# Registration - I Shashidhar Reddy Puchakayala (Shashi) 

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- What is registration?
- Why registration?



## Formulation of problem

Find feasible transformations $\varphi$,

$$
\varphi \in \prod_{1}^{d}\left(R^{d}\right), \text { such that } \quad D[\varphi]=\min
$$

$$
D[R, T ; \varphi]=D[R, T o \varphi]
$$

## Distance Measures?

- Uni Modality
- Intensity based.
- Correlation

- Multi Modality
- Mutual Information and joint Entropy
- Maximum Likelihood
- Kullback-Leibler Divergence



## Intensity Based

- Minimisation of squared differences

$$
\begin{aligned}
& D^{S S D}[R, T]:=\frac{1}{2}\|T-R\|_{L_{2}}^{2} \\
& \frac{1}{2} \int_{R^{d}}(T(x)-R(x))^{2} d x \\
& D^{S S D}[R, T ; \varphi]=D^{S S D}[R, T o \varphi]
\end{aligned}
$$

## Results



## Mutual Information



## 2-D Histogram

- How does a 2-D histogram of two same images look like?

Image 1
Registration compensates for different head position at acquisition.


Image 2

sagittal slices
256 x 256 x 9
1.2 X $1.2 \times 4 \mathrm{~mm}$

registered


Histogram


Difference image

## Histogram dispersion

Registered
A
B



CT intensity

Not registered


CT intensity

## Registration criterion



Not registered


the statistical dependence of corresponding voxel intensities is maximal at registration

## Maximization of

## mutual information

## Interpretation

$\mathrm{I}_{\mathrm{AB}}(\alpha) \quad$ mutual information of A and B

$$
\mathrm{I}_{\mathrm{AB}}(\alpha)=\mathrm{H}_{\mathrm{A}}(\alpha)+\mathrm{H}_{\mathrm{B}}(\alpha)-\mathrm{H}_{\mathrm{AB}}(\alpha)
$$

"Find as much of the complexity in the separate datatsets (maximizing $\mathrm{H}_{\mathrm{A}}$ and $\mathrm{H}_{\mathrm{B}}$ ) such that at the same time they explain each other well (minimizing $\mathrm{H}_{\mathrm{AB}}$ )."

$$
\mathrm{I}_{\mathrm{AB}}(\alpha)=\mathrm{H}_{\mathrm{A}}(\alpha)-\mathrm{H}_{\mathrm{A} \mid \mathrm{B}}(\alpha)
$$

"Find as much of the complexity in datatset A (maximizing $\mathrm{H}_{\mathrm{A}}$ ) while minimizing the residual complexity of A knowing B (minimizing $\mathrm{H}_{\mathrm{A} \mid \mathrm{B}}$ )."

## Maximization of

 mutual information

$$
I(A, B)=\sum_{a, b} p_{A B}(a, b) \log _{2} \frac{p_{A B}(a, b)}{p_{A}(a) \cdot p_{B}(b)}
$$

$$
\boldsymbol{\alpha}^{*}=\arg \max _{\boldsymbol{\alpha}} I(A, B)
$$

## Application

Radiotherapy treatment planning of the prostate from CT and MR images (Oyen et al.)


- summary


## Groups





