Introduction (1)

- The active contour model, or snake, is defined as an energy-minimizing spline.
- Active contours results from work of Kass et al. in 1987.
- Active contour models may be used in image segmentation and understanding.
- The snake's energy depends on its shape and location within the image.
- Snakes can be closed or open.

Introduction (2)

Aorta segmentation using active contours

Introduction (3)

- First an initial spline (snake) is placed on the image, and then its energy is minimized.
- Local minima of this energy correspond to desired image properties.
- The snake is active, always minimizing its energy functional, therefore exhibiting dynamic behavior.
- Also suitable for analysis of dynamic data or 3D image data.

Examples (1)

Hands

Examples (2)

Highway

Heart

Modeling

- The contour is defined in the \((x, y)\) plane of an image as a parametric curve
  \(v(s) = (x(s), y(s))\)
- Contour is said to possess an energy \(E_{snake}\) which is defined as the sum of the three energy terms.
  \[ E_{snake} = E_{internal} + E_{external} + E_{constraint} \]
- The energy terms are defined so that the desired final position of the contour will have a minimum energy \(E_{min}\).
- Therefore our problem of detecting objects reduces to an energy minimization problem.

What are these energy terms which do the trick for us??
Internal Energy ($E_{int}$)
- Depends on the intrinsic properties of the curve.
- Sum of elastic energy and bending energy.

Elastic Energy ($E_{elastic}$):
- The curve is treated as an elastic rubber band possessing elastic potential energy.
- It discourages stretching by introducing tension.
  \[
  E_{elastic} = \frac{1}{2} \int \alpha(s) |v_s|^2 ds
  \]
- Weight $\alpha(s)$ allows us to control elastic energy along different parts of the contour. Considered to be constant $\alpha$ for many applications.
- Responsible for shrinking of the contour.

Bending Energy ($E_{bending}$):
- The snake is also considered to behave like a thin metal strip giving rise to bending energy.
- It is defined as sum of squared curvature of the contour.
  \[
  E_{bending} = \frac{1}{2} \int \beta(s) |v_s|^2 ds
  \]
- $\beta(s)$ plays a similar role to $\alpha(s)$.
- Bending energy is minimum for a circle – for a closed snake, or a line for an open one.
- Total internal energy of the snake can be defined as
  \[
  E_{int} = E_{elastic} + E_{bending} = \frac{1}{2} \int \left[ \alpha |v_s|^2 + \beta |v_s|^2 \right] ds
  \]

External energy of the contour ($E_{ext}$)
- It is derived from the image.
- Define a function $E_{image}(x,y)$ so that it takes on its smaller values at the features of interest, such as boundaries.
  \[
  E_{ext} = \int E_{image}(v(s)) ds
  \]
- Key rests on defining $E_{image}(x,y)$: Some examples
  - $E_{image}(x,y) = -|\nabla I(x,y)|^2$
  - $E_{image}(x,y) = -|\nabla (G_{\sigma}(x,y)*I(x,y))|^2$

Energy and force equations
- The problem at hand is to find a contour $v(s)$ that minimize the energy functional
  \[
  E_{total} = \frac{1}{2} \int \left[ \alpha |v_s|^2 + \beta |v_s|^2 \right] |v_s|^2 + E_{image}(v(s)) ds
  \]
- Using variational calculus and by applying Euler-Lagrange differential equation we get following equation
  \[
  \alpha v_{tt} - \beta v_{xx} - \nabla E_{image} = 0
  \]
- Equation can be interpreted as a force balance equation.
- Each term corresponds to a force produced by the respective energy terms. The contour deforms under the action of these forces.

Elastic force
- Generated by elastic potential energy of the curve.
  \[
  F_{elastic} = \alpha v_{ss}
  \]
- Characteristics (refer diagram)

Bending force
- Generated by the bending energy of the contour.
- Characteristics (refer diagram):
  - Initial curve (High bending energy)
  - Final curve deformed by bending force (low bending energy)
- Thus the bending energy tries to smooth out the curve.
External force

\[ F_{\text{ext}} = -\nabla E_{\text{image}} \]

- It acts in the direction so as to minimize \( E_{\text{ext}} \)

Discretizing

- The contour \( \gamma(s) \) is represented by a set of control points \( v_1, v_2, \ldots, v_n \).
- The curve is piecewise linear obtained by joining each control point.
- Force equations applied to each control point separately.
- Each control point allowed to move freely under the influence of the forces.
- The energy and force terms are converted to discrete form with the derivatives substituted by finite differences.

Solution and Results

Method 1:

\[ \alpha v_s - \beta v_{\text{ext}} - \gamma \nabla E_{\text{image}} = 0 \]

- \( \gamma \) is a constant to give separate control on external force.
- Solve iteratively.

Method 2:

- Consider the snake to also be a function of time i.e. \( v(s, t) \)

\[ \alpha v_s - \beta v_{\text{ext}} - \nabla E_{\text{image}} = v_s(s, t) \quad \text{and} \quad v(s, t) = \frac{\partial v(s, t)}{\partial t} \]

- If RHS=0 we have reached the solution.
- On every iteration update control point only if new position has a lower external energy.
- Snakes are very sensitive to false local minima which leads to wrong convergence.

- Noisy image with many local minima
  - WGN sigma=0.1
  - Threshold=15
Weakness of traditional snakes (Kass model)

- Extremely sensitive to parameters.
- Small capture range.
- No external force acts on points which are far away from the boundary.
- Convergence is dependent on initial position.

Weakness (contd…)

- Fails to detect concave boundaries. External force can’t pull control points into boundary concavity.

Gradient Vector Flow (GVF)

(A new external force for snakes)

- Detects shapes with boundary concavities.
- Large capture range.

Model for GVF snake

- The GVF field is defined to be a vector field \( V(x, y) = (u(x, y), v(x, y)) \)

- \( V(x, y) \) is defined such that it minimizes the energy functional

\[
E = \int \int \mu (u_x^2 + u_y^2 + v_x^2 + v_y^2) + | \nabla f |^2 | V - \nabla f |^2 \, dx \, dy
\]

\( f(x,y) \) is the edge map of the image.

- GVF field can be obtained by solving the following equations:

\[
\mu \nabla^2 u - (u - f_x)^2 + f_z^2 = 0
\]

\[
\mu \nabla^2 v - (v - f_y)^2 + f_z^2 = 0
\]

\( \nabla^2 \) is the Laplacian operator.

- The above equations are solved iteratively using the time derivative of \( u \) and \( v \).