MEDIAL AXIS TRANSFORMATION
Motivation: Describe a shape

- Ways to describe shape:
  - Edges, Contours: Local properties, need to be linked to global structures by edge linking and grouping
  - Hough transform: Global grouping, restrictive shape
  - Topology: Very general
  - Geometry: Very restrictive
**Shape Description using MAT**

**Example:** Qualitative & quantitative characterization

<table>
<thead>
<tr>
<th># segments</th>
<th>Point types</th>
<th>Shape</th>
<th>Axis</th>
<th>Axis curvature</th>
<th>Axis curvature change</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>ENN...NNE</td>
<td>Worm</td>
<td>Straight</td>
<td>φ</td>
<td>neg/pos</td>
</tr>
<tr>
<td>1</td>
<td>L</td>
<td>Worm</td>
<td></td>
<td>φ</td>
<td>pos</td>
</tr>
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<td></td>
<td></td>
<td>pos</td>
</tr>
</tbody>
</table>
Advantages of MAT

• Higher flexibility
• More natural description
• Hierarchical description
• Figure-sub Figure relationship, Graphical Description

Properties of MAT

• same length, no junctions
• radius is constant
• only difference is course of axis
• for eg. worm
Prairie Fire Analogy

- Propagation of fire fronts
- Begins at the boundary, constant speed
- Radius of curvature
- Circle: single point of symmetry
- Oval: SA with constant SD
- Dots arranged in circle:
- Symm Axis branches in the ground
- First row: open contours
- Second row: closed contours
- Third row: Parallel boundary, grassfire disappears all at once
- Triangle: 3 corners propagate, disappear at center of inscribed circle
- Ellipse: shortest to longest radius of curvature
  Symmetry, acceleration
Alternative Description of Skeleton

Geometrical description:

Skeleton: Locations of center points of all largest disks fully inside the object

(smaller disks fully covered by larger disks are removed!)

Reconstruction:

Superposition of all disks defined by the skeleton.
Disk: center & radius
SHAPE DESCRIPTION USING SAT's

CONDITIONS FOR SHAPES UNDER STUDY
1. Boundary has tangent and curvature everywhere except at finite no. of places
2. Boundary has a finite no. of connected pieces (infinite holes can't exist).

POINT TYPES
1. E - End Points,
2. N - Normal Points,
3. B - Branch points

SEGMENTATION
Simplified Segments
Intuitive for biological shapes
Divides on Boundary or knot

FIGURE
a) Elementary Shape Decomposition
b) Relational Graph Structure
c) Simplified segmentation
Elements of Shape Language

Object Width = 2*radius
Object Opening = Triangle of tangents at touching points
Axis of Curvature
Object Curvature : Change in Object Angle along Axis
Elements of Shape Language
1. Axis of Curvature Change
2. Mathematical Characterization
Voronoï-Diagramm des Objektinneren
A Voronoi diagram is a geometric structure that represents proximity information about a set of points or objects. Given a set of sites or objects, the plane is partitioned by assigning to each point its nearest site. The points whose nearest site are not unique, form the Voronoi diagram. That is, the points on the Voronoi diagram are equidistant to two or more sites.

so for a set $S$ of $n$ sites:

Voronoi diagram $VD(S)$: the partition of the plane into blocks of points with the same nearest site or sites.
REGULARIZATION : Concept of Residuals

Types of Residuals

1. Potential
2. Circularity
3. Bi-Circularity
4. Chord

Example : Key

Example : Eliminate Lines from consecutive points
SKELETAL PYRAMID

Skeletons: ExoSkeleton and EndoSkeleton

Pruning of Skeletal Pyramid

1. Increasing the threshold without bound: branches are removed, other branches are also trimmed
2. Topological Hierarchy is computed

Figure 6: The Skeleton Pyramid: Two First Order VSK for Different Parameter Settings. (a) “maple leaves”. (b) Voronoi skeletons, $\Delta R_G$, $T = 3.0$. The complex boundary introduces numerous rather irrelevant branches. The shaded area (as in (c) and (d)) depicts the result of a vertex-based reconstruction of the shape, namely by drawing the largest inscribed disk at each vertex of the VSK. (c) The first order VSK of “maple leaf” shapes. Parameters $\tau_H = 0.10$, $\tau_C = 0.10$. (d) First order VSK with $\tau_H = 0.02$, $\tau_C = 0.10$. 
Applications

1. Separation of Overlapping objects: (bottleneck in VD)
   - can be identified as local minima of disk radii along a branch.
2. Object Recognition: Fastest one at nodes,
   others do contour matching, some weight with residual
3. Extraction of Line Graphs
4. Understanding of Road Maps