Continuous Medial Representations for Geometric Object Modeling

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Methods to define skeleton of object

- maximal inscribed ball
- Hierarchal Voronoi
- Shock of boundary evolution
- core tracking.
- m-rep
- cm-rep
M-rep: Inverse skeletonization

- Traditional: the intersection of the normal.
- Now: a medial point with two boundary points.

\[
U^{\pm 1} = R \begin{bmatrix} \cos(\theta) \\ \pm \sin(\theta) \end{bmatrix},
\]

\[
y^{\pm 1} = x + r U^{\pm 1},
\]

\[
m = \{x, r, R, \theta\}
\]
M-rep: sampling

- Medial points are sampled on the axis.
- A sparse representation.
- Sampled medial points are fixed relative to the axis.

http://midag.cs.unc.edu/defmreps/jl3_movie2.gif
Continuous m-rep: cm-rep

• Can move the medial points along the axis.

• Generate a synthetic given set of b-spline control points.

• Maximize overlapping between this shape and template by adjusting control point.
Continuous medial axis

\( m \) Medial surface

\( r \) Radial scalar field

\( u, v \) Parametrization of \((m, r)\)

\( b^t \) Boundary counterparts of \((m, r)\)

\( t \) Indexes the two parts \((-1, 1)\) of the implied boundary.

\( u^t \) Unit normal to the boundary, also the direction from a point on \( m \) to its boundary counterpart.

\( n \) Unit normal to the medial surface.
When $\|\text{del } r\| = 0$, meaning $r$ does not change, $U = N$. 

\[ f(x, u, v) = \|x - m(u, v)\|^2 - r(u, v)^2 = 0 \]
\[ f = 0, \quad f_u = 0, \quad f_v = 0 \]

\[ b^t = m + ru^t, \]
\[ u^t = -\nabla r + t\sqrt{1 - \|\nabla r\|^2}n \]

\[ \nabla r = \begin{bmatrix} m_u & m_v \end{bmatrix} I_m^{-1} \begin{bmatrix} r_u \\ r_v \end{bmatrix} \quad \|\nabla r\| \leq 1 \]

- When $\|\text{del } r\| = 0$, meaning $r$ does not change, $U = N$. 

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\(03/18/10\) Continuous medial axis
B-spline interpolation

\[ m(u, v) = \sum_{i=0}^{d_1} \sum_{j=0}^{d_2} N_i^3(u) N_j^3(v) \mathbf{m}_{ij} \]

\[ r(u, v) = \sum_{i=0}^{d_1} \sum_{j=0}^{d_2} N_i^3(u) N_j^3(v) \mathbf{r}_{ij} \]

- **m** Medial surface
- **r** Radial scalar field
- **u, v** Parametrization of \((m, r)\)
- **b** Boundary counterparts of \((m, r)\)
- **t** Indexes the two parts \((-1, 1)\) of the implied boundary.
- **u** Unit normal to the boundary, also the direction from a point on \(m\) to its boundary counterpart.
- **n** Unit normal to the medial surface.
constraints

- Closed
- Connected
- non-singular
Parameter estimation

- object
- boundary representation
- target object
- skeletonization
- inverse skeletonization
- update model parameters
- parametric medial model

match
Parameter estimation

- Obj $F = \text{prior} + \text{Energy}$
- Prior: prefer low curvature.

\[ (|\mathbf{\tilde{m}}_{i+1} - \mathbf{\tilde{m}}_i| - |\mathbf{\tilde{m}}_i - \mathbf{\tilde{m}}_{i-1}|)^2 \]

Energy: mean square distance between interpolated boundary and target shape, summed over all sampled points.
\[
\mathbf{I}_{b^t} = \begin{bmatrix}
    b_{uu}^t \cdot u^t & b_{uv}^t \cdot u^t \\
    b_{vuu}^t \cdot u^t & b_{vv}^t \cdot u^t
\end{bmatrix} = - \begin{bmatrix}
    b_{u}^t \cdot u_u^t & b_{v}^t \cdot u^t \\
    b_{u}^t \cdot u_v^t & b_{v}^t \cdot u_v^t
\end{bmatrix}
\]

\[
\nabla r = \frac{dr}{ds} t = \frac{r'}{\sqrt{x'^2 + y'^2}} t,
\]

Narrated Quicktime movie about model construction in 3D (.mov with audio, 3 min, 11mb)

Narrated Quicktime movie about fitting models to images in 3D (.mov with audio, 90 sec, 18mb)

Movie of a spinning 3D model of the hippocampus (.avi loop, 200kb)
backup(2)
backup(3)