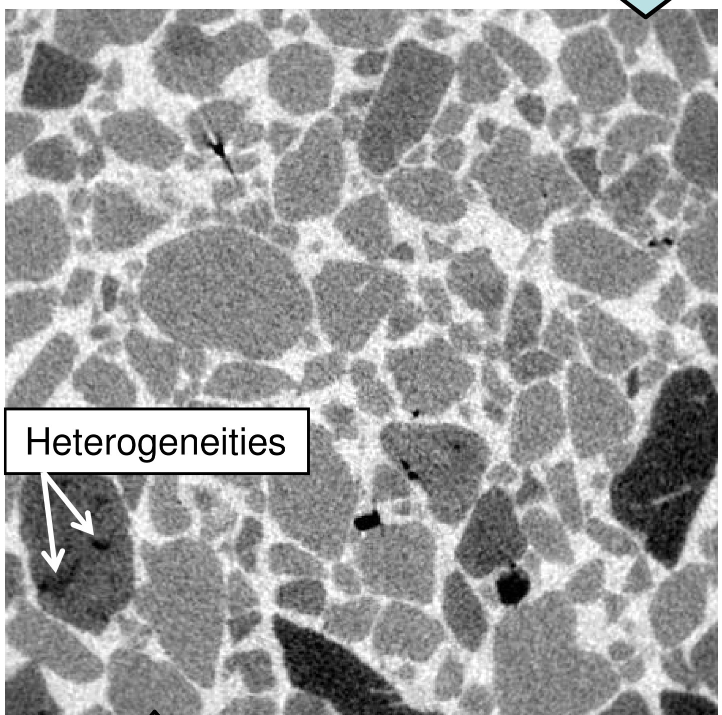
Grayscale value indicates density

Simulation of sand using Material Point Method (MPM)

Sulsky and Schreyer (2004), Daphalapurkar *et al.* (2007), Coker *et. al.* (2005) Sulsky (2003), Bardenhagen and Brackbill (2000), Roessig *et al.* (2002), Bardenhagen (1998), Bardenhagen *et al.* (2001), Voyiadjis *et al.* (2005)

Image from X-Ray μ-CT. Grayscale value indicates relative density.

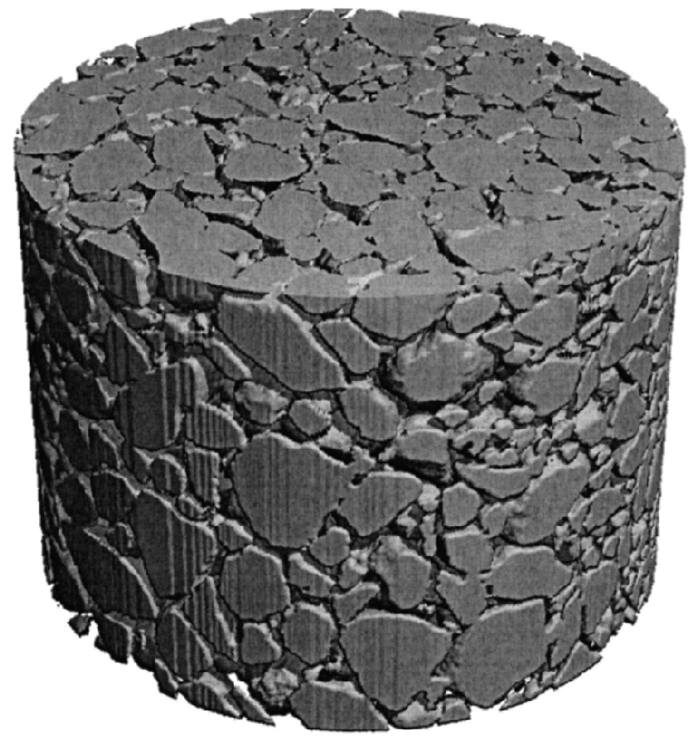
Sand grains material constitutive model and properties



Contact algorithm

Simulations at granular scale

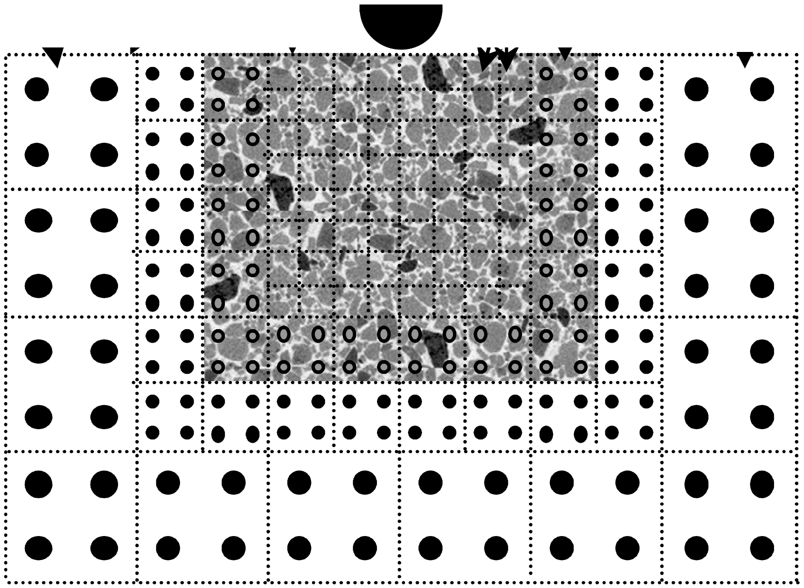
Image from X-Ray μ-CT , after thresholding and 3D reconstruction.



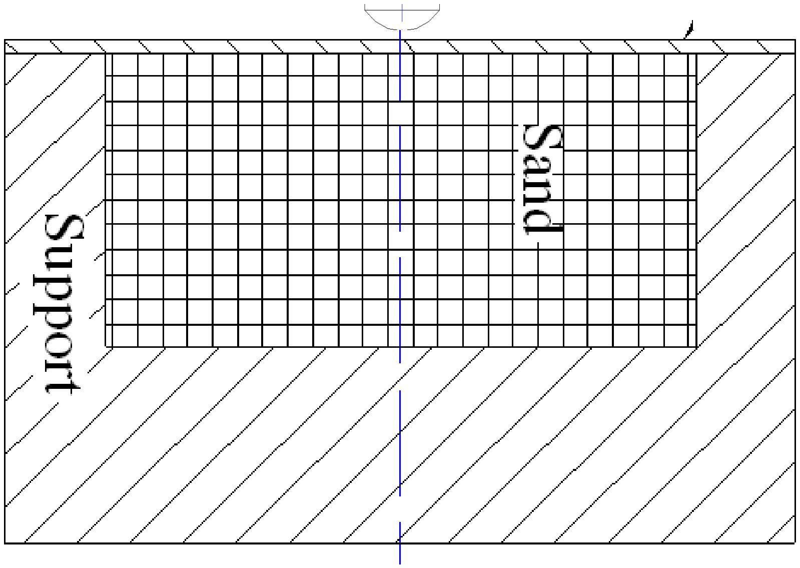
Representative 3D Model for high strain-rate compression, shear, and high-speed penetration into sand

3D multiscale simulations of sand using MPM and correlating with experimental results from SHPB

Sectional views



Multiscale simulation using MPM



Schematic of high-speed penetration experiments using Split Hopkinson Pressure Bar (SHPB)

Conclusions from the study on mechanical properties of sand grains

- Nanoindentation is an effective technique for characterization of mechanical properties of sand grains.

- Mechanical properties of sand at granular scale, mainly Young's modulus, hardness, stress-strain relation and fracture toughness, were determined.

- Within a single sand grain, the mechanical properties were found to vary, indicating the anisotropy, heterogeneity and presence of defects. Further, X-ray μ-CT results confirmed the observations. Representative Young's modulus for the sand grains was found to be 84.7 GPa (range 44.5 to 107 GPa), hardness to be 11 GPa (range 5.2 to 15.3 GPa), and fracture toughness to be 1.82 MPa-m$^{0.5}$ (range 0.75 to 3.4 MPa-m$^{0.5}$).

- μ-CT of sand was carried out to determine the granular structure. Mechanical properties determined using nanoindentation will be assigned to the sand grains. These will act as inputs for MPM simulations at granular scale.