Interpreting Complex Images Using Appearance Models

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- Tim Cootes
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- Rhodri Davies (IPMI 2003 and PhD Thesis)

Overview

- Problem definition / motivation
- Modelling shape
- Modelling appearance
- Interpreting images using appearance models
- Practical applications

Problem Definition / Motivation

Complex and Variable Objects

- Faces
- Medical images
- Manufactured assemblies

Understanding Images

- Relating image to a conceptual model
  - high-level interpretation
- ‘Explaining’ the image
  - class of valid interpretations
- Labelling structures
  - basis for analysis

What Makes a Good Approach?

- Principled
  - makes assumptions explicit
  - uses domain knowledge systematically
- Generic
  - can be applied directly to new problems
- Computationally tractable
  - practical using standard PC/workstation

Interpretation by Synthesis

Interpret images using generative models of appearance – ‘explain’ the image

Generative Models

- High-level description
  - shape
  - spatial relationships
  - grey-level appearance (texture map)
- Compact
- Parameterised

Modelling Issues

- General
  - deformable to represent any example of class
- Specific
  - only represent ‘legal’ examples of class
- Learn from examples
  - knowledge of how things vary
  - generic

Modelling Shape

Modelling

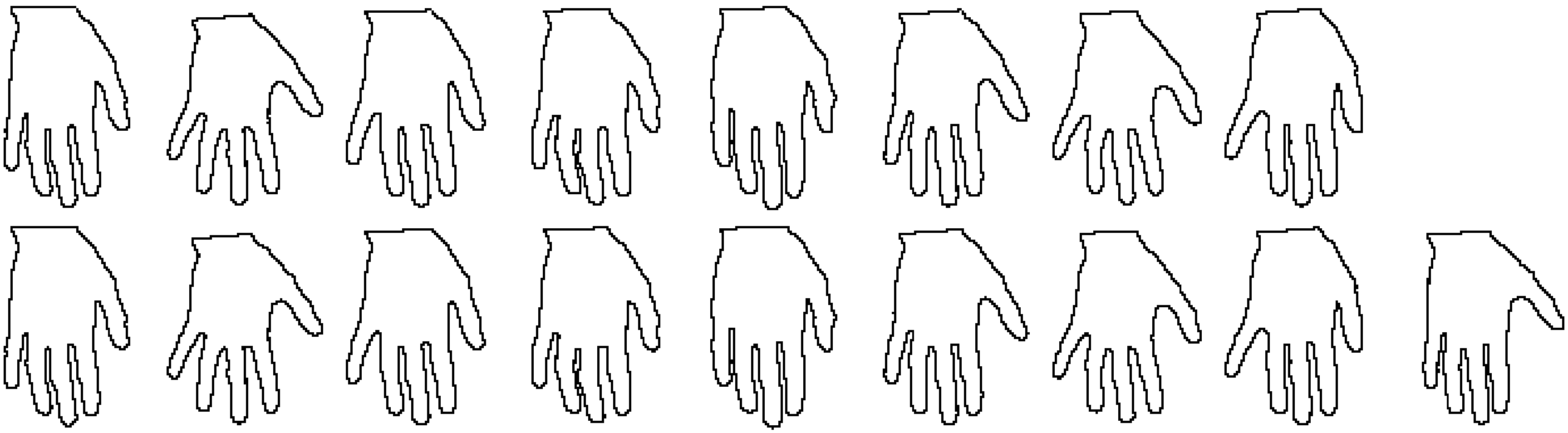

Figure 3.1. A training set of hand outlines.

From: PhD thesis Rhodri Davies

Modelling

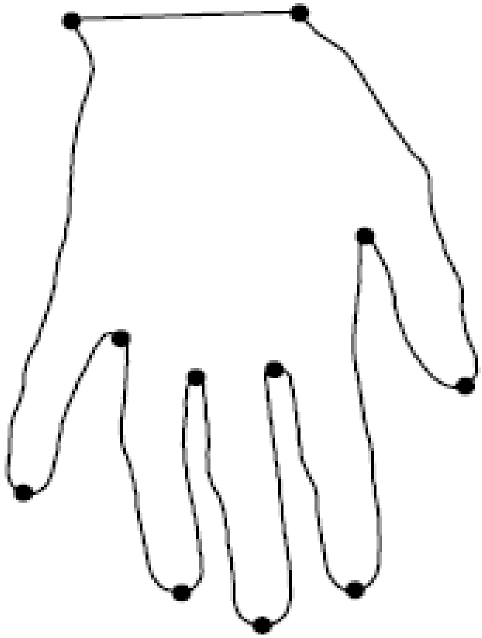

Figure 3.2. How manual landmarks were placed on the training boundaries.

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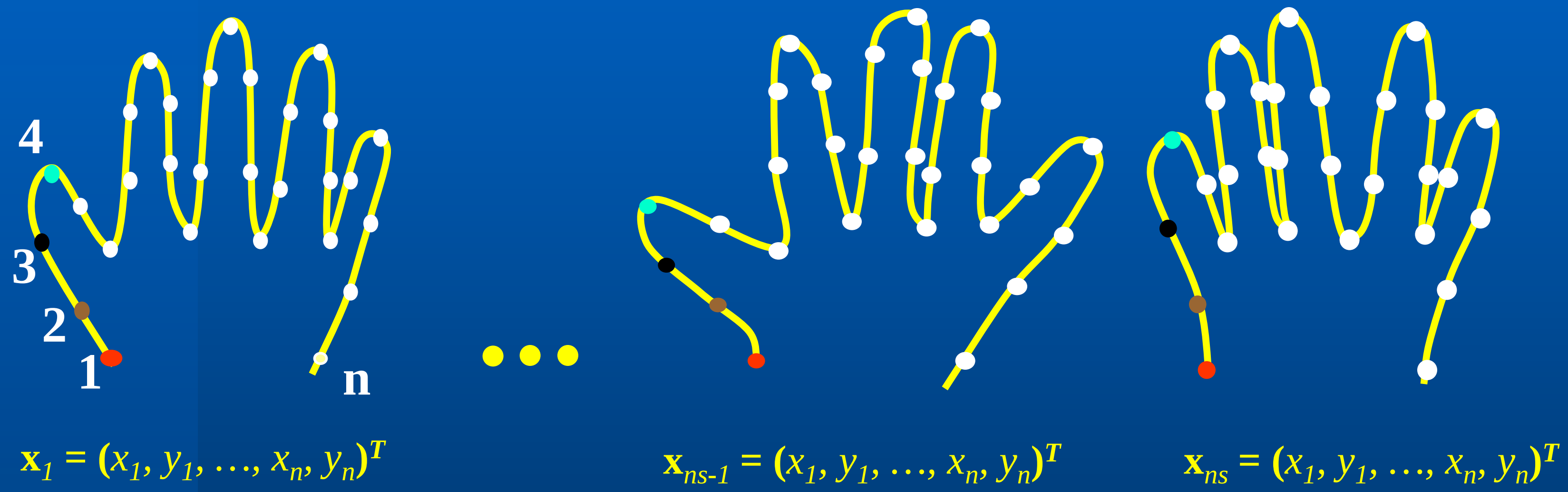
Modelling Shape

- Define each example using points
- Each (aligned) example is a vector

$$\mathbf{x}_i = \{x_{i1}, y_{i1}, x_{i2}, y_{i2} \ldots x_{in}, y_{in}\}$$

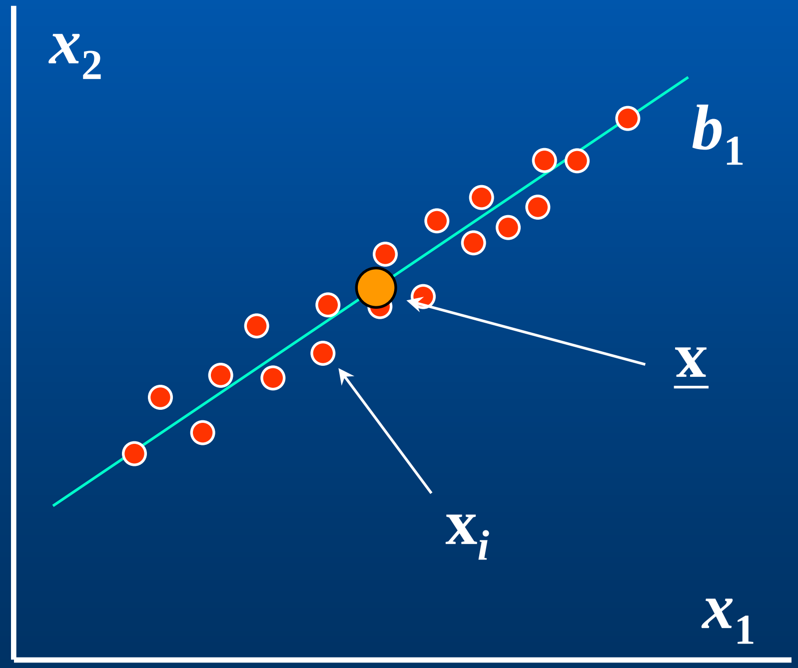
Statistical Shape Models



- Shape vector
  - statistical analysis
  - correspondence problem

Modelling Shape

- Points tend to move in correlated ways



Statistics of Shape Variability

Let $\mathbf{x}_i$ be a vector describing the n points of the i^{th} shape in the set;

$$\boldsymbol{x}_i = (x_{i0},\ y_{i0},\ x_{i1},\ y_{i1},\ \ldots,\ x_{ik},\ y_{ik},\ \ldots,\ x_{in-1},\ y_{in-1})^T$$

where (x_{ij}, y_{ij}) is the j^{th} point of the i^{th} shape.

The mean shape, $\bar{\boldsymbol{x}}$, is calculated using

$$\bar{\boldsymbol{x}} = \frac{1}{N}\sum_{i=1}^{N}\boldsymbol{x}_i$$

Statistics of Shape Variability

The modes of variation, the ways in which the points of the shape tend to move together, can be found by applying principal component analysis to the deviations from the mean as follows [22]. For each shape in the training set we calculate its deviation from the mean, $d\mathbf{x}_i$, where

$$d\boldsymbol{x}_i = \boldsymbol{x}_i - \bar{\boldsymbol{x}} \quad (2)$$

We can then calculate the $2n$ x $2n$ covariance matrix, $\mathbf{S}$, using

$$\boldsymbol{S} = \frac{1}{N}\sum_{i=1}^{N} d\boldsymbol{x}_i d\boldsymbol{x}_i^T \quad (3)$$

Statistics of Shape Variability

The modes of variation of the points of the shape are described by $\boldsymbol{p}_k$ $(k = 1..2n)$, the unit eigenvectors of **S** such that

$$\boldsymbol{S}\boldsymbol{p}_k = \lambda_k \boldsymbol{p}_k \qquad (4)$$

(where λ_k is the k'th eigenvalue of **S**, $\lambda_k \geq \lambda_{k+1}$)

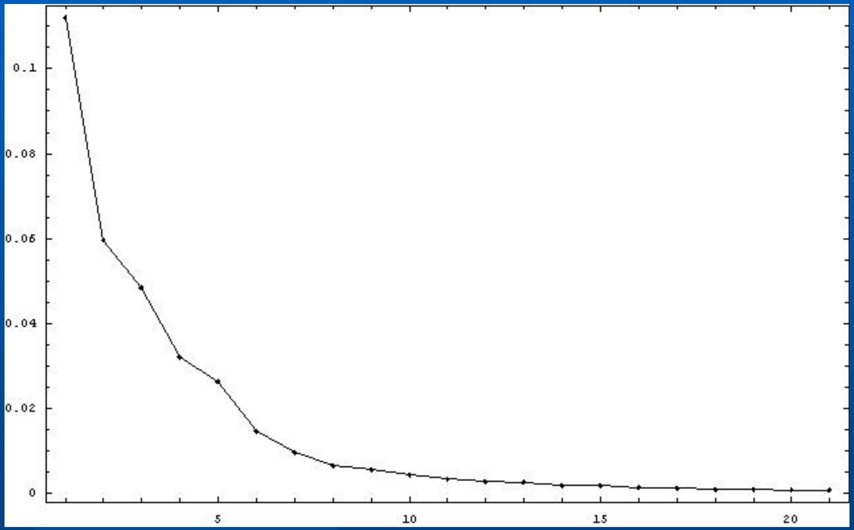
$$\boldsymbol{p}_k^T \boldsymbol{p}_k = 1 \qquad (5)$$

Statistics of Shape Variability

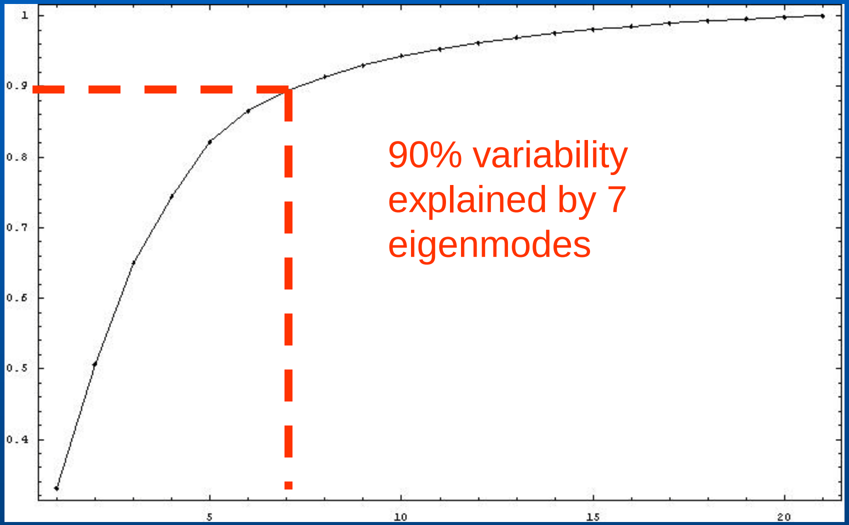
Most of the variation can usually be explained by a small number of modes, t $(<2n)$. One method for calculating t is to choose the smallest number of modes such that the sum of their variances explain a sufficiently large proportion of λ_T, the total variance of all the variables, where

$$\lambda_T = \sum_{k=1}^{2n} \lambda_k \qquad (6)$$

Statistics of Shape Variability



Eigenvalues (example with 22 shapes)



Cumulative function of eigenvalues, normalized

Statistics of Shape Variability

Any shape in the training set can be approximated using the mean shape and a weighted sum of these deviations obtained from the first t modes

$$\boldsymbol{x} = \bar{\boldsymbol{x}} + \boldsymbol{P}\boldsymbol{b} \tag{8}$$

where $\boldsymbol{P} = (\boldsymbol{p}_1 \ \boldsymbol{p}_2 \ \ldots \ \boldsymbol{p}_t)$ is the matrix of the first t eigenvectors,

$\boldsymbol{b} = (b_1 \ b_2 \ \ldots \ b_t)^T$ is a vector of weights, one for each eigenvector

the eigenvectors are orthogonal, $\boldsymbol{P}^T\boldsymbol{P} = \boldsymbol{I}$ so

$$\boldsymbol{b} = \boldsymbol{P}^T(\boldsymbol{x} - \bar{\boldsymbol{x}}) \tag{9}$$

- Each shape x with dimensionality 2n can be expressed with a b-vector with dimensionality t, t << 2n)

The above equations allow us to generate new examples of the shapes by varying the parameters (**b**) within suitable limits, so the new shapes will be similar to those in the training set.

Modelling Shape

- Principal component analysis (PCA)

$$\mathbf{x} = \overline{\mathbf{x}} + \mathbf{P}\mathbf{b}$$

$$\mathbf{P} = \text{ modes of variation}$$

$$\mathbf{b} = \text{ shape vector}$$

- Reduced dimensionality
  - typically 10 - 50 shape parameters

Statistical Shape Models

- Principal components analysis (PCA)
- Generative shape model:

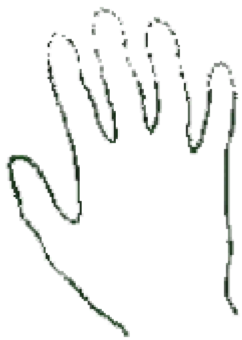
$$\mathbf{x}_i = \overline{\mathbf{x}} + \mathbf{P}\mathbf{b}_i$$

$\overline{\mathbf{x}}$ **:** mean shape
$\mathbf{P}$ **:** modes of variation
$\mathbf{b}_i$ **:** shape parameters

- Reduced dimensionality
  - typically 10 - 50 shape parameters

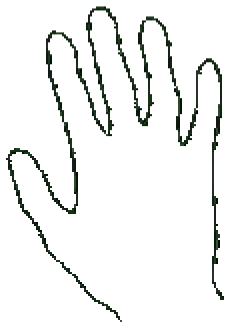
Hand Model

- Modes of shape variation



b_1

b_2



$\mathbf{b}$

Hand Model: Eigenmodes of Variation

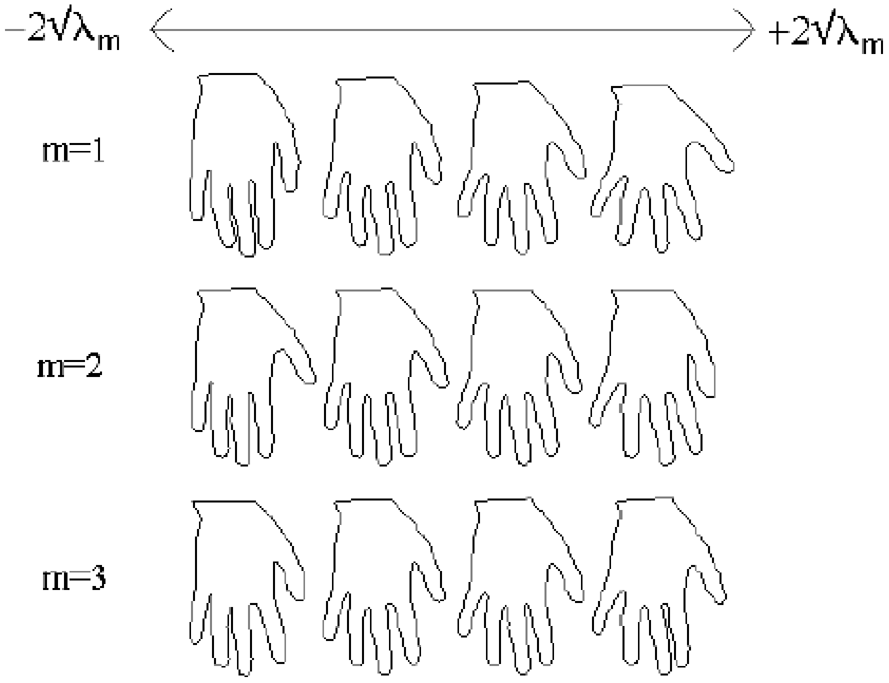


Figure 3.3. The first three modes of variation of the manually constructed hand outline model. Each parameter (b_m, $m = 1 \ldots 3$) is varied independently by $\pm 2\sqrt{\lambda_m}$.

From: PhD thesis Rhodri Davies

Importance of Correspondence

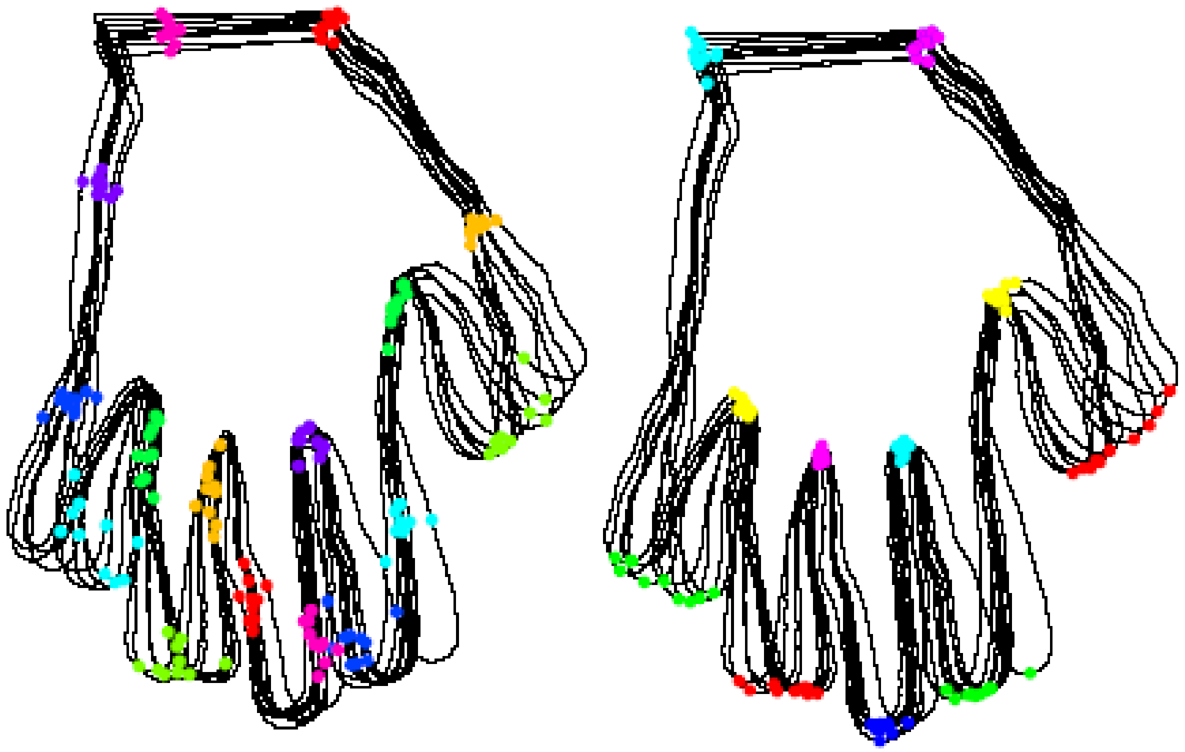


Figure 5.3. A subset of the correspondences and the alignment used to build the equally spaced Hand model.

Figure 5.4. The manual landmarks and alignment used to build the manual Hand model.

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Left: Arc-length parametrization

Right: Manual placement of corresponding landmarks

Correspondence and quality of shape model

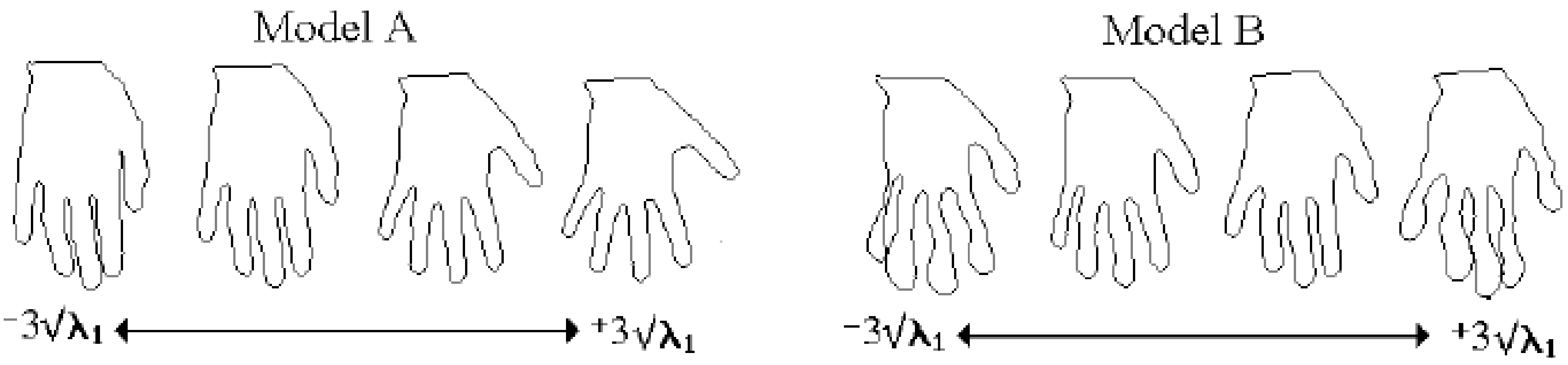

Figure 3.4. The first mode of variation of models A and B. The first parameter (b_1) is varied by $\pm 3\sqrt{\lambda_m}$.

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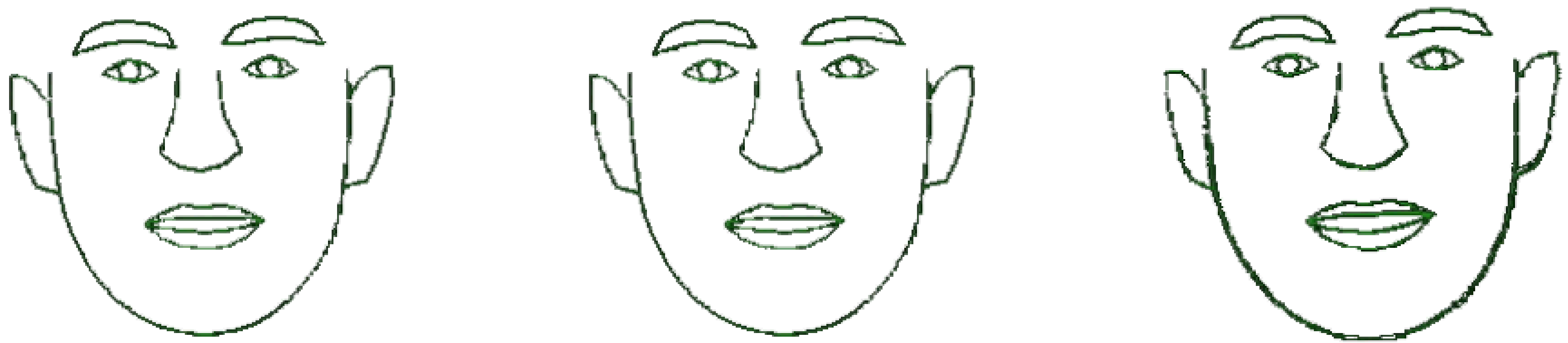
Left: Manual placement, Right: Arc-length parametrization

Face Model

- Shape and spatial relationships



b_1 b_2 $\mathbf{b}$