Mechanical Properties of Snow as a Random Heterogeneous Material using Uintah

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Outline

• Motivation
• Stochastic reconstruction of snow microstructure
• Representative Volume Element
  – Elastic
  – Viscoplastic
• Application – Microscale plane strain indentation
• Future work
Motivation: large uncertainties in properties

• Environmental conditions dictate snow metamorphism ('sintering' process)
• Metamorphism determines microstructure
• Microstructure determines properties
  – Mechanical (elasticity, viscoplasticity, damage, fracture…)
  – Physical (thermal conductivity, permeability, dielectric constant…)
• Density alone is insufficient to characterize properties
• Process -> microstructure -> properties
Applications

• Vehicle-snow interaction (ground and air vehicles)
• Civil infrastructure (foundation, pavement, runway …)
• Avalanche
• Sports
• Geophysical
• Extraterrestrial (comets …)
Vehicle-Snow Interaction

Tire/Vehicle Dynamics

Tire-Snow Interactions

Tires

Snow
Tire Models

- Smooth
- Grooved
- Treaded
2-D Tire-Snow Interaction: Abaqus, 200 kg/m^3, Drucker-Prager (CRREL) model

Density distribution, grey region indicates density larger than 700 kg/m3
Stochastic Reconstruction of Snow Microstructure from X-Ray Tomography Images

• Properties of snow strongly depend on microstructure – one major source of uncertainties
• Structure-property relationships needed to understand physical mechanisms of deformations and failure
• Build digital stochastic models to represent snow microstructures
• Stochastic geometry and mechanics
What is stochastic reconstruction?

Generate a simulated microstructure having the same statistical characteristics as the real one.
Porosity (pore volume fraction)

Probability that two points a distance $r$ apart will lie in pore space

Two-point probability function
Reconstruction Steps using Gaussian Random Fields

1. Find one-point and two-point correlation functions from snow images
2. Solve for level cut parameter
   \[ \alpha = \sqrt{2} \cdot \text{erf}^{-1} \left( 1 - 2 p_{\text{expt}} \right) \]

Determine function \( g \) from experimental one-point and two-point correlation functions by solving:
\[
I ( g(r_i) ) = 2\pi \left( p_{\text{expt}} - p_{\text{expt}}^{(2)} (r_i) \right)
\]

3. Solve three unknown parameters in \( g \): \( \xi, r_c, d \)
4. Numerically generate Fourier transform coefficients
5. Perform 3D inverse FFT to generate discrete GRF
6. Perform one-level cut to get phase function in spatial domain

\[
p_{\text{expt}}, p_{\text{expt}}^{(2)} (r_i) \rightarrow \alpha, g(r_i) \rightarrow \xi, r_c, d \quad (\text{i.e. } \rho(k)) \rightarrow Y_{\alpha\beta\gamma} \rightarrow y_{lmn} \rightarrow \phi(r_{lmn})
\]
Skyscan 1172 Microtomography
Snow Sample Holder

Diameter 1 cm
Grey-level Cross-Sectional Image
Sieved Snow < 1 mm Grain Size

7.344 mm by 7.344 mm, density 387 kg/m$^3$
Resolution: 1225 by 1225, Pixel size: 6 micron

Brighter pixels represent ice
3-D Visualization of a Cube of Snow Microstructure
Side Length = 3.618 mm
Reconstruction results

Comparison of two-point correlation functions

Translation distance, \( r \), micron

- ▲ real snow microstructure
- □ reconstructed one
Reconstruction Results

Reconstructed microstructure

Link to real one
Representative Volume Element (RVE)

• Definition

• Elastic Properties
  – Theoretical bounds
  – Initial results

• Viscoplastic Properties
  – SUVIC-I
  – Initial results
Representative Volume Element (RVE) for Mechanical Properties

• Definition (Nemat-Nasser and Hori):
  – *RVE for a material point of a continuum mass is a material volume which is statistically representative of the infinitesimal material neighborhood of that material point.*
  – RVE is the volume element over which homogenization can be performed.
  – Size of an RVE depends on the physical or mechanical properties of interest.
  – Size of an RVE requires a tolerance.
  – Size of an RVE should be independent of boundary conditions.
  – Size of volume smaller than RVE is called an SVE (statistical volume element).
RVE of Elastic Moduli

- Numerical calculation of elastic moduli of scanned images and reconstructed volume.
- Using elastic material properties so ‘error’ due to creep or time-dependent effects won’t be present.
- Relatively ‘easy’ to conduct.
- Several numerical methods available – finite element method using voxel-based or solid-based mesh.
- Material Point Method (MPM) used:
  - Snow is considered as a semi-granular material.
Elastic Moduli using Uintah
MPM Implicit

- Unconfined compression
- Load-displacement -> Macroscopic stress and strain -> Young’s modulus and Poisson’s ratio
- Largest size - 2.8 million cells, 83 million particles
- Nominal density 387 $\frac{kg}{m^3}$
- Ice properties
  - Young’s modulus 9.3 GPa
  - Poisson’s ratio 0.325
Hashin-Sritkman Upper Bound

\[ \phi_1 = \text{volume fraction of air} \]

\[ \phi_2 = \text{volume fraction of ice} \]

\[ G, K = \text{Shear and bulk modulus of ice}. \]

\[ K_U = K \phi_2 - \frac{\phi_1 \phi_2 K^2}{3} \]

\[ G_U = G \phi_2 - \frac{\phi_1 \phi_2 G^2}{3} \]

\[ H_2 = G \left[ \frac{3}{2} K + \frac{4}{3} G \right] \]
Uintah Results - Young’s Modulus

Test data (CR 97): 10MPa - 0.8GPa
Uintah Results - Poisson’s Ratio

Test data (CR 97): 0.22 - 0.35
Viscoplasticity
SUVIC-I (Aubertin and Lee)

• Strain rate history-dependent Unified Viscoplastic model with Internal variables for Crystalline materials – Ice

• Isotropic polycrystalline ice at

\[ T \geq -55^0 C; 10^{-8} \leq \varepsilon \leq 10^{-2} \, s^{-1}; 0.04 \text{MPa} \leq \sigma_{equiv} \leq 20 \text{MPa} \]

• Unified model – plasticity, creep and their interactions are modeled in the same way

• Three internal variables: back stress (kinematic hardening), yield and drag stress (isotropic hardening)

• Evolution of the state variables: combined action of hardening, dynamic recovery

• Viscoplastic – introduction of a yield surface makes a clear distinction between elastic and inelastic behavior.
SUVIC-I - continued

• Part of the inelastic strains are recoverable – grain boundary sliding, reverse motion of dislocations (backstress)
• Hardening has mixed (kinematic and isotropic) nature related to the existence of internal stresses
• Kinematic hardening due to backstress created by directional obstacles to dislocations motion.
Summary of SUVIC-I

\[ \dot{\varepsilon}_{ij} = \dot{\varepsilon}_{ij}^e + \dot{\varepsilon}_{ij}^l; \quad \dot{\varepsilon}_{ij}^e = \frac{\dot{S}_{ij}}{2G} + \frac{\dot{\sigma}_{kk}}{9K} \delta_{ij}. \]

\[ \dot{\varepsilon}_{ij}^l = A \left( \frac{X_{ae} - R}{K} \right)^N n_j \exp \left( -\frac{Q}{R_g T} \right), \quad n_j = \frac{3}{2} \frac{S_{ij} - B_{ij}}{X_{ae}}, \]

\[ X_{ae} = \frac{3}{2} (S_{ij} - B_{ij})(S_{ij} - B_{ij}) \]

\[ \dot{\varepsilon}_{ij}^l = \frac{2}{3} \dot{\varepsilon}_{ij}^l \dot{\varepsilon}_{ij}^l = A \left( \frac{X_{ae} - R}{K} \right)^N \exp \left( -\frac{Q}{R_g T} \right) \]

\[ \dot{B}_{ij} = \frac{2}{3} A \dot{\varepsilon}_{ij}^l - A \frac{1}{B_e^e} B_{ij}, \quad B_e = B_0 \left( \frac{\dot{\varepsilon}_e^l}{\dot{\varepsilon}_0^l} \right)^{\frac{1}{n}} \]

\[ \dot{R} = A_3 \dot{\varepsilon}_e^l \left( 1 - \frac{R}{R'} \right), \quad \dot{K} = A_3 \dot{\varepsilon}_e^l \left( 1 - \frac{K}{K'} \right). \]

\[ R' = R_0 \left( \frac{\dot{\varepsilon}_e^l}{\dot{\varepsilon}_0^l} \right)^{\frac{1}{n}}, \quad K' = \frac{\dot{\varepsilon}_e^l}{A \exp \left( -\frac{Q}{R_g T} \right)} \left( \frac{X_{ae} - R'}{R'} \right) \]

\[ X_{ae}' = \sigma' - B_e'; \quad \sigma' = \sigma_0 \left( \frac{\dot{\varepsilon}_e^l}{\dot{\varepsilon}_0^l} \right)^{\frac{1}{n}} \]
Numerical Integration

\[
\Delta \varepsilon^i = \Delta t \left[ (1 - \theta) \dot{\varepsilon}_t^i + \theta \dot{\varepsilon}_{t+\Delta t}^i \right]; \quad 0 \leq \theta \leq 1
\]

\[
\dot{\varepsilon}_t^i = \dot{\varepsilon}_t^i + \sum_i \frac{\partial \dot{\varepsilon}_t^i}{\partial \beta_i} \beta_i \Delta t.
\]

\[
\dot{\varepsilon}_{t+\Delta t}^i = \dot{\varepsilon}_t^i + \sum_i \frac{\partial \dot{\varepsilon}_t^i}{\partial \beta_i} \beta_i \Delta t.
\]

\[
\dot{\sigma}_{ij} = \left[ \dot{\varepsilon}_{ij} - \frac{\xi}{1 + \xi} \frac{1}{H} P_{ij} Q_{kl} \right] \dot{\varepsilon}_{kl} - \frac{\dot{\varepsilon}_t^i}{1 + \xi} P_{ij}.
\]
Viscoplastic Behavior – Uintah MPM Implicit SUVIC-I

Compression of 2.4mm-sidelength Cube (4.2e-04 1/sec)

Stress (MPa) vs. Strain

Scanned
Gaussian Random Field
Microscale Plane Strain Indentation (1/3)  
(7.344mm x 7.344mm x 0.012 mm)  
Uintah MPM Implicit SUVIC-I
Microscale Plane Strain Indentation (2/3)
Plane Strain Indentation @20% strain (3/3)
Future Work

- Obtain statistical distributions of the elastic and tangent moduli of real and simulated snow
- Optimize the code for SUVIC-I
- Implement damage, failure models of ice into Uintah
- Conduct simulations of triaxial cell and micropenetrometer (snow pen)
- Conduct microscale tension/compression experiments inside MicroCT and compare with simulations
- Develop continuum constitutive laws
- …
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Questions?