Generalized Hough Transform

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Original Hough Transform

- $(x-x_0)^2 + (y-y_0)^2 = r^2$
- Each point is evidence for a circle.
- Given $(x,y,r)$ increment bins in all satisfying $(x_0,y_0)$
- Find local maxima in accumulator
Gradient Information $\phi(x)$

\[
\frac{df}{dx}(x, a) = 0
\]

\[
\frac{dy}{dx} = \tan[\phi(x) - \frac{\pi}{2}]
\]
Arbitrary Shape

• Reconstruction of the reference origin by adding all displacement vectors to all boundary points
• \( R(\phi(x)) \) table holds all reference points that a certain gradient is evidence of.
Scale(s) / Rotation(\(\theta\))

\[ T_s[R(\Phi)] = sR(\Phi) \]

• Vote in all bins ranging over scale

\[ T_\theta[R(\Phi)] = Rot\{R[(\Phi - \theta) mod 2\pi], \theta\} \]

• Vote in all bins ranging over rotation

• Accumulator space is now 4D
  – \(A((x,y),\theta,s)\)
Composite Images

• Sub-shapes \( S_k \) of shape \( S \) can be found by taking voting on the union of R-tables of sub-shapes.

\[
R_s(\phi) = T_s \left\{ T_\theta \left[ \bigcup_{k=1}^{N} R_{S_k}(\phi) \right] \right\}.
\]
Strength in numbers

- Dynamic Programming
- Connected Components
- Weight Functions

Fig. 12. Dynamic Hough transform.