Mechanical Properties of Snow as a Random Heterogeneous Material using Uintah

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Outline

- Motivation
- Stochastic reconstruction of snow microstructure
- Representative Volume Element
  - Elastic
  - Viscoplastic
- Application – Microscale plane strain indentation
- Future work

Motivation: large uncertainties in properties

- Environmental conditions dictate snow metamorphism (‘sintering’ process)
- Metamorphism determines microstructure
- Microstructure determines properties
  - Mechanical (elasticity, viscoplasticity, damage, fracture…)
  - Physical (thermal conductivity, permeability, dielectric constant…)
- Density alone is insufficient to characterize properties
- Process -> microstructure -> properties

Applications

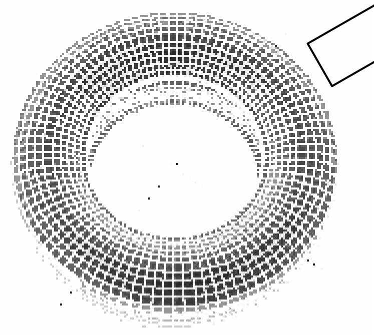
- Vehicle-snow interaction (ground and air vehicles)
- Civil infrastructure (foundation, pavement, runway …)
- Avalanche
- Sports
- Geophysical
- Extraterrestrial (comets …)

Vehicle-Snow Interaction

Tire/Vehicle Dynamics

Tire-Snow Interactions

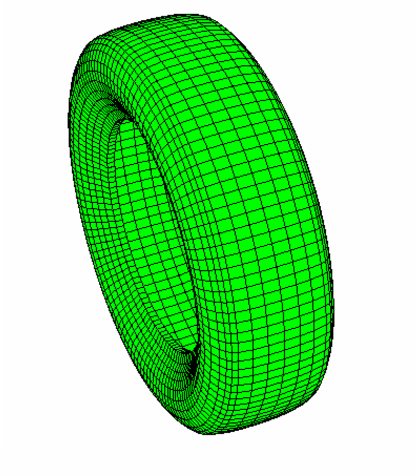
Tires



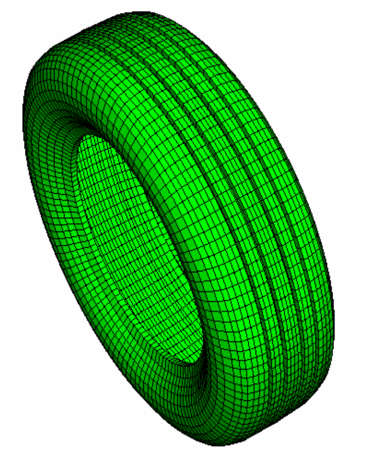
Snow

Tire Models

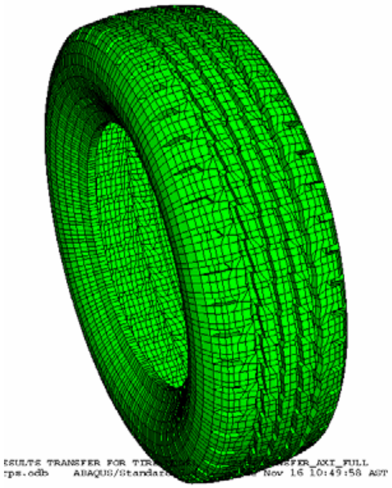
Smooth



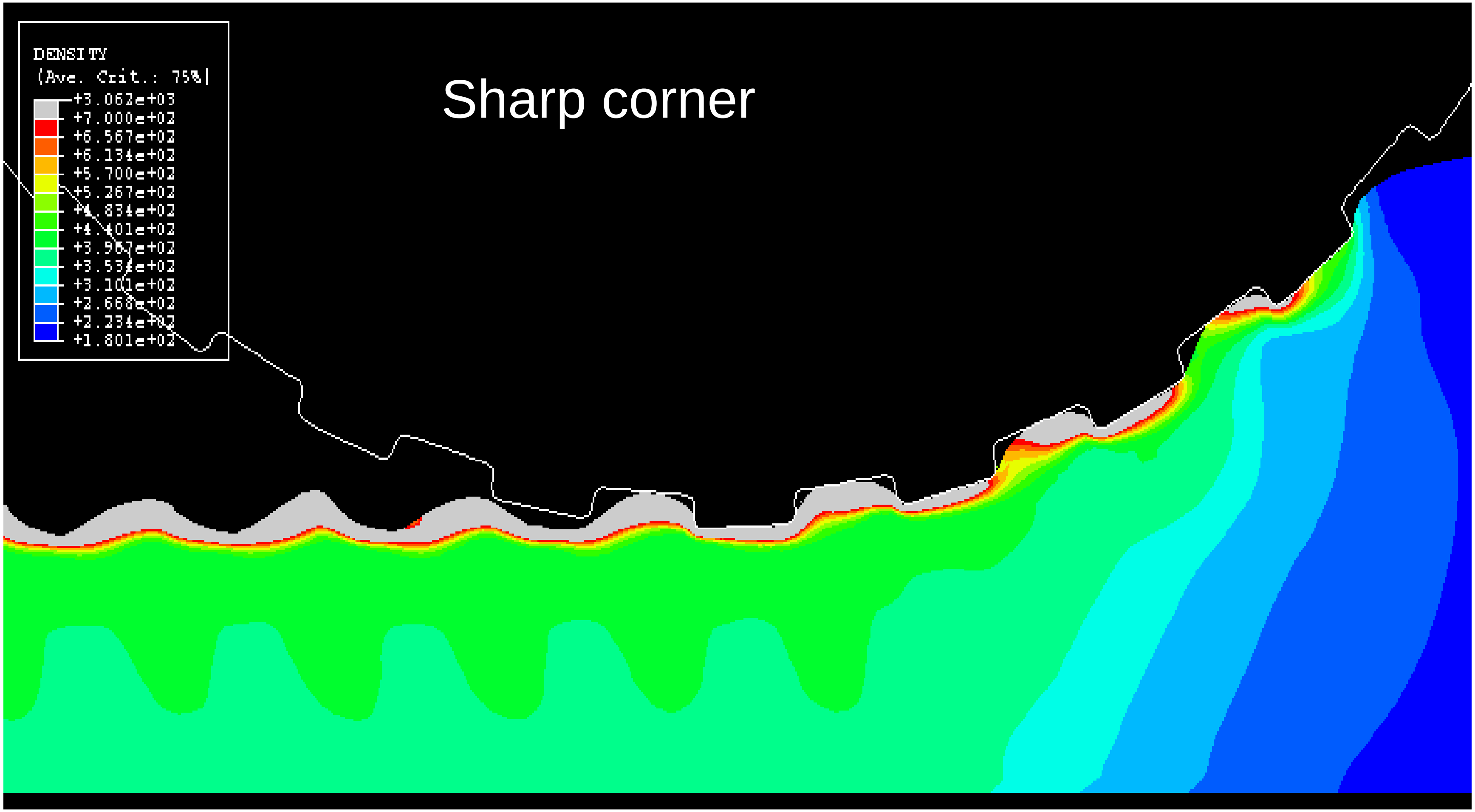
Grooved



Treaded



2-D Tire-Snow Interaction: Abaqus, 200 kg/m^3, Drucker-Prager (CRREL) model

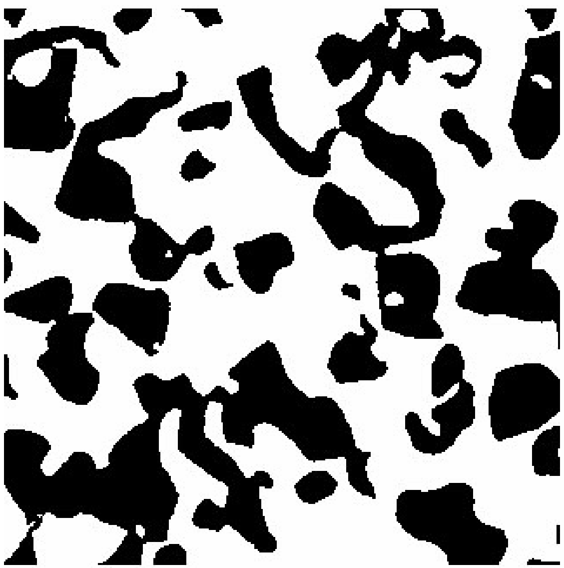


Density distribution, grey region indicates density larger than 700 kg/m3

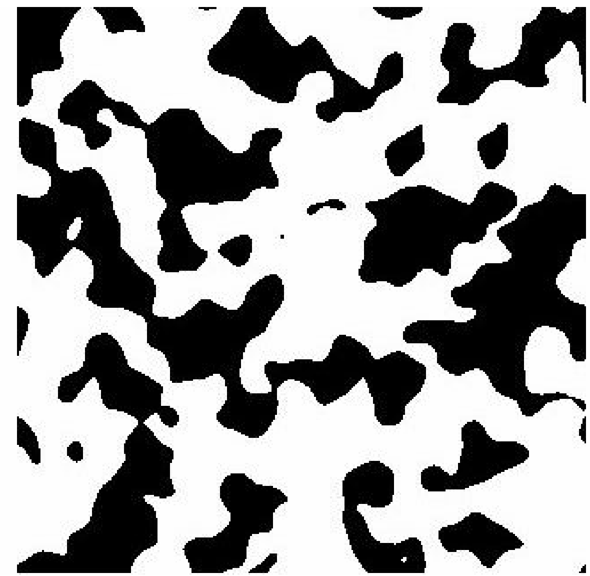
Stochastic Reconstruction of Snow Microstructure from X-Ray Tomography Images

- Properties of snow strongly depend on microstructure – one major source of uncertainties
- Structure-property relationships needed to understand physical mechanisms of deformations and failure
- Build digital stochastic models to represent snow microstructures
- Stochastic geometry and mechanics

What is stochastic reconstruction?



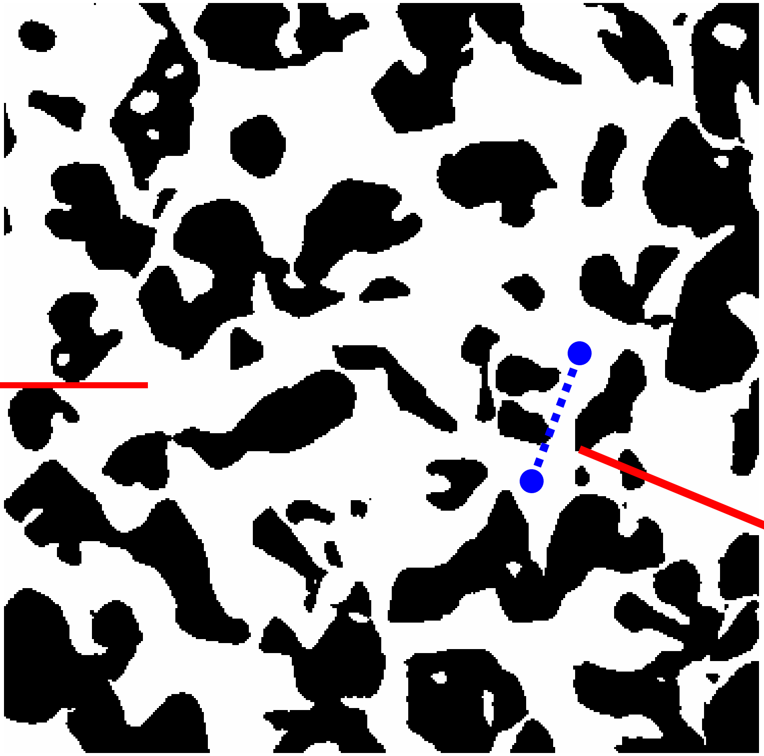
Real microstructure



Simulated one

Generate a simulated microstructure having the same statistical characteristics as the real one

Statistical Information from Snow Microstructure



Porosity (pore volume fraction)

Probability that two points a distance r apart will lie in pore space

Two-point probability function

Reconstruction Steps using Gaussian Random Fields

1. Find one-point and two-point correlation functions from snow images
2. Solve for level cut parameter

$$\alpha = \sqrt{2}\ \mathrm{erf}^{-1}\left(1 - 2\,p_{\mathrm{expt}}\right)$$

Determine function g from experimental one-point and two-point correlation functions by solving:

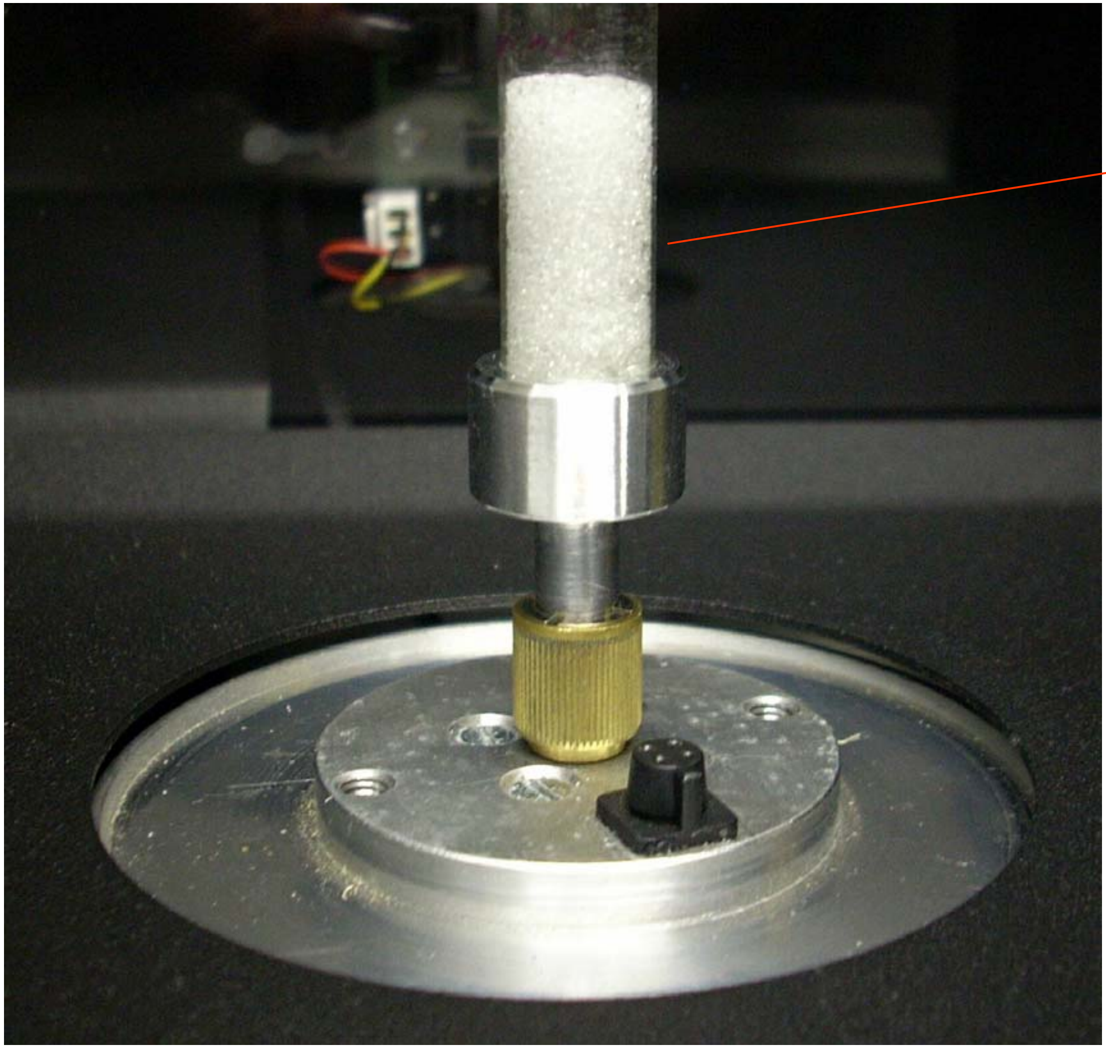
$$I\left(g(r_i)\right) = 2\pi\left(p_{\mathrm{expt}} - p^{(2)}_{\mathrm{expt}}(r_i)\right)$$

3. Solve three unknown parameters in g: ξ , r_c , d
4. Numerically generate Fourier transform coefficients
5. Perform 3D inverse FFT to generate discrete GRF
6. Perform one-level cut to get phase function in spatial domain

$$\boxed{p_{\mathrm{expt}}, p^{(2)}_{\mathrm{expt}}(r_i)} \rightarrow \boxed{\alpha, g(r_i)} \rightarrow \boxed{\xi, r_c, d \ \ (\text{i.e. } \rho(k))} \rightarrow \boxed{Y_{\alpha\beta\gamma}} \rightarrow \boxed{y_{lmn}} \rightarrow \boxed{\phi(\mathbf{r}_{lmn})}$$

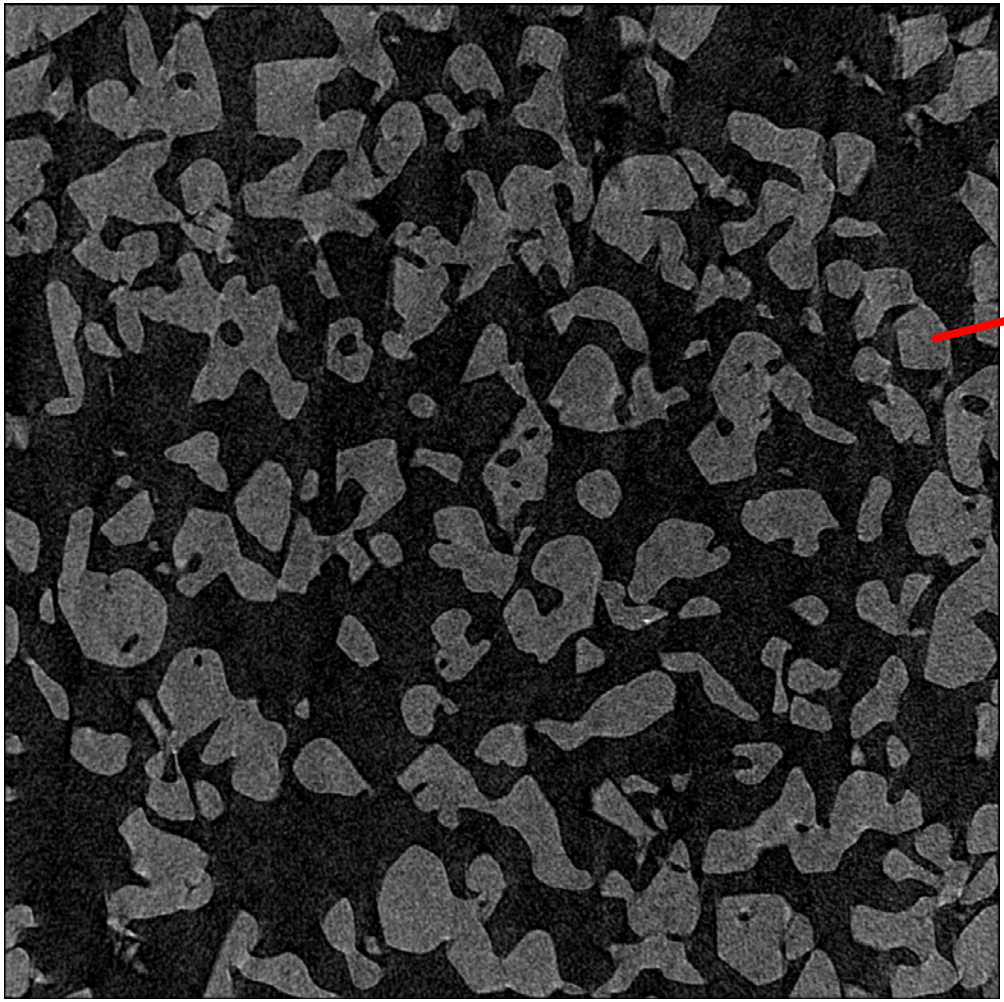
Skyscan 1172 Microtomography

Snow Sample Holder



Diameter 1 cm

Grey-level Cross-Sectional Image
Sieved Snow < 1 mm Grain Size

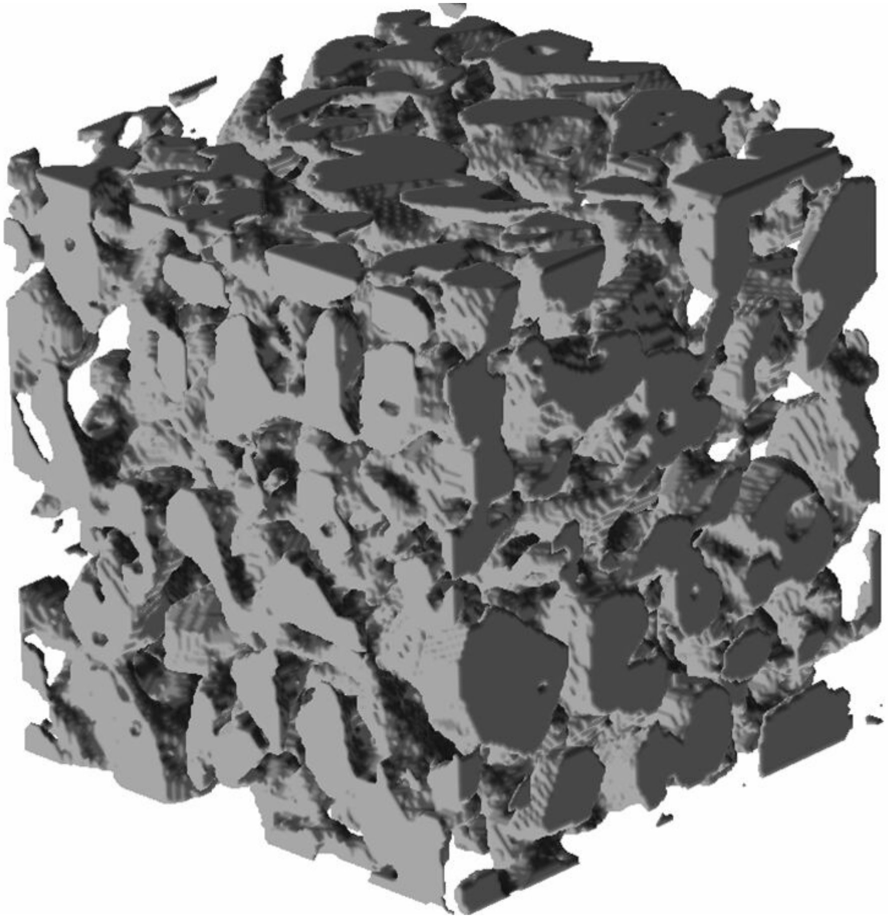


Brighter pixels represent ice

7.344 mm by 7.344 mm, density 387 kg/m^3

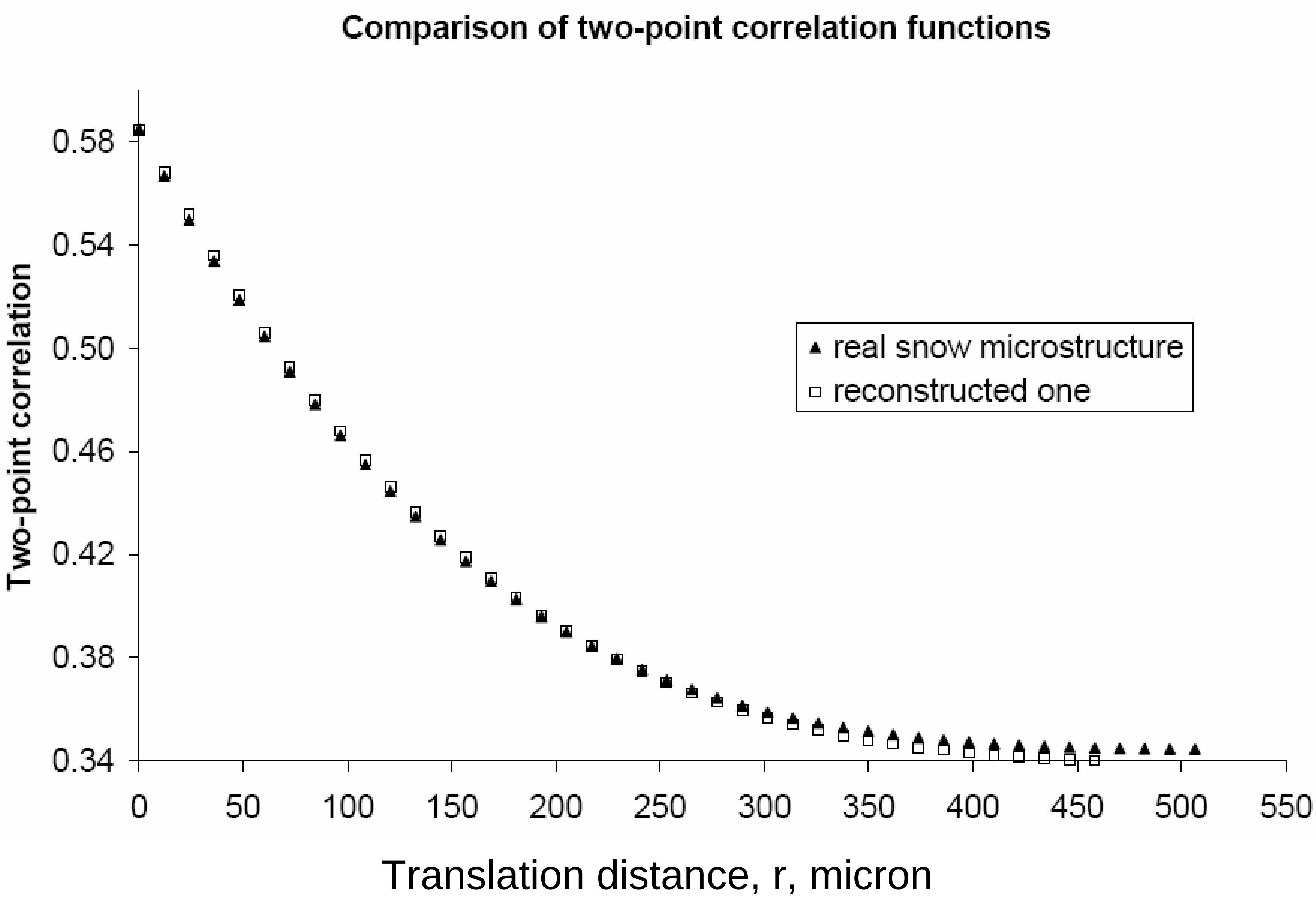
Resolution:1225 by 1225, Pixel size: 6 micron

3-D Visualization of a Cube of Snow Microstructure
Side Length = 3.618 mm

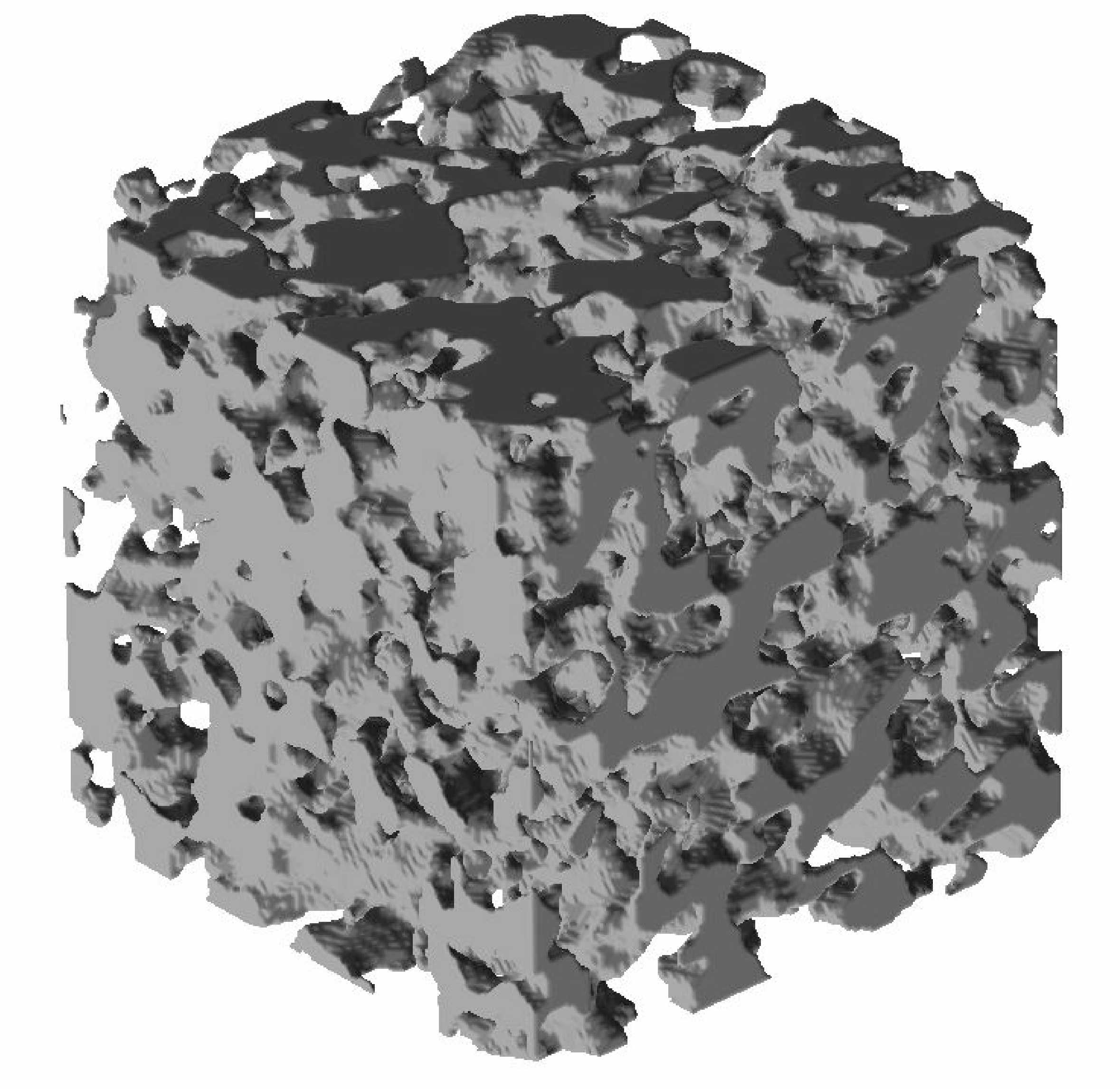


Link to reconstructed

Reconstruction results



Reconstruction Results



Reconstructed microstructure

Link to real one

Representative Volume Element (RVE)

- Definition
- Elastic Properties
  - Theoretical bounds
  - Initial results
- Viscoplastic Properties
  - SUVIC-I
  - Initial results

Representative Volume Element (RVE) for Mechanical Properties

- Definition (Nemat-Nasser and Hori):
  - *RVE for a material point of a continuum mass is a material volume which is statistically representative of the infinitesimal material neighborhood of that material point*.
  - RVE is the volume element over which homogenization can be performed.
  - Size of an RVE depends on the physical or mechanical properties of interest.
  - Size of an RVE requires a tolerance.
  - Size of an RVE should be independent of boundary conditions.
  - Size of volume smaller than RVE is called an SVE (statistical volume element).

Return

RVE of Elastic Moduli

- Numerical calculation of elastic moduli of scanned images and reconstructed volume.
- Using elastic material properties so ‘error’ due to creep or time-dependent effects won’t be present.
- Relatively ‘easy’ to conduct.
- Several numerical methods available – finite element method using voxel-based or solid-based mesh.
- Material Point Method (MPM) used:
  - Snow is considered as a semi-granular material.

Elastic Moduli using Uintah MPM Implicit

- Unconfined compression
- Load-displacement -> Macroscopic stress and strain ->Young's modulus and Poisson's ratio
- Largest size - 2.8 million cells, 83 million particles
- Nominal density 387 $\frac{kg}{m^3}$
- Ice properties
  - Young's modulus 9.3 GPa
  - Poisson's ratio 0.325

Hashin-Sritkman Upper Bound

ϕ_1 = volume fraction of air

ϕ_2 = volume fraction of ice

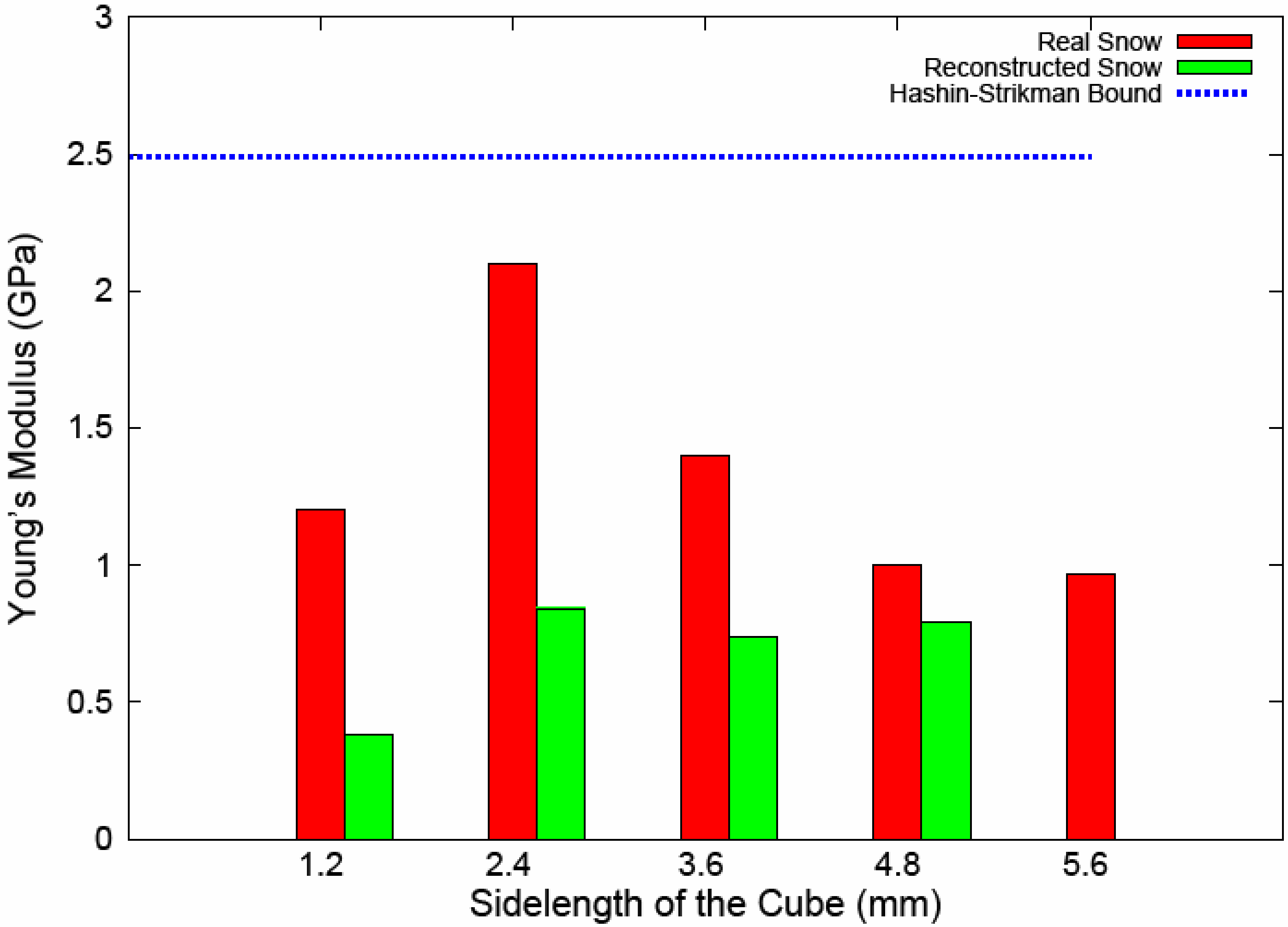
G, K = Shear and bulk modulus of ice.

$$K_U = K\phi_2 - \frac{\phi_1\phi_2 K^2}{K\phi_1 + \frac{4}{3}G}$$

$$G_U = G\phi_2 - \frac{\phi_1\phi_2 G^2}{G\phi_1 + H_2}$$

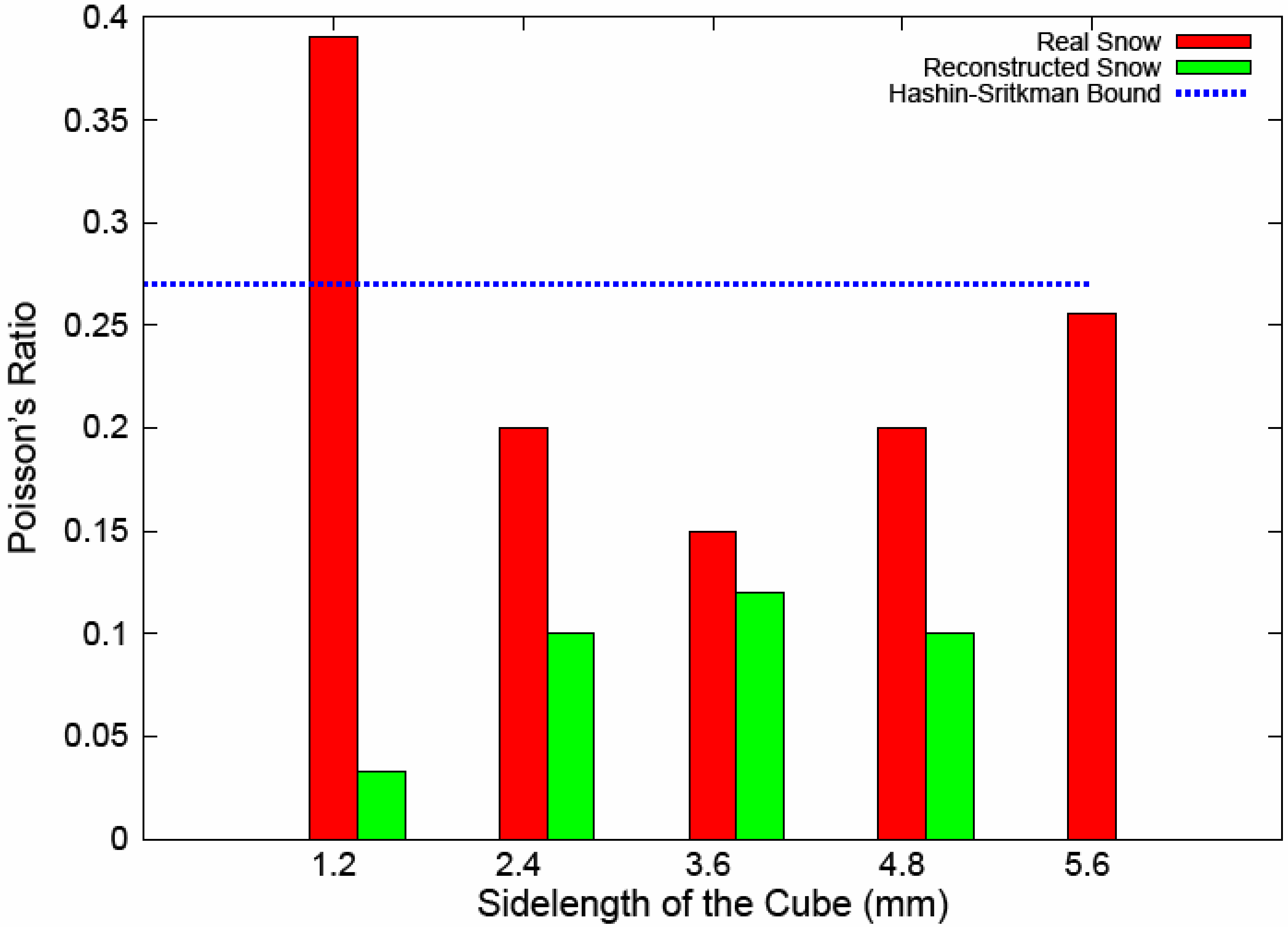
$$H_2 = G\left[\frac{\frac{3}{2}K + \frac{4}{3}G}{K + 2G}\right]$$

Uintah Results - Young's Modulus



Test data (CR 97): 10MPa - 0.8GPa

Uintah Results - Poisson's Ratio



Test data (CR 97): 0.22 - 0.35

Viscoplasticity
SUVIC-I (Aubertin and Lee)

- Strain rate history-dependent Unified Viscoplastic model with Internal variables for Crystalline materials – Ice
- Isotropic polycrystalline ice at

$$T \geq -55^0 C; 10^{-8} \leq \varepsilon \leq 10^{-2} s^{-1}; 0.04\,\mathrm{MPa} \leq \sigma_{eqiv} \leq 20\,\mathrm{MPa}$$

- Unified model – plasticity, creep and their interactions are modeled in the same way
- Three internal variables: back stress (kinematic hardening), yield and drag stress (isotropic hardening)
- Evolution of the state variables: combined action of hardening, dynamic recovery
- Viscoplastic – introduction of a yield surface makes a clear distinction between elastic and inelastic behavior.

SUVIC-I - continued

- Part of the inelastic strains are recoverable – grain boundary sliding, reverse motion of dislocations (backstress)
- Hardening has mixed (kinematic and isotropic) nature related to the existence of internal stresses
- Kinematic hardening due to backstress created by directional obstacles to dislocations motion.

Summary of SUVIC-I

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}^e_{ij} + \dot{\varepsilon}^i_{ij}; \qquad \dot{\varepsilon}^e_{ij} = \frac{\dot{S}_{ij}}{2G} + \frac{\dot{\sigma}_{kk}}{9K}\delta_{ij}.$$

$$\dot{\varepsilon}^i_{ij} = A\left\langle \frac{X_{ae} - R}{K} \right\rangle^N n_{ij} \exp\left(-\frac{Q}{R_g T} \right), \; n_{ij} = \frac{3}{2}\frac{S_{ij} - B_{ij}}{X_{ae}},$$

$$X_{ae} = \sqrt{\frac{3}{2}\left(S_{ij} - B_{ij}\right)(S_{ij} - B_{ij})}$$

$$\dot{\varepsilon}^i_e = \sqrt{\frac{2}{3}\dot{\varepsilon}^i_{ij}\dot{\varepsilon}^i_{ij}} = A\left\langle \frac{X_{ae} - R}{K} \right\rangle^N \exp\left(-\frac{Q}{R_g T} \right)$$

$$\dot{B}_{ij} = \frac{2}{3}A_1\dot{\varepsilon}^i_{ij} - \frac{A_1}{B^{'}_e}B_{ij}, B^{'}_e = B_0\left(\frac{\dot{\varepsilon}^i_e}{\dot{\varepsilon}_0} \right)^{\frac{1}{n}}$$

$$\dot{R} = A_3\dot{\varepsilon}^i_e\left(1 - \frac{R}{R'} \right), \dot{K} = A_5\dot{\varepsilon}^i_e\left(1 - \frac{K}{K'} \right).$$

$$R' = R_0\left(\frac{\dot{\varepsilon}^i_e}{\dot{\varepsilon}_0} \right)^{\frac{1}{n}}, K' = \left(\frac{\dot{\varepsilon}^i_e}{A\exp\left(-\frac{Q}{R_g T} \right)} \right)^{-\frac{1}{N}} \left(X^{'}_{ae} - R' \right)$$

$$X^{'}_{ae} = \sigma' - B^{'}_e; \; \sigma' = \sigma_0\left(\frac{\dot{\varepsilon}^i_e}{\dot{\varepsilon}_0} \right)^{\frac{1}{n}}$$

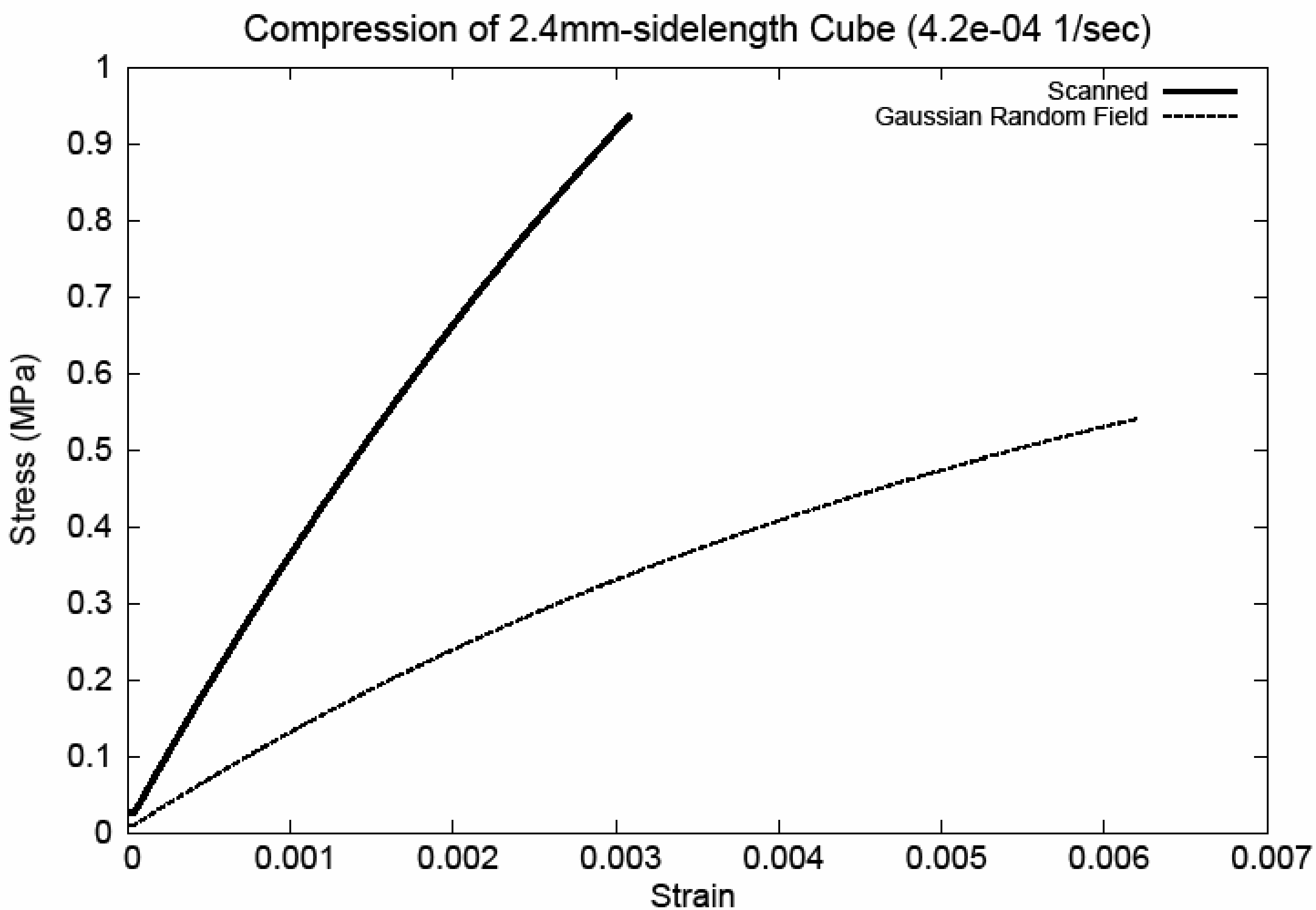
Numerical Integration

$$\Delta\varepsilon^{i} = \Delta t\left[(1-\theta)\dot{\varepsilon}_{t}^{i} + \theta\dot{\varepsilon}_{t+\Delta t}^{i}\right];\ 0 \le \theta \le 1$$

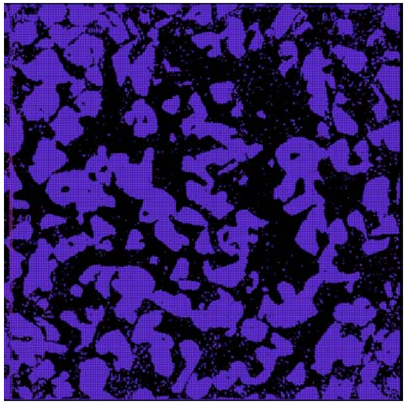
$$\dot{\varepsilon}_{t+\Delta t}^{i} = \dot{\varepsilon}_{t}^{i} + \sum_{i}\frac{\partial\dot{\varepsilon}^{i}}{\partial\beta_{i}}\dot{\beta}_{i}\Delta t.$$

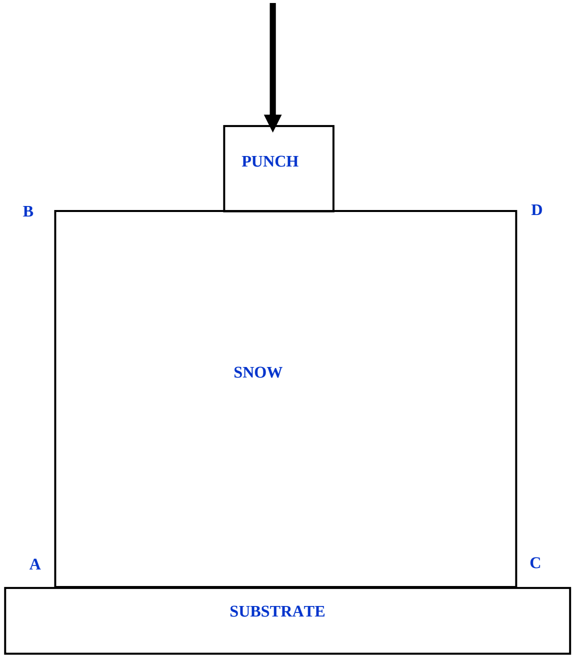
$$\dot{\sigma}_{ij} = \left[L_{ijkl} - \frac{\xi}{1+\xi}\frac{1}{\overline{\overline{H}}}P_{ij}Q_{kl}\right]\dot{\varepsilon}_{kl} - \frac{\dot{\varepsilon}_{t}^{i}}{1+\xi}P_{ij}.$$

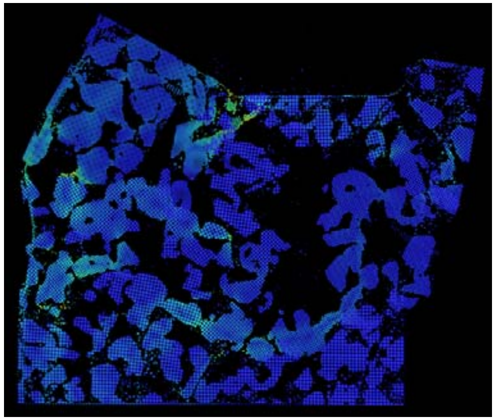
Viscoplastic Behavior – Uintah MPM Implicit SUVIC-I



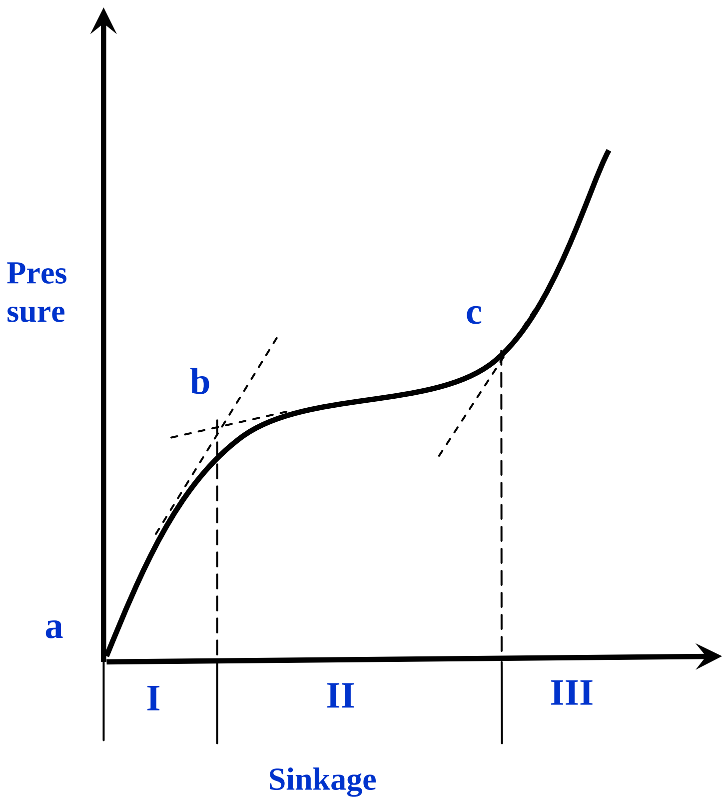
Microscale Plane Strain Indentation (1/3)
(7.344mm x 7.344mm x 0.012 mm)
Uintah MPM Implicit SUVIC-I

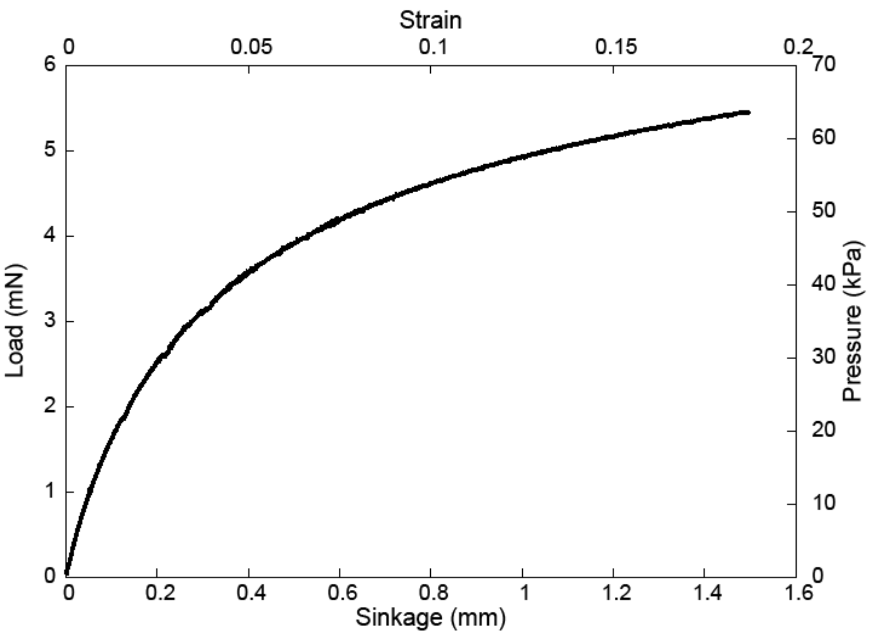




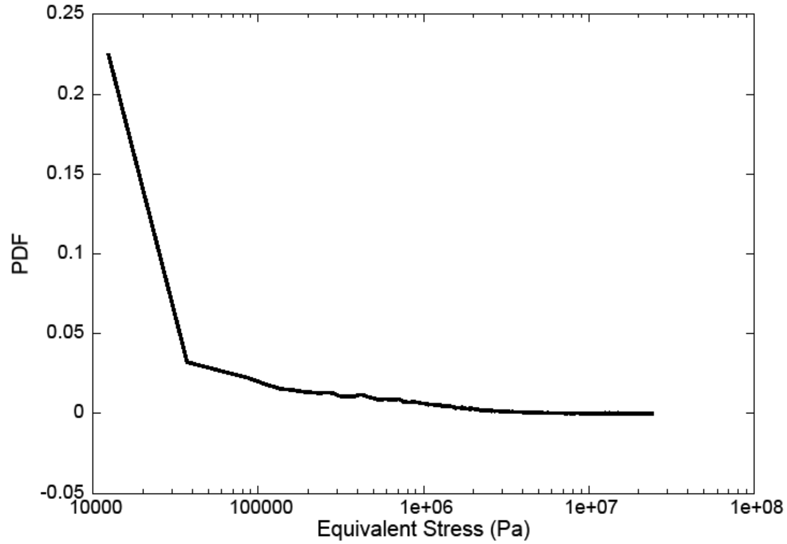


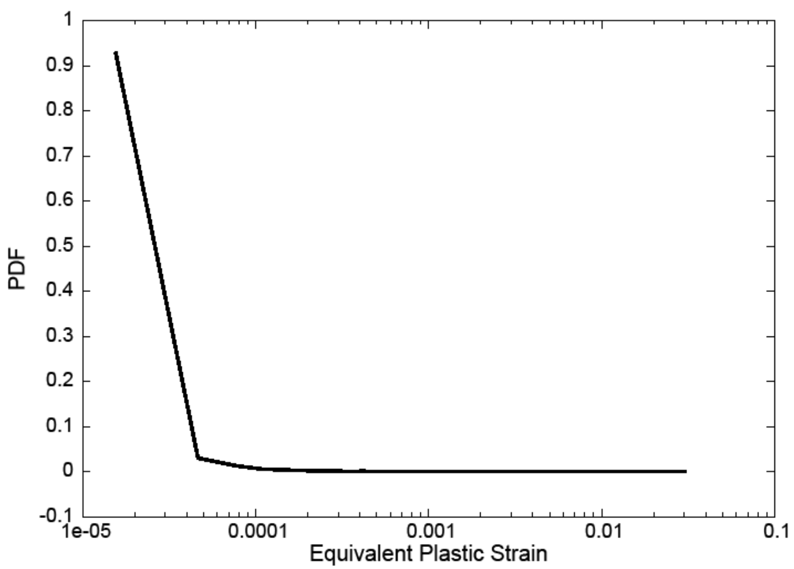
Microscale Plane Strain Indentation (2/3)

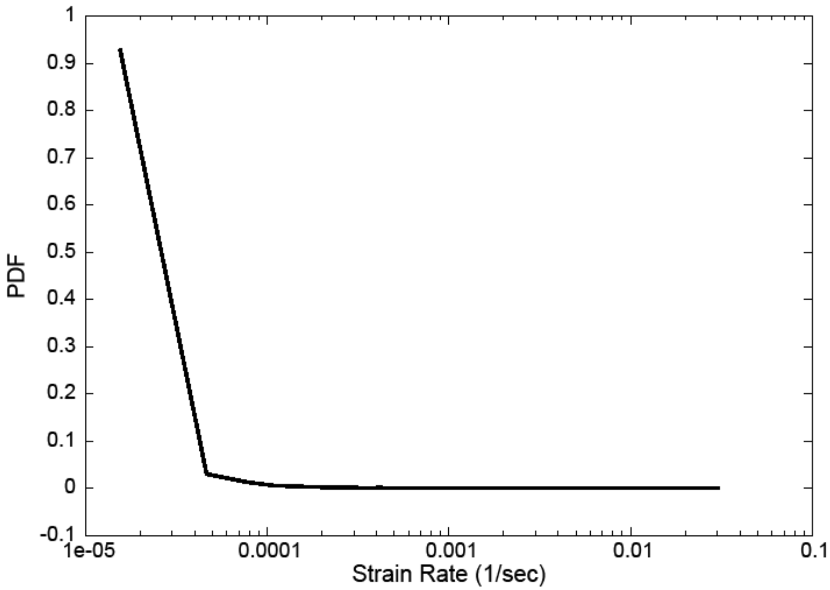




Plane Strain Indentation @20% strain (3/3)







Future Work

- Obtain statistical distributions of the elastic and tangent moduli of real and simulated snow
- Optimize the code for SUVIC-I
- Implement damage, failure models of ice into Uintah
- Conduct simulations of triaxial cell and micropenetrometer (snow pen)
- Conduct microscale tension/compression experiments inside MicroCT and compare with simulations
- Develop continuum constitutive laws
- …

Acknowledgements

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- UAF Arctic Regions Supercomputing Center (ARSC)

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Thank You!

Questions?