Nonrigid Registration using Free-Form Deformations

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Paper Presented: Rueckert et al. , TMI 1999: Nonrigid registration using free-form deformations: Application to breast MR images

Overview

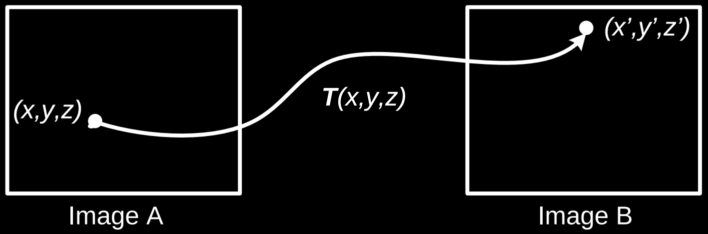
- Introduction – image registration
- Tasks within medical image analysis
- **TMI paper 1999:**
  - Nonrigid registration using free-form deformations
    - Global model
    - Local model
    - Regularisation
    - Similarity measures
    - Optimisation
  - Results on Breast MRI

Introduction – image registration

- Image registration - general definition:
  - determining a mapping between the coordinates in one image and those in another,
  - to achieve biological, anatomical or functional correspondence
- Purpose of image registration in medical image analysis
  - Monitoring of changes in an individual
  - Fusion of information from multiple sources
  - Comparison of one subject to another
  - Comparison of one group to another

Introcution – image registration

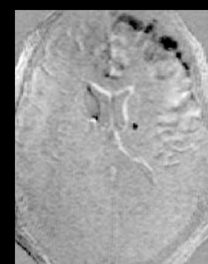
- Registration of one image to the coordinate system of another image by a transformation, ***T***: $(x,y,z) \mapsto (x', y', z')$

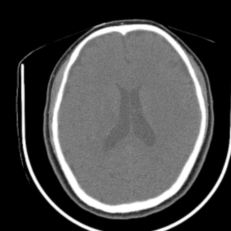


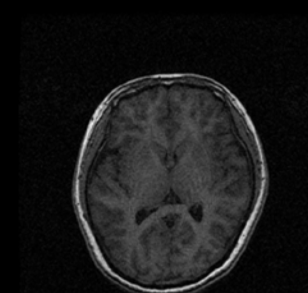
- Types of transformation
  - Rigid – rotation, translation
  - Affine – rotation, translation, scaling, shearing
  - Nonrigid – All sorts of nonlinear deformations

Tasks within medical image analysis I

- Rigid registration
  - Bones
- Affine registration
  - If scale changes are expected
    - Growth
    - Inter-subject registration
- Nonrigid registration
  - Correction for tissue deformation
    - Breast MRI
    - Liver MRI
    - Brain shift modelling
  - Modelling of tissue motion
    - Cardiac motion
    - Respiratory motion
  - Modelling of growth and atrophy
    - Brain development
    - Dementia or schizophrenia
  - Fusion of different modalities







Nonrigid registration using free-form deformations: Application to breast MR images

By D. Rueckert, L. I. Sonoda, C. Hayes, D.L. G. Hill,
M. O. Leach and D.J. Hawkes

IEEE Transactions on Medical Imaging 1999

Nonrigid registration using free-form deformations

- A combined transformation consisting of both a local and a global transformation
- $\boldsymbol{T}(x,y,z) = \boldsymbol{T}_{global}(x,y,z) + \boldsymbol{T}_{local}(x,y,z)$
- Global: Accounts for the overall motion of the object
- Local: Accounts for local deformations of the object
- Cost function: $C = C_{similarity} + \lambda \cdot C_{smooth}$

Global motion model

- Simplest choice: Rigid transformation
  - Rotation, translation $\Rightarrow$ 6 degrees of freedom (d.o.f)
- More general: Affine transformation
  - Rotation, translation, scaling and shearing $\Rightarrow$ 12 d.o.f

$$\mathrm{T}_{global}(x, y, z) = \begin{pmatrix} \theta_{11} & \theta_{12} & \theta_{13} \\ \theta_{21} & \theta_{22} & \theta_{23} \\ \theta_{31} & \theta_{32} & \theta_{33} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} \theta_{14} \\ \theta_{24} \\ \theta_{34} \end{pmatrix}$$

Local motion model I

- Free form deformations (FFDs) based on cubic B-splines
- Basic idea: To deform an object by manipulating an underlying $n_x \times n_y \times n_z$ mesh of control points Φ, with spacing δ.
  - Control points can be displaced from their original location
  - Control points provide a compact parameterisation of the transformation

$$\Omega = \{(x, y, z) | 0 \leq x < X, 0 \leq y < Y, 0 \leq z < Z\}$$

$$\mathrm{T}_{local}(x, y, z) = \sum_{l=0}^{3} \sum_{m=0}^{3} \sum_{n=0}^{3} B_l(u) B_m(v) B_n(w) \phi_{i+l, j+m, k+n}$$

$$i = \lfloor x/n_x \rfloor - 1, \quad j = \lfloor y/n_y \rfloor - 1, \quad k = \lfloor z/n_z \rfloor - 1$$
$$u = x/n_x - \lfloor x/n_x \rfloor, \quad v = y/n_y - \lfloor y/n_y \rfloor, \quad w = z/n_z - \lfloor z/n_z \rfloor$$

Local motion model II

- B_i represents the i^{th} basis function of the B-spline

  $B_0(u) = (1-u)^3/6$

  $B_1(u) = (3u^3-6u^2+4)/6$

  $B_2(u) = (-3u^3+3u^2+3u+1)/6$

  $B_3(u) = u^3/6$

- B-splines are locally controlled – computationally efficient

Local motion model III

- Hierarchical approach
- A hierarchy of control point meshes $\Phi^1,\ldots\Phi^L$ at increasing resolutions
- At each resolution we have a transformation $\mathbf{T}^l_{local}$

$$\mathrm{T}_{local}(x,y,z) = \sum_{l=1}^{L} \mathrm{T}^l_{local}(x,y,z)$$

- Represented by a single B-spline FFD
  - Control point mesh progressively refined
  - New control points inserted at each level
  - Spacing is halved in every step

Regularisation of the local transformation

- Constrain to a smooth transformation
  - Penalty term:

$$C_{smooth} = \frac{1}{V}\int_0^X \int_0^Y \int_0^Z [\left(\frac{\partial^2 \mathbf{T}}{\partial x^2}\right)^2 + \left(\frac{\partial^2 \mathbf{T}}{\partial y^2}\right)^2 + \left(\frac{\partial^2 \mathbf{T}}{\partial z^2}\right)^2 + 2\left(\frac{\partial^2 \mathbf{T}}{\partial xy}\right)^2 + 2\left(\frac{\partial^2 \mathbf{T}}{\partial xz}\right)^2 + 2\left(\frac{\partial^2 \mathbf{T}}{\partial yz}\right)^2] dx dy dz$$

Similarity measures I

- How do we know when we have a good fit between two images??
- Depends on the type of images you are registering
- Similarity assumptions
  - Identity
    - Single-modality, only differ by gaussian noise
  - Linear
    - Single-modality, differ by constant intensity
  - Information theoretic/probabilistic
    - multi-modality, intensity changing, related by some statistical or functional relationship

Similarity measures II – Information theoretic

$p(a)$	:	Probability of intensity a in image A
$p(b)$	:	Probability of intensity b in image B
$p(a,b)$	:	Probability of intensity a in image A occurring at the same place as intensity b in image B

- Entropy

$$H(A) = -\sum_a p(a) log p(a)$$

- Joint entropy

$$H(A,B) = -\sum_a \sum_b p(a,b) log p(a,b)$$

- Mutual information (MI)

$$I(A,B) = H(A) + H(B) - H(A,B)$$
$$= -\sum_a \sum_b p(a,b) log \frac{p(a,b)}{p(a)p(b)}$$

Similarity measures III – Information theoretic

- Mutual information is still sensitive to overlapping
- Normalised mutual information

$$I(A,B) = \frac{H(A) + H(B)}{H(A,B)}$$

  - Is robust to the amount of overlap between images

Final Cost function

$$C(\Theta, \Phi) = -C_{similarity}(I(t_0), \mathbf{T}(\mathbf{I}(\mathbf{t}))) + \lambda C_{smooth}(\mathbf{T})$$

$$C_{similarity} = \frac{H(A) + H(B)}{H(A, B)}$$

$$C_{smooth} = \frac{1}{V} \int_0^X \int_0^Y \int_0^Z [\left(\frac{\partial^2 \mathbf{T}}{\partial x^2}\right)^2 + \left(\frac{\partial^2 \mathbf{T}}{\partial y^2}\right)^2 + \left(\frac{\partial^2 \mathbf{T}}{\partial z^2}\right)^2 + 2\left(\frac{\partial^2 \mathbf{T}}{\partial xy}\right)^2 + 2\left(\frac{\partial^2 \mathbf{T}}{\partial xz}\right)^2 + 2\left(\frac{\partial^2 \mathbf{T}}{\partial yz}\right)^2] dx dy dz$$

Optimization

calculate the optimal affine transformation parameters Θ by maximising $C_{similarity}$

initialise the control points Φ

repeat

calculate the gradient vector of the cost function, $C(\Theta,\Phi)$ with respect to the nonrigid transformation parameters, Φ: $\nabla C = \delta C(\Theta,\Phi^l)/\delta\Phi^l$

while $||\nabla C|| > \varepsilon$ **do**

recalculate the control points $\Phi = \Phi + \mu \nabla C / ||\nabla C||$

recalculate the gradient vector ∇C

increase the control point resolution by calculating new control points Φ^{l+1} from Φ^l

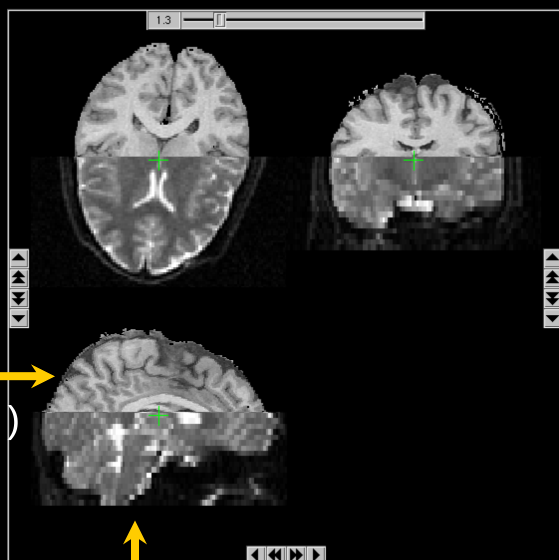
increase the image resolution

until finest level of resolution is reached

Rview warping

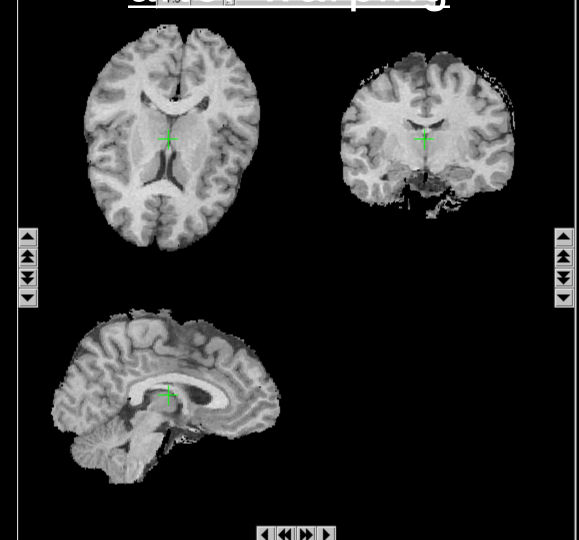
- http://www.doc.ic.ac.uk/~dr/software/

- Images after affine registration



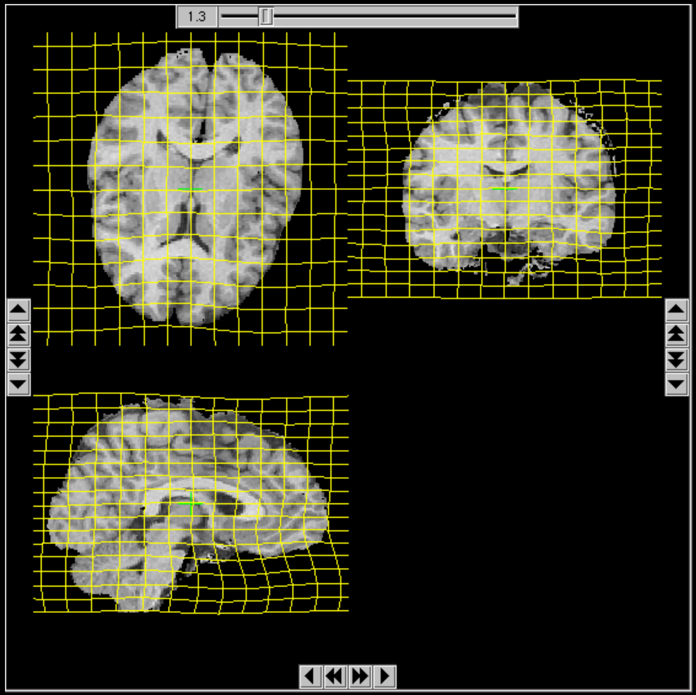
- Target image
- (T1 weighted MRI)
- Source image
- (Baseline DTI image)

- Brain deformation after warping



Rview warping

- Visualization of the deformation with the deformation grid



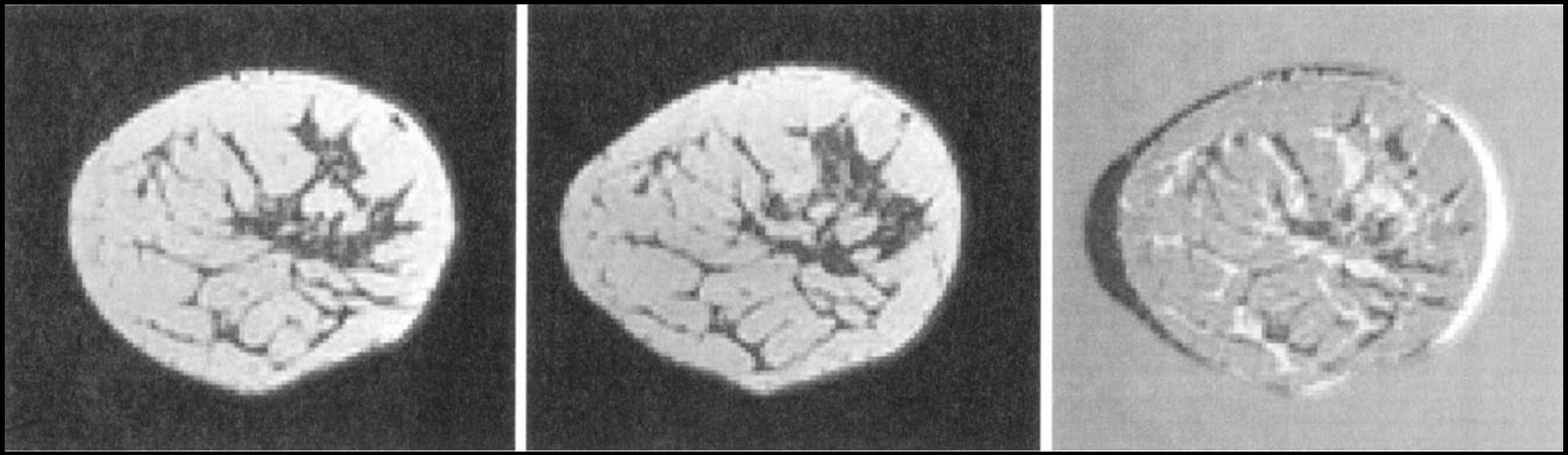
- The grid shows the deformation from the target image to the source image.

Application: Breast MRI

- 9.5% of women in the UK develop breast cancer
- Examination
  - Currently: X-ray mammography
  - Two 3D MR scans
  - Pre- and post-contrast
  - Rate of uptake is determined by the difference between the two different scans
- Problems: Motion of the patient, respiratory and cardiac motion
- Registration of the pre- and post contrast images is required

Breast MRI without contrast agent

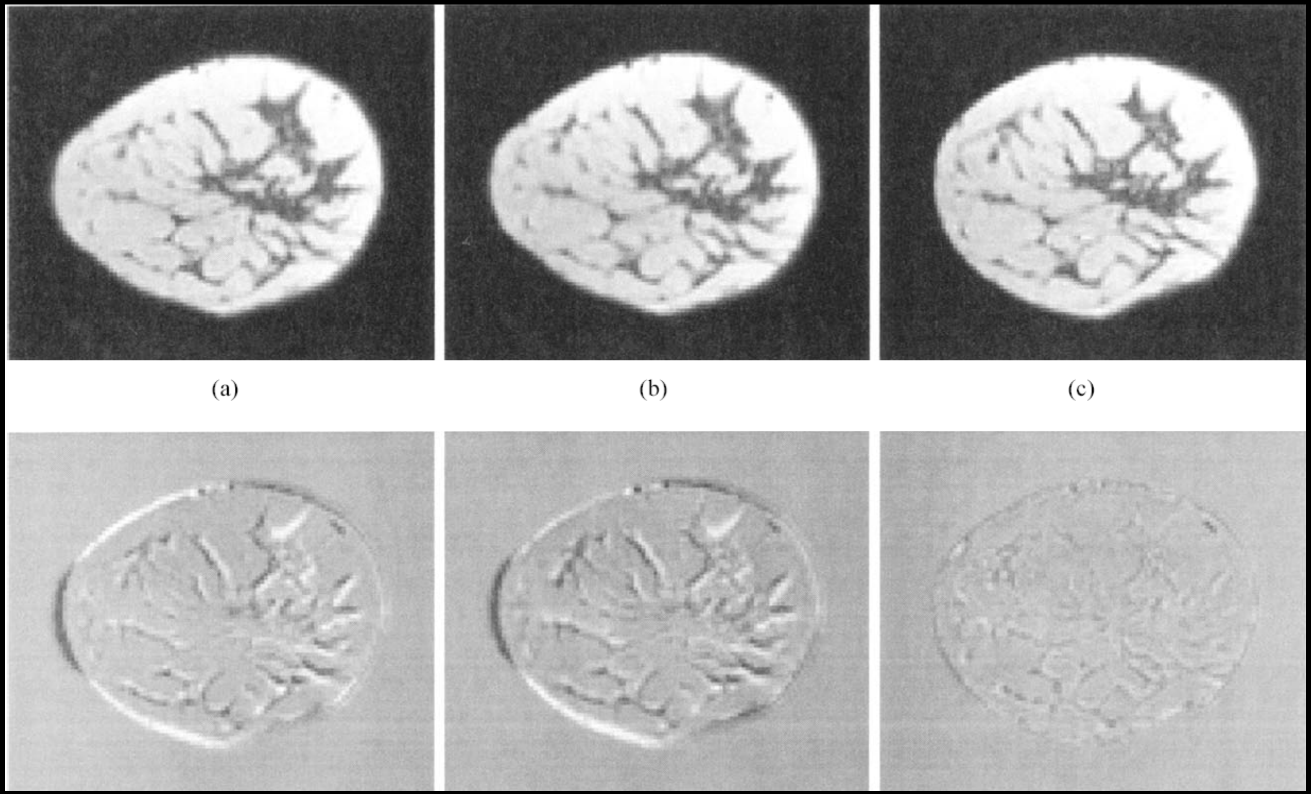
No registration



Before motion After motion Difference

Breast MRI without contrast agent

Registration



Trans-
formed
image

Difference

Rigid Affine Nonrigid

Similarity measures

- Sums of squared differences

$$SSD = \frac{1}{N}\sum_i (I_A(\mathrm{p_i}) - I_B(\mathrm{T}(\mathrm{p_i})))^2$$

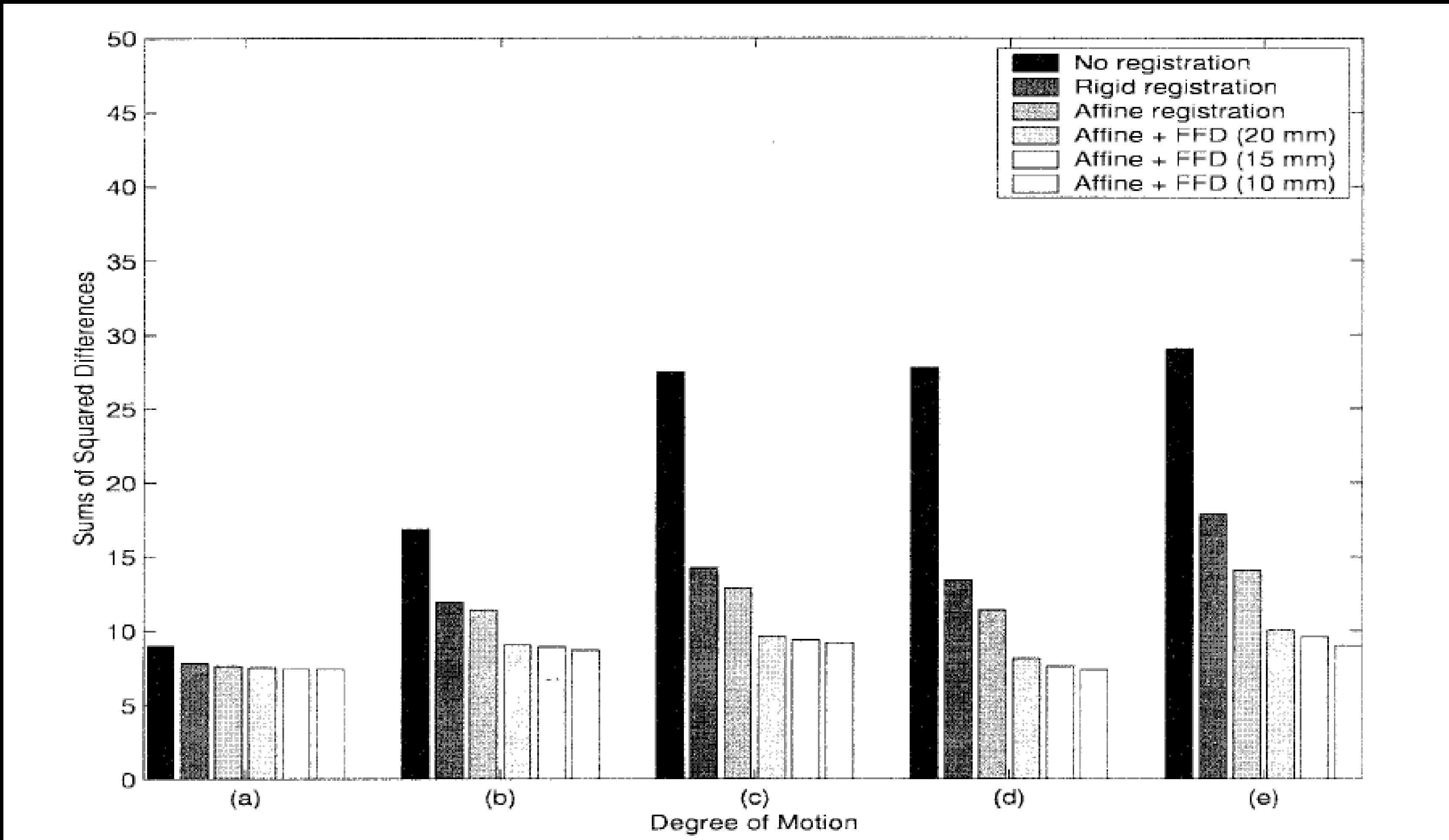
I_A	:	Image A
I_B	:	Image B
$\mathrm{p_i}$	:	Pixel/voxel i
μ_A	:	Mean intensity in image A
μ_B	:	Mean intensity in image B

- Normalised cross correlation

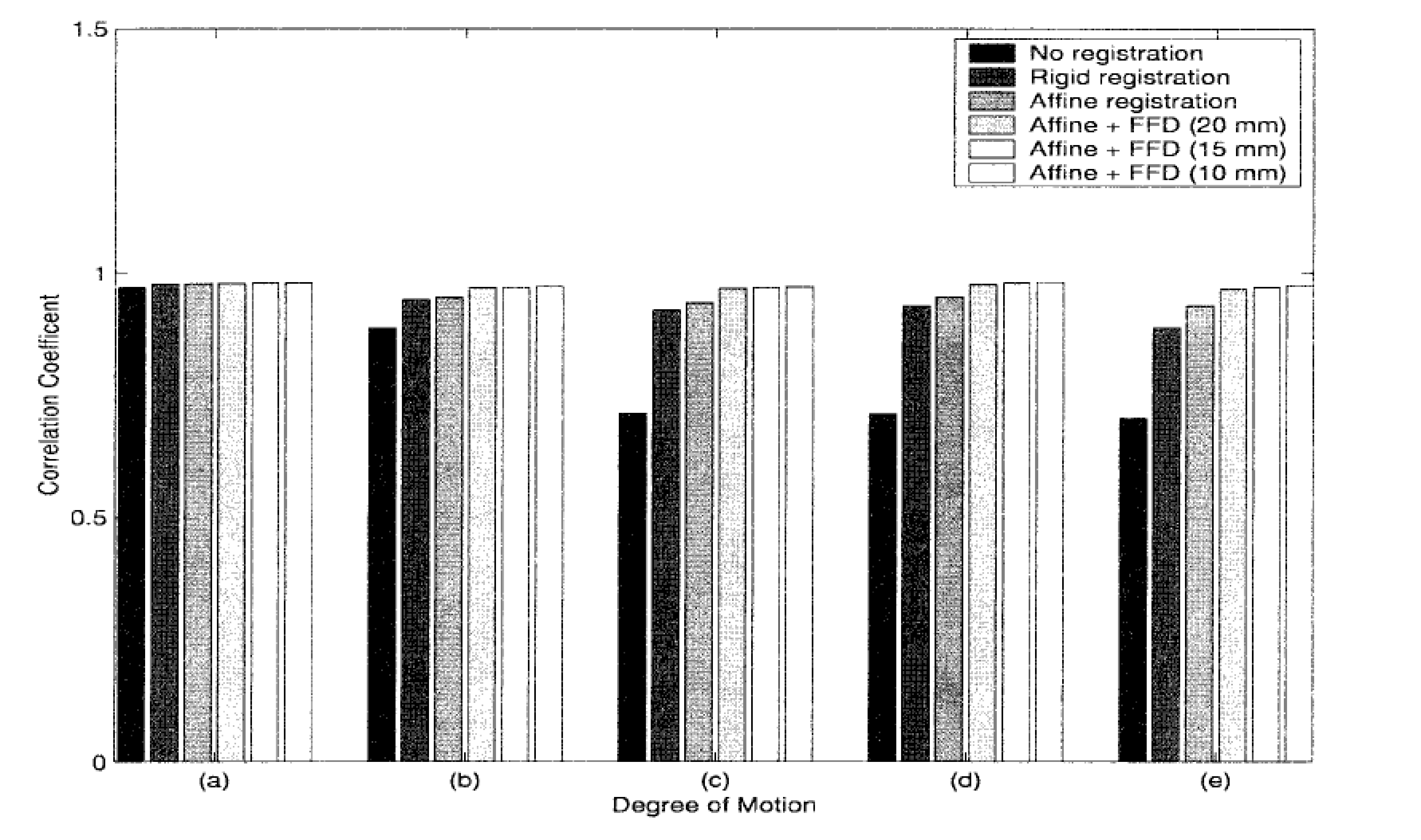
$$CC = \frac{\sum_i (I_A(\mathrm{p_i}) - \mu_A)(I_B(\mathrm{T}(\mathrm{p_i})) - \mu_B))}{\sqrt{\left(\sum_i (I_A(\mathrm{p_i}) - \mu_A)^2\right)\left(\sum_i (I_B(\mathrm{T}(\mathrm{p_i})) - \mu_B)^2\right)}}$$

Breast MRI with contrast agent – tumour detection
No registration

Pre-contrast Post-contrast Difference



Comparison of the registration error in terms of SSD for different degrees of volunteer motion. (a) No voluntary movement. (b) Cough. (c) Move head. (d) Move arm. (e) Lift out of coil and back.



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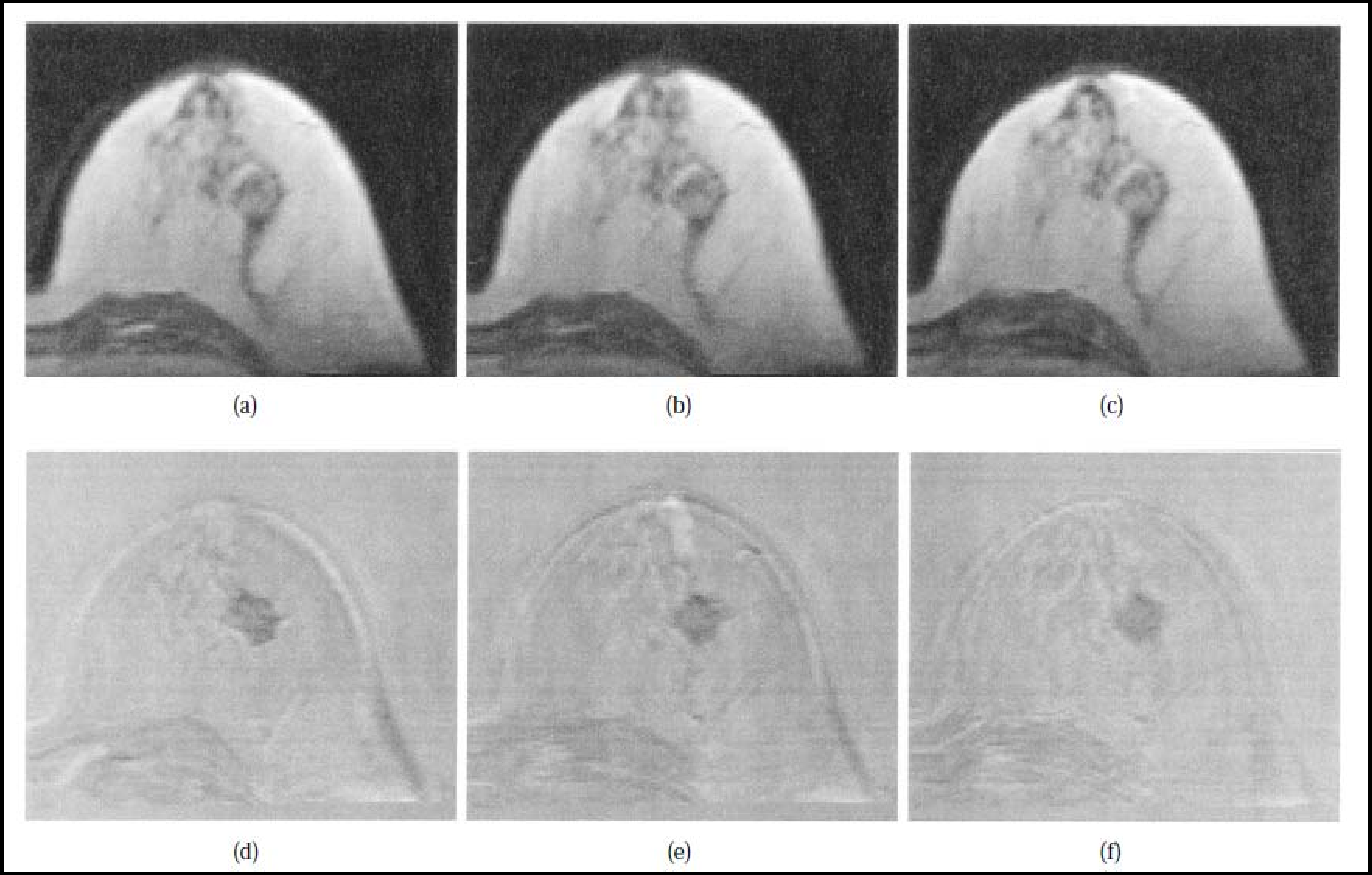

(a) After rigid. (b) After affine. (c) After nonrigid registration. The corresponding difference images are shown in (d)–(f).

Breast MRI – with contrast – tumour detection
Maximum intensity projection