Arctic Sea Ice Modeling with MPM

MPM Workshop

18 March 2008

K. Peterson[1]

D. Sulsky[1]

H. Schreyer[2]

[1]Department of Mathematics and Statistics

[2]Department of Mechanical Engineering

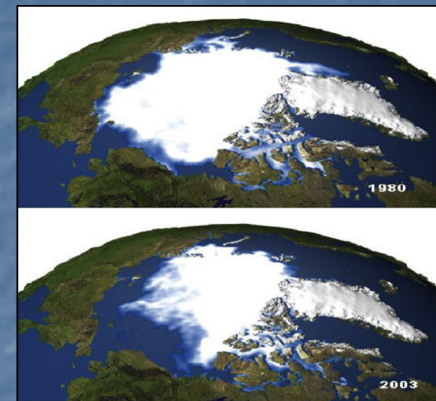
University of New Mexico

Outline

- Background
  - Motivation – Why Sea Ice?
  - Satellite Data
  - Goal
- Components of Sea Ice Model
  - Momentum equation
  - Constitutive model
  - Ice thickness distribution
  - Thermodynamics
- Numerical Implementation - MPM
- Satellite Data Based Kinematics
- Beaufort Sea Calculations
- Conclusions

Sea Ice Model Uses

- Climate Modeling



science.hq.nasa.gov


www.climateprogress.org

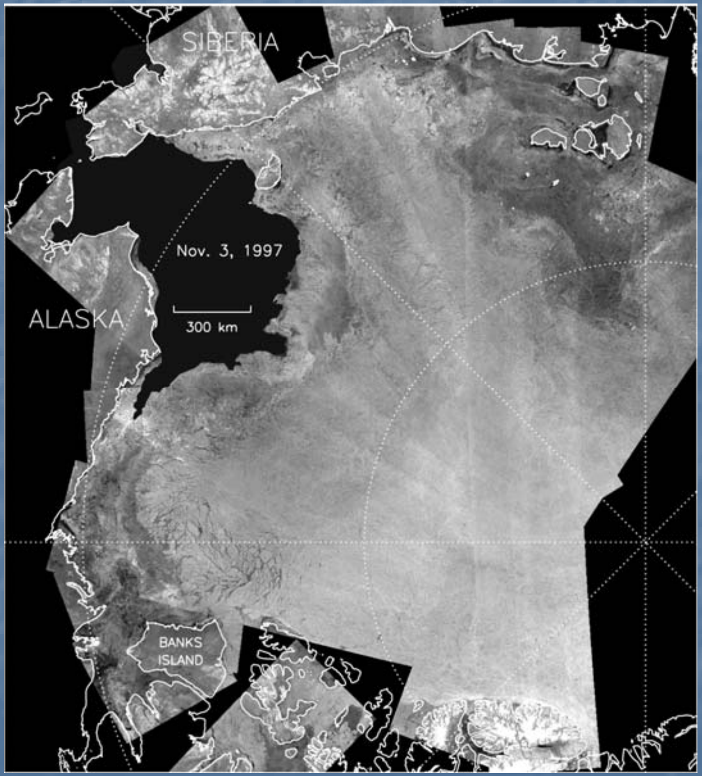
- Forecasting

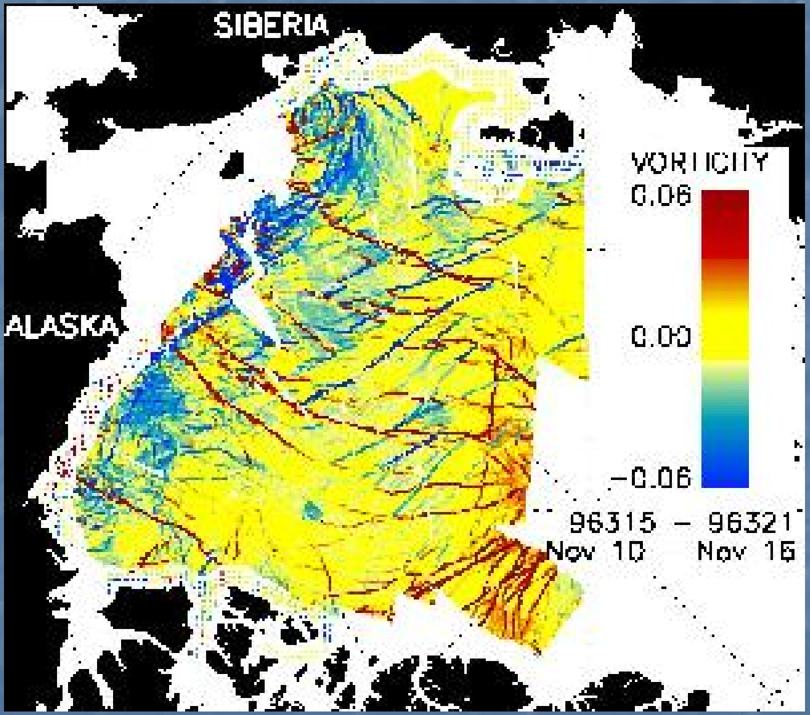
- Ice Structure Interactions


www.sandwell.com

Satellite Data

RADARSAT Geophysical Processor System (RGPS)





*Kwok, R., D. A. Rothrock, H. L. Stern and G. F. Cunningham, Determination of Ice Age using Lagrangian Observations of Ice Motion, *IEEE Trans. Geosci. Remote Sens.*, Vol. 33, No. 2, pp. 392-400, 1995.

Goal

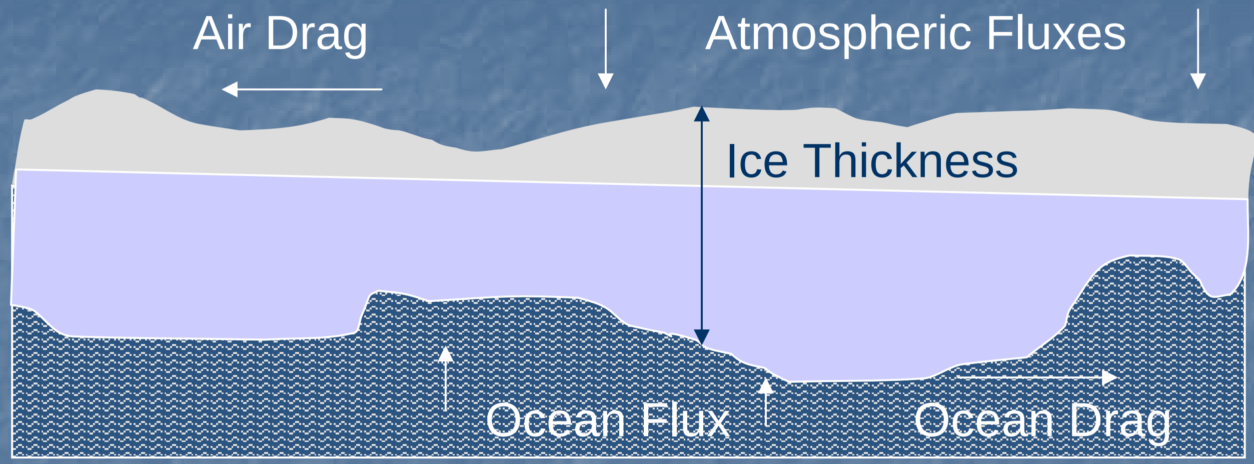
Want a numerically efficient sea ice model that includes observational features such as leads and ridges and uses available satellite data for verification

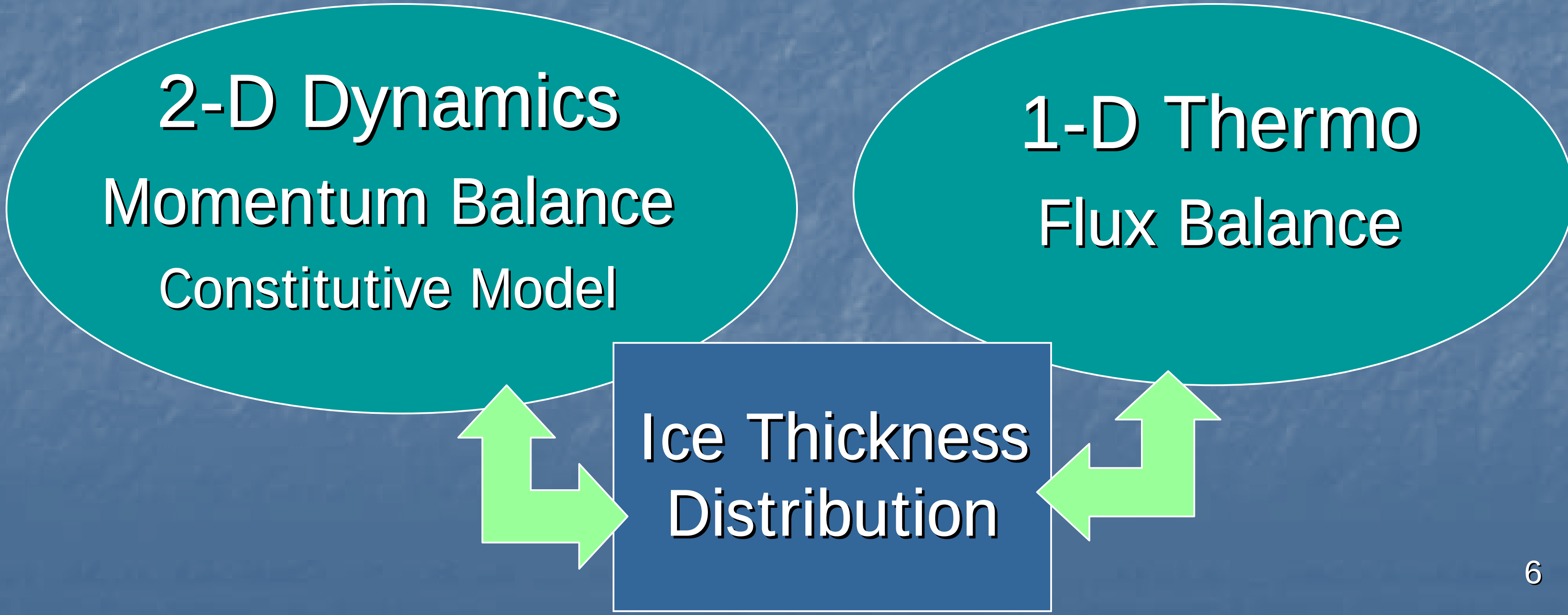
Lead

Ridge

- Existing Sea Ice Models
  - Isotropic constitutive model
  - Generally use Eulerian numerical schemes
- Our Model
  - Anisotropic constitutive model
  - Lagrangian material points

Components of Sea Ice Model





Momentum Equation

$$\rho h \frac{d\mathbf{v}}{dt} = \nabla \cdot (\sigma h) + \tau_a + \tau_w - \rho h f_c (\mathbf{e}_3 \times \mathbf{v})$$

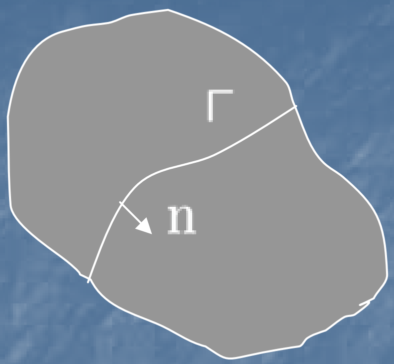
$$\mathbf{t}_a = c_a \rho_a \|\mathbf{v}_a\| \mathbf{R}_a \mathbf{v}_a$$

$$\mathbf{t}_w = c_w \rho_w \|\mathbf{v} - \mathbf{v}_w\| \mathbf{R}_w (\mathbf{v} - \mathbf{v}_w)$$

ρ = ice density
h = ice thickness
$\mathbf{v}$ = ice velocity
σ = stress tensor

τ_a = air drag
τ_w = water drag
f_c = Coriolis parameter

Elastic-Decohesive Constitutive Model



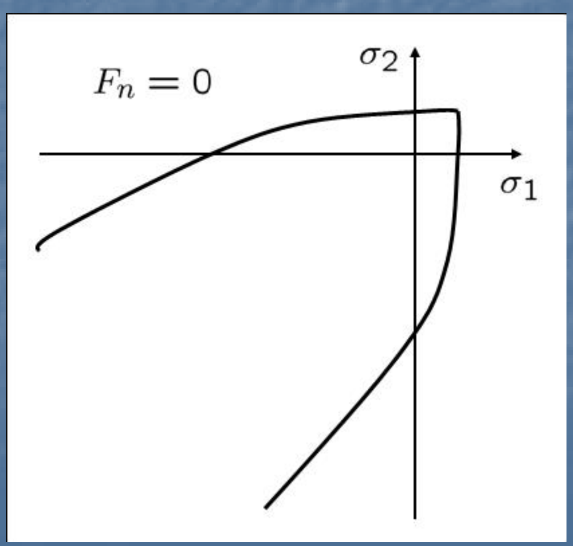
Strain Rate $\dot{\varepsilon} = \frac{1}{2}\left(\nabla \mathbf{v} + (\nabla \mathbf{v})^T\right)$

$$\varepsilon = \varepsilon^e + \varepsilon^d$$

Elasticity

$$\dot{\sigma} = \mathbb{E}\dot{\varepsilon}^e$$

Failure Function $F_n(\sigma)$



Decohesion

$$\varepsilon^d = ([\![\mathbf{u}]\!] \otimes \mathbf{n})^s \delta_\Gamma$$

Flow Rules $[\![\dot{u_n}]\!] = \omega \frac{\partial F}{\partial \tau_n}$

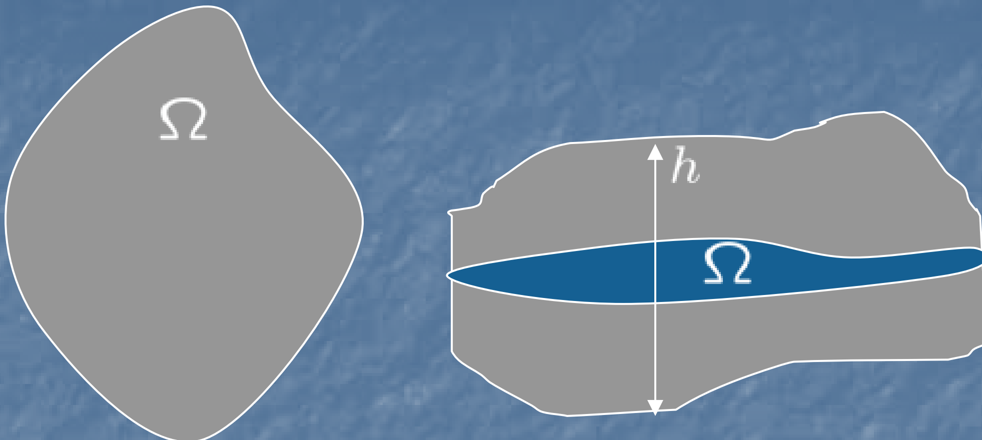
$$[\![\dot{u_t}]\!] = \omega \frac{\partial F}{\partial \tau_t}$$

$\tau_n = \mathbf{n} \cdot \sigma \cdot \mathbf{n}$ - normal traction

$\tau_t = \mathbf{n} \cdot \sigma \cdot \mathbf{t}$ - tangential traction

Schreyer, H., L. Monday, D. Sulsky, M. Coon, R. Kwok (2006), Elastic-decohesive Constitutive Model for Sea Ice, *J. of Geophys. Res.,* 111, C11S26, doi:10.1029/2005JC003334.

Ice Thickness Distribution



$$\int_{\Omega} d\Omega = R$$

$$\int_0^h g(h,t)dh = \frac{1}{R}A(h,t)$$

$$\int_0^\infty g(h,t)dh = 1$$

g = thickness distribution

R = ice region area

$A(h,t)$ = ice area with thickness less than h at time t

Evolution Equation

$$\frac{dg}{dt} = (-\nabla \cdot \mathrm{v})g - \frac{\partial}{\partial h}(fg) + \psi$$

$f = dh/dt$ = growth rate

ψ = mechanical redistribution (ridging)

Ridging Function

$$\psi = \delta(h) r_{op} + w_r r_{cl}$$

$$w_r(h) = \frac{-a(h) + n(h)}{-\int_0^{h_{max}} (-a(h) + n(h)) dh}$$

thickness distribution of ice participating in ridging: $a(h) = b(h) g(h)$

thickness distribution of newly ridged ice: $n(h)$

$$n(h) = \int_0^{h_{max}} a(\tilde{h}) \gamma(\tilde{h}, h) d\tilde{h}$$

Thermodynamics

Balance of Fluxes

Top

$$(1-\alpha)F_R - I_0 + F_L - \epsilon_L \sigma T_0^4 + F_s + F_l + k_0 \left(\frac{\partial T}{\partial z}\right)_0 = \left\{ \begin{array}{ll} 0 & \text{for } T_0 < 0^\circ C \\ -q_s \frac{dh}{dt} & \text{for } T_0 = 0^\circ C \end{array} \right\}$$

Bottom

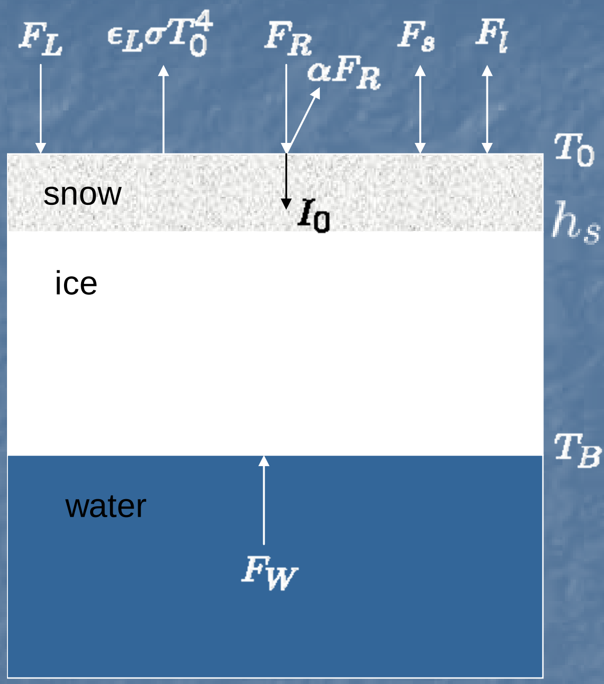
$$k_B \left(\frac{\partial T}{\partial z}\right)_B - F_w = q_B \frac{dh}{dt}$$

Snow/Ice Interface

$$k_s \left(\frac{\partial T}{\partial z}\right)_{h_s} = k_i \left(\frac{\partial T}{\partial z}\right)_{h_s}$$

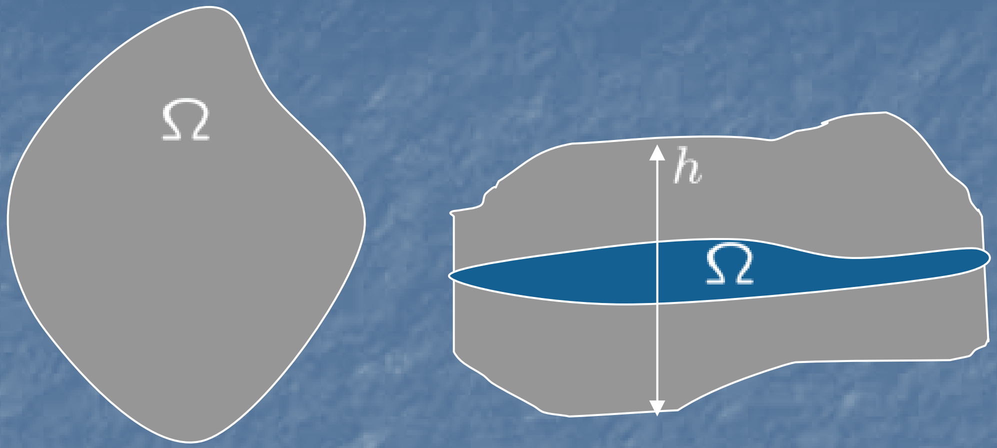
Diffusion

$$(\rho c)\frac{\partial T}{\partial t} = \frac{\partial}{\partial z} k \frac{\partial T}{\partial z} + \kappa I_0 e^{-\kappa z}$$



h = thickness
T = temperature
q_s = energy of melting at top
q_B = energy of melting at bottom
c = heat capacity
k = conductivity
κ = extinction coefficient

MPM for Sea Ice



$$\Omega = \bigcup_{p=1}^{N_p} \Omega_p$$

$$m_p \approx \rho_p \bar{h}_p \Omega_p$$

$$m_i \mathbf{a}_i = F_i^{int} + F_i^{ext}$$

$$(F_x)_i^{int} = \sum_{p=1}^{N_p} \Omega_p \bar{h}_p \left((\sigma_{xx})_p \frac{\partial N_i}{\partial x}(\mathbf{x}_p) + (\sigma_{xy})_p \frac{\partial N_i}{\partial y}(\mathbf{x}_p) \right)$$

$$(F_y)_i^{int} = \sum_{p=1}^{N_p} \Omega_p \bar{h}_p \left((\sigma_{xy})_p \frac{\partial N_i}{\partial x}(\mathbf{x}_p) + (\sigma_{yy})_p \frac{\partial N_i}{\partial y}(\mathbf{x}_p) \right)$$

$$F_i^{ext} = \sum_{p=1}^{N_p} \left(\Omega_p \tau_a + \Omega_p \tau_w + m_p f_c (\mathbf{e}_3 \times \mathbf{v}_p) \right) N_i(\mathbf{x}_p)$$

Ice Thickness Distribution in MPM

Discrete ice thickness categories

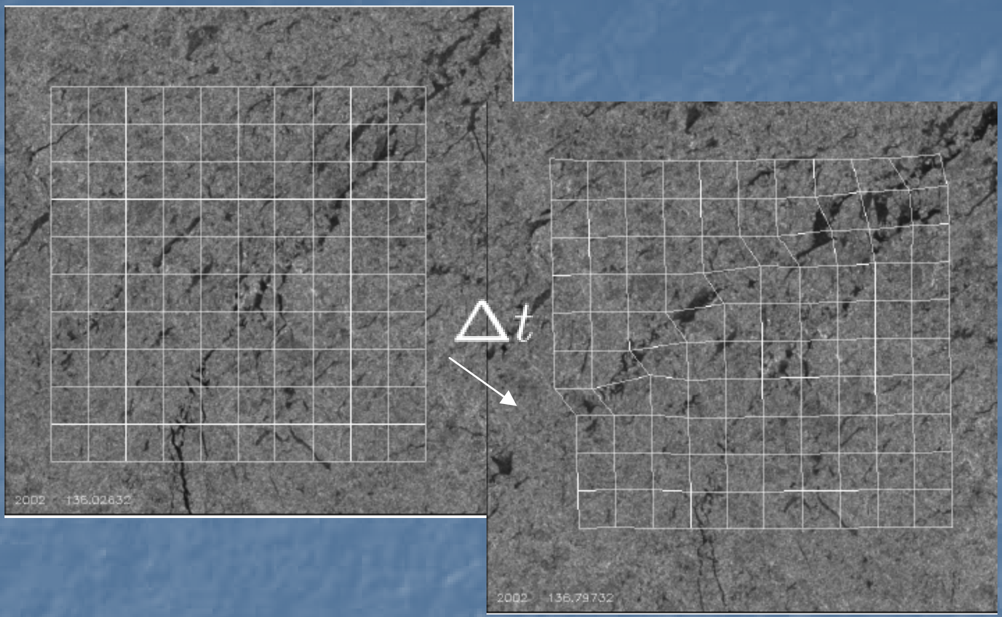
$$g_{p,n} = \int_{h_{n-1}}^{h_n} g_p(h,t)\,dh \qquad 1 = \sum_{n=0}^{N_h} g_{p,n} \qquad \bar{h}_{p,n} = \frac{v_{p,n}}{g_{p,n}}$$

$$v_{p,n} = \int_{h_{n-1}}^{h_n} h g_p(h,t)\,dh \qquad \bar{h}_p = \frac{1}{N_h}\sum_{n=0}^{N_h} v_{p,n}$$

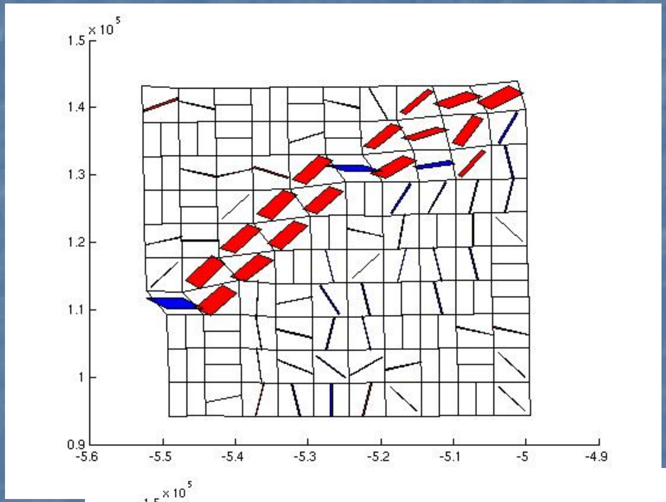
Solve in three pieces

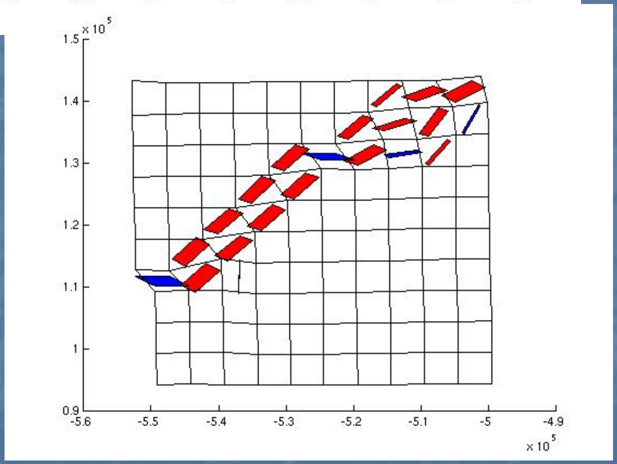
- Horizontal Transport $\dfrac{dg_{n,p}}{dt} = (-\nabla \cdot \mathbf{v})g_{n,p}$
- Transport in Thickness Space $\dfrac{dg_{n,p}}{dt} = -\displaystyle\int_{h_{n-1}}^{h_n} \frac{\partial(gh)}{\partial h}\,dh$
- Redistribution $\dfrac{dg_{n,p}}{dt} = \psi_n$

Kinematics

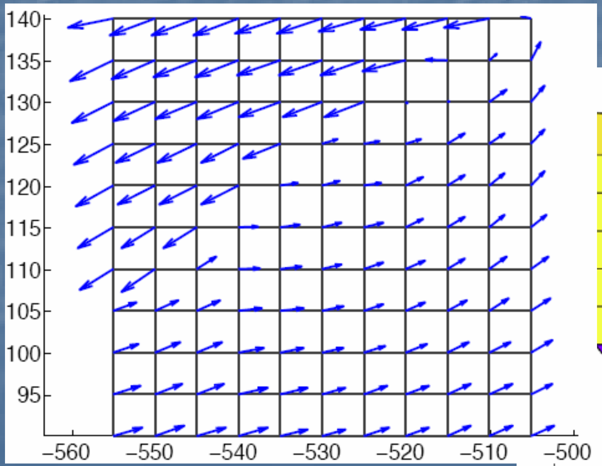


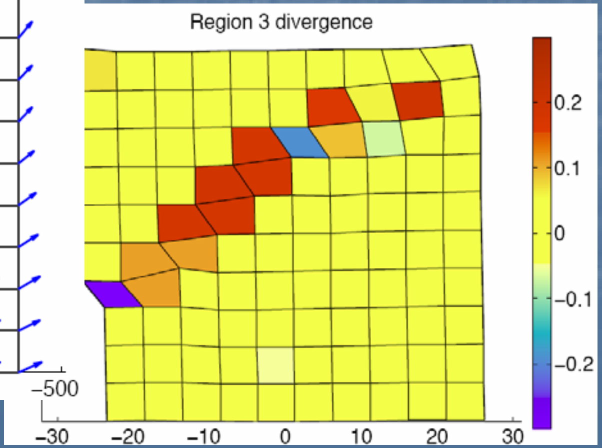
No Cutoff





400 m Cutoff



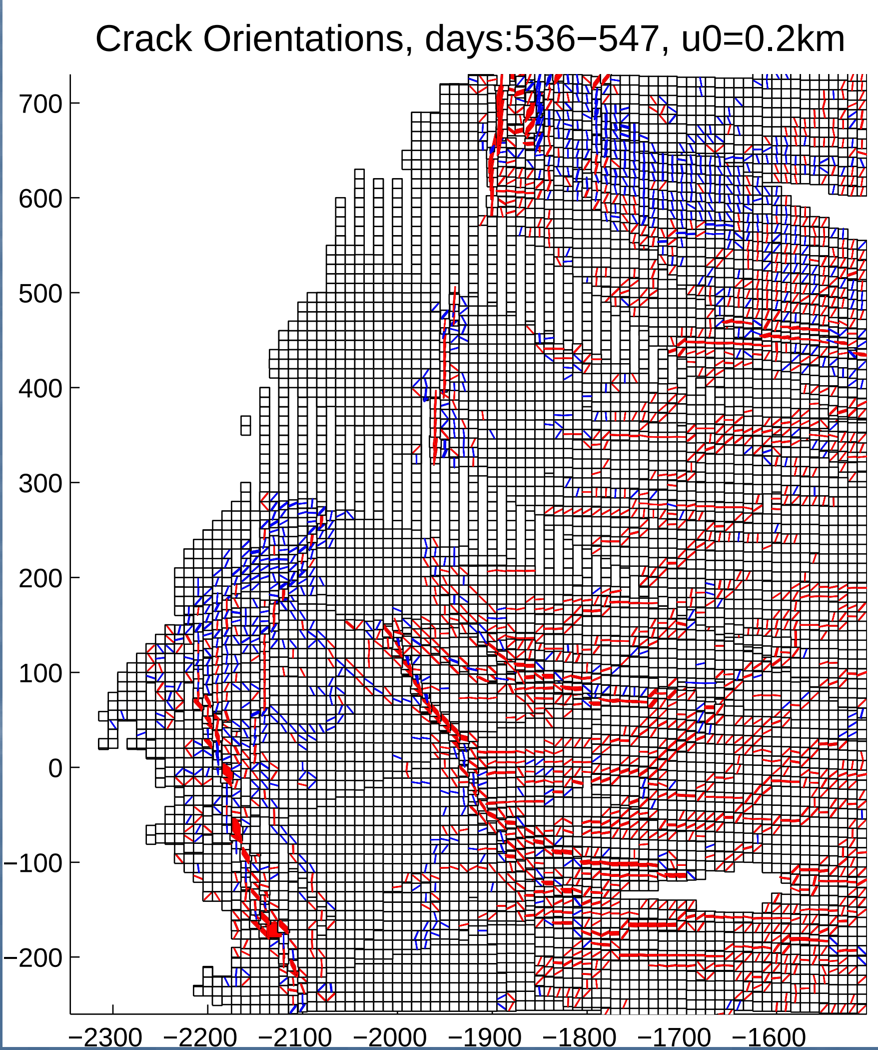


Beaufort Sea Calculations

- Using RGPS data for 23 Feb – 10 Mar 2004 in Beaufort Sea region
- Calculation setup
  - 10 km square background grid
  - 4 material points per cell
  - Rigid material points for land boundary
  - Including wind, ocean, and Coriolis forces
  - Boundary conditions are RGPS velocities linearly interpolated in time
  - RGPS data used to initialize leads in calculation

Beaufort Sea Calculations

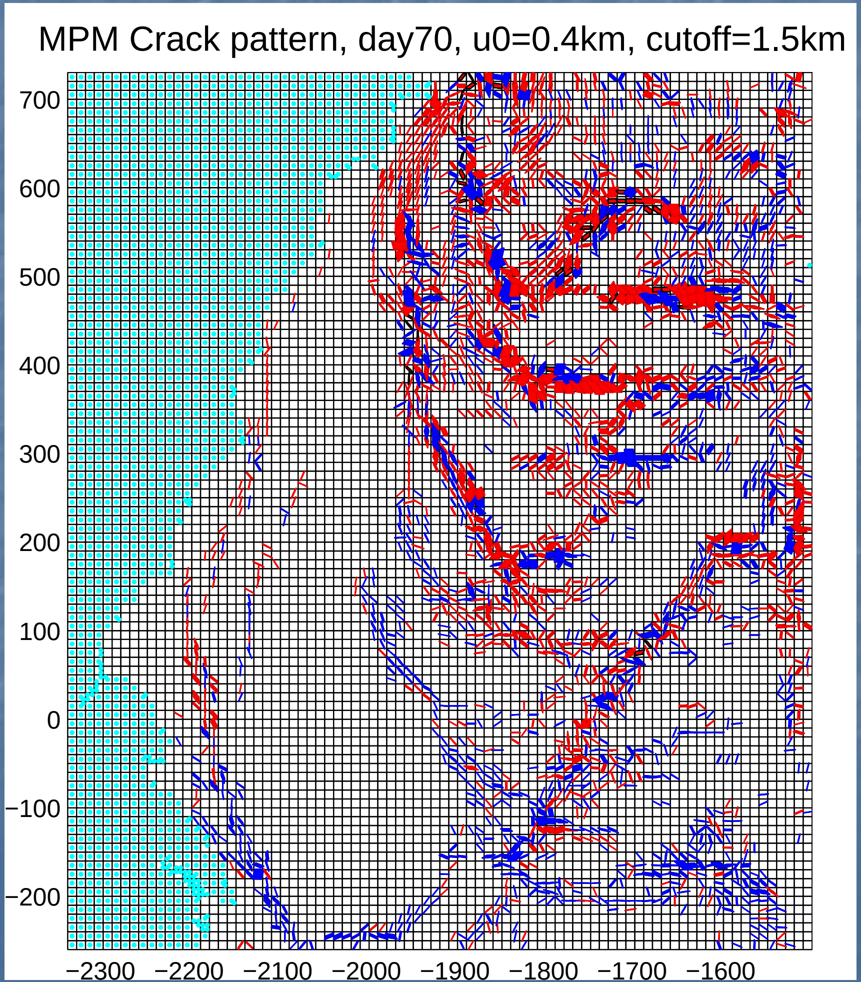
Initialization Day 54 (Feb 23)

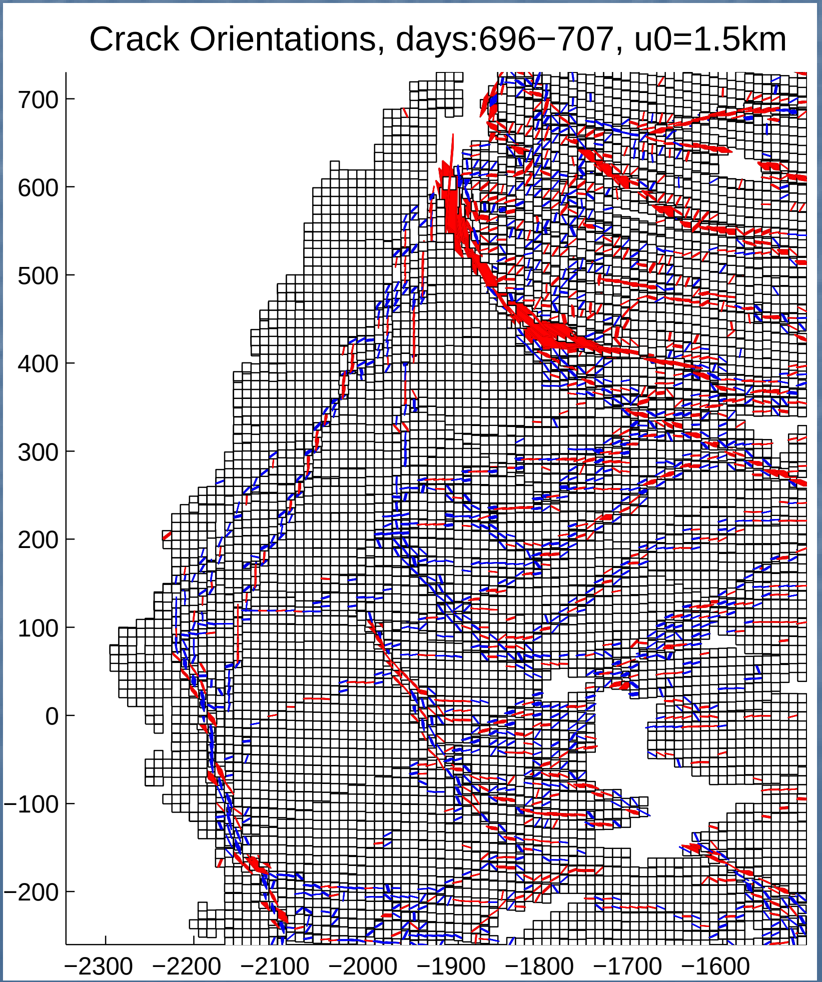


Beaufort Sea Calculations

Leads Day 70 (March 11)

MPM Kinematics

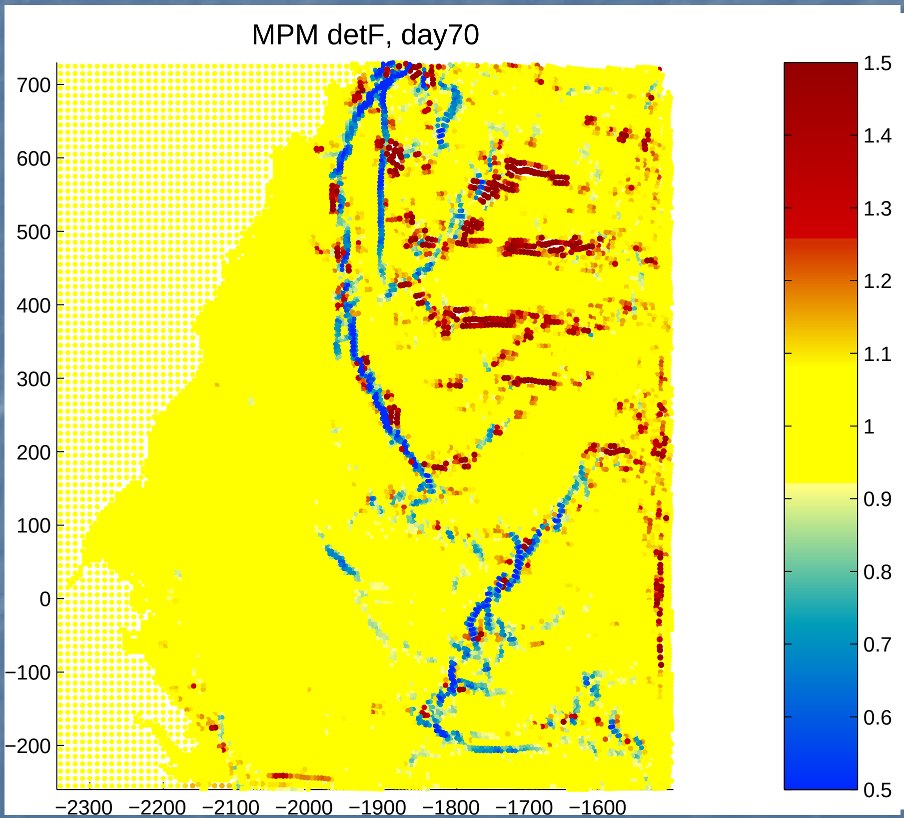




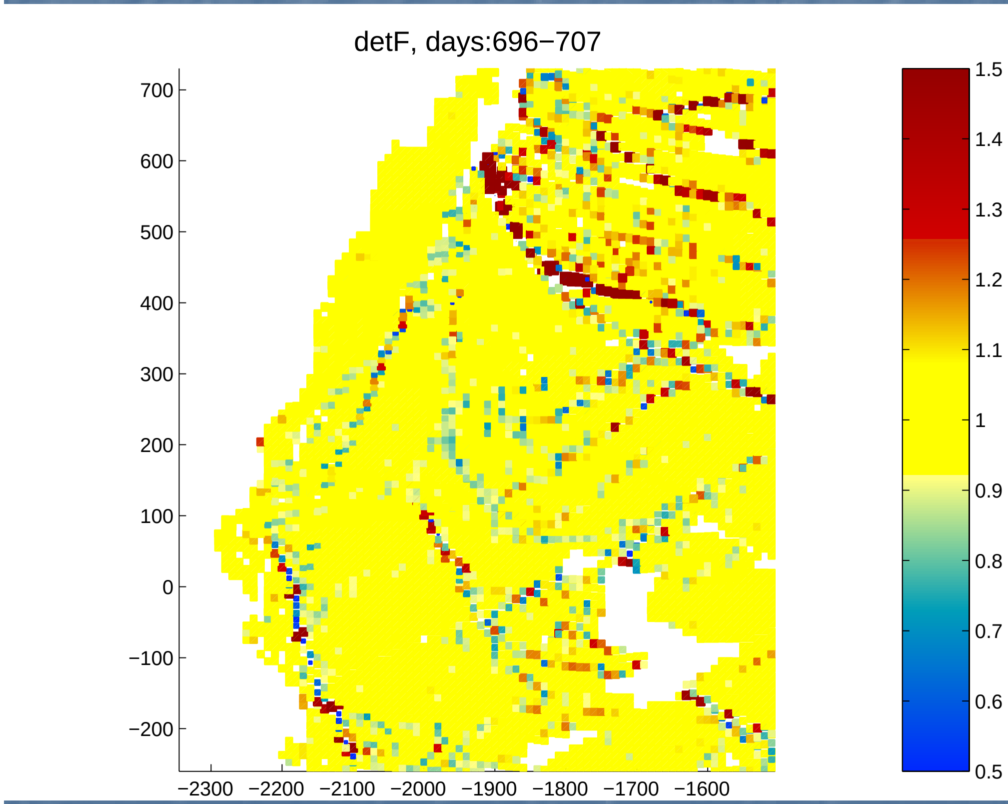
Beaufort Sea Calculations

Det (F) Day 70 (March 11)

MPM



Kinematics



Conclusions

- Have shown sea ice model using MPM with Elastic-Decohesive constitutive model
- Advantages over other models
  - MPM handles advection naturally
  - Elastic-Decohesive Model allows explicit calculation of lead evolution
- Currently implementing ice thickness distribution
- Future work
  - Implement thermodynamic model
  - Connect to ocean and atmospheric models

Acknowledgments

- Prof. Deborah Sulsky
  - UNM Dept. of Mathematics and Statistics
- Prof. Howard Schreyer
  - UNM Dept. of Mechanical Engineering

- This work has been supported by the NSF under Grant No. DMS-0222253

www.jupiterimages.com